THEORY, DESIGN AND MEASUREMENT
OF THE BROOKHAVEN NARROW QUADRUPOLES*

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Summary. A novel approach is presented which results in mechanically simple quadrupoles producing high field gradients with modest power consumption and with very small aberrations. These lenses can be assembled either in four-fold geometrically symmetrical versions or in narrow "septum" versions without side yokes. The asymmetrical version conserves the very valuable space available for small angle secondary beams. There is no deterioration of optical properties since the four-fold magnetic symmetry is maintained in the aperture region. In addition, the fringing field is quite small. A unique magnet profile was adopted. Most general purpose iron quadrupoles are derived from the concept of four identical hyperbolic pole profiles which are quadrupole equipotentials. However, for a practical design this geometry must be severely distorted to provide space for reasonable current density coils. The present conceptual approach is to abandon the hyperbolic idea completely and treat the problem from the viewpoint of its four-fold symmetry which permits only $r^2 \sin 2\theta$, $r^6 \sin 6\theta$, $r^{10} \sin 10\theta$, etc., potential terms. Since these terms increase as the fourth power in their spatial variations, their magnitudes can be separately controlled with properly chosen physical perturbations. Using this method a quadrupole of high optical purity was produced employing a simple iron profile composed only of plane surfaces.

### A. Theory of Narrow Quadrupoles

The primary purpose of this development was to produce a lens of small overall width for a given aperture acceptance and which would have negligible fringing fields beyond its sides. Such lenses are used with beams set up at small angles to the synchrotron. In addition much smaller angles are possible than with conventional quadrupoles between separate beams looking at the same target. This is also a very valuable property.

It is clear that two gradient type C-magnets can be located to face each other in a manner to produce a predominantly quadrupole field. Such a pair could be used in this case to provide a quadrupole field and still allow complete access on the horizontal symmetry plane. However, with the requirement that the overall width be small, it appeared that severe octupole aberrations would be hard to avoid, and that appreciable fringing fields would result.

The solution arrived at was by analogous argument to the similar septum dipole problem where one wants to locate a dipole field close to a region where its fringing field must be very small. If a C-type dipole magnet with parallel pole surfaces has the current return located in the gap near the outside, it terminates

the magneto-motive force and little fringing field occurs provided the yoke reluctance is very small compared to the gap reluctance.

This analogous idea was called a septum quadrupole (Fig. 1). In this quadrupole case, the current elements in the top and bottom slots can be considered to produce the driving magneto-motive force and the two returns in the coil slots on the horizontal symmetry plane again terminate the magneto-motive force at the sides of the magnet.

This conception follows from recognizing that with four identical poles and four identical slots, the two yoke plates on the top and bottom are sufficient to provide the necessary low reluctance path between the bases of the poles (Fig. 1). The important point, however, is that the four-fold magnetic symmetry is maintained to high accuracy for all practical values of excitation, assuming the pole bases are broad enough to provide a low reluctance isolation region between the aperture region and the asymmetrical yokes. Even if the two yoke plates develop appreciable saturation (which is a situation that need not happen, since vertical space is generally not tightly limited) the aperture region will see no asymmetry and, therefore, no octupole. (However, since one desires very small fringing fields outside the magnet, it is necessary to keep the reluctance of the yoke small, but this argument is separate from the beam optical arguments).

By popular usage, these quadrupoles over the years have been renamed "narrow quadrupoles".[+]

A model quadrupole was constructed with these features. This quadrupole had an 8-in. aperture with a 24-in. core length. (Fig. 1). At the same time a different approach to profile design was *incorporated*.

### B. Development of Pole Contour Based on Symmetry Arguments

#### I. General Theory of Pole Contour

In the design of practical quadrupoles, the goal is to produce a two dimensional quadrupole field in the interior of the magnet, with little or no contributions from other field multipolarities. The aberrations produced by end effects are generally accepted, although sometimes

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+We wish to acknowledge that in wrestling with the same problem, a similar idea of using two yokes in place of four was independently arrived at by E.J. Wilson of CERN and Harwell "A Quadrupole Magnet of Large Acceptance", MPS/EP-29, 11/29/62. It is clear that he thought of this approach several months before us. He describes the idea by the flux distribution, calling it a "figure of eight" quadrupole.

attempts are made to reduce them as a separate problem. The basic design concerns solving the two dimensional field problem.

For air core quadrupoles, current distributions are practical which produce directly the required magnetic potential $r^2 \sin 2\theta$: where $\theta = 0$ is midway between poles, and $r$ is the radius.

For iron core quadrupoles, the conceptually simple version of a pole profile shaped as a rectangular hyperbola with infinite permeability iron produces the required field. This approach is clearly out of the question for a real magnet on various theoretical and practical grounds.

Most quadrupole designs are, however, derived from this concept with at least a reasonable approximation to a rectangular hyperbolic pole surface in the vicinity of the pole center. The geometry is then severely distorted by terminating this profile completely and cutting the poles back for the conductor slots. Magnetic design then reduces to doing this in a manner which produces only modest contributions from higher multipolarities over the useful region of the aperture.

The alternative is to take the more aesthetically appealing road to quadrupole design by creating the quadrupole potential with a combination of iron core and distributed surface currents. However, this approach has many problems, so that the modified hyperbolic equipotential approach is used for most general purpose quadrupoles.

The underlying idea developed in the present work was that indeed the hyperbolic equipotential pole is so distorted in practice that the reason just about any quadrupole with four identical poles does not behave too badly follows simply from the fact that the four-fold symmetry restricts the aberrations which can appear to the orders $2(2n+1)$. The field terms allowed have radial components $a_2 \, r \sin 2\theta$, $a_6 \, r^5 \sin 6\theta$, $a_{10} r^9 \sin 10\,\theta$, $a_{14} \, r^{13} \sin 14\theta$, etc: when $a_N$ is the coefficient of the Nth harmonic. The spatial distribution of these field aberrations varies widely: the radial dependence of the successive terms increases as $r^4$ and the angular variation changes drastically as well. Considering the field as a superposition of these multipolarities one can expect to be sucessful at separating the variables and perturbing the terms individually. Only the first few terms are of practical importance, because of the rapidly increasing radial dependence.

## II. Description of Developmental Model

The approach used was to tentatively make the simplest quadrupole employing a rectangular coil slot and treat its field as an expansion in terms of the harmonic coefficients $a_N$. Fig. 1 shows this simple quadrupole with square cross section poles except for the plane profile surfaces cut at 45 degrees to the slots. A further condition imposed was that the aperture radius to the pole tip center and to the center of the inner surface of the exciting coils be essentially the same. The deviation of this flat profile from a rectangular hyperbola coincident at its center is re-

latively small over the width of the pole tip; thus the rectangular coil slot is the most fundamental deviation of the contour from the ideal rectangular hyperbola. That is, the variation of the most basic parameter, the slot width W, should principally change the coefficient of the lowest power, $a_2$, while a local "bump" on the flat face, appropriately located, should affect more strongly the higher order terms.

The starting point was to attempt to set the parameter W to make $a_2$ vanish. If the argument is correct it is simple to see that if W equals the radius of the magnet, the vertical component of field on the midplane (Fig. 2) both in the coil slot and in the aperture would meet at the interface, if projected linearly. This is a simple calculation using $.4\pi \ NI = \int H \, . dl$. This geometry should give a smaller value of $a_2$ than if the projections are quite discontinuous at the boundary.

Based on this argument the magnet was made with W = 4.435 inches. The coil package width was 2.960 inches since existing coils from a standard AGS 8Q24 magnet were used; thus one is able to perturb W by inserting iron shims on either side of the coil package. A "kit" was designed which provided two more perturbations; a small "bump" of various widths and depths could be added or subtracted either at the center of the pole or at the edge adjoining the slot. Note that a bump at the center of the pole is in phase for all terms, but at the edge is approximately $23^\circ$ to the pole center. This compares to the allowed phase angles of $30^\circ$ for $6\theta$, $18^\circ$ for $10\theta$ and $13^\circ$ for $14\theta$. Since the small bumps applied are localized, the cross sectional area added rather than its actual shape is important, to first order; and therefore only simple rectangular shaped bumps were used. In this manner, if the flat pole becomes more complicated at least it only involves plane cut surfaces parallel to the main surface.

Such an approach would be very difficult to compute, even for the case of infinite permeability. The approach taken was to rely on accurate field measurements.

## III. Description of Harmonic Coil and Pole Perturbation Measurements

The measurements were taken with a harmonic coil which measures the radial field components of the multipolarities present at very close to maximum radius. Both internal two dimensional and azimuthally integrated field analysis are made. The correct amplitudes and phases of the multipolarities present can be accurately computed for the effective search coil radius provided a true axis of rotation exists during the time necessary for a full revolution of measurement. A small error in estimating this radius of measurement has only a very small effect. This method minimizes the problem of positional errors introduced into individual data points, which produce false contributions to harmonic coefficients.

It is also not necessary to have the device

at the center of the quadrupole to get the correct result. It can be shown that the coefficients of the multipolarities are unaffected by centering errors, except of course for the dipole term.

The amplitude and phase angle of the various two dimensional aberration terms in the quadrupole field were measured at an intermediate field ($B_{tip} \simeq 5$ kG) at a radius of 4.020-in. (Magnet radius = 4.314-in). Case A (Col. III) in Table I shows the results for the magnet with a plane profile cut at 45° to the slot. The ratio of the first three aberration terms to the quadrupole term is given and the signs give the relative phases of the terms. It can be seen from the Table that the ratio of the first aberration term to the quadrupole term ($6\theta/2\theta$) is less than 1% which tends to substantiate the basic symmetry assumptions in design stated previously. In order to get some quantitative information of the effect of the pole perturbations on the aberration terms, the harmonic coil was used to measure the field with an 1/8-in. x 1/2-in. cut on each end of the pole (Case B) and then with a 1/8-in. x 3/4-in. bump in the center of the pole (Case C) and the results are tabulated in the Table and illustrated in Fig. 3. The combination of Case B with Case C is called Case D and represents a closer approximation to a rectangular hyperbola and a more reasonable shape. As can be seen from the Table, Case D indeed gives very low higher order ($10\theta/2\theta$ and $14\theta/2\theta$) aberrations but gives about 5% $6\theta/2\theta$. However, the fact that one gets 5% $6\theta/2\theta$ is not surprising. In the process of reducing the higher order terms by rounding the pole, both perturbations introduced tend to generate $6\theta$ in phase with $2\theta$ at the pole.

To counteract this, appropriately narrower coil slots were required. This was accomplished by placing two iron shims of width 0.120-in. in each coil slot, against the main quadrants of the magnet. This coil slot perturbation was measured in combination with two different pole profiles (Cases B and D) and the results are tabulated in Table I. From the Table one can see that the perturbation, as expected, affects principally the $6\theta/2\theta$ aberration and causes it to decrease by approximately 1.85%; the effect is essentially the same for both the pole profiles (B and D) that were tested. This proves the basic assumption made with respect to separating the variables. Based on the information obtained from the perturbation, it could be predicted that one wanted to put 0.250-in. of iron shim in the coil slots in combination with a (3/4-in. x 1/8-in.) center rectangular bump and an end negative bump (or "hole") of cross section 1/8-in. x 1/2-in.; this particular combination gave a very low harmonic content (Table I). While this was a good pole profile magnetically speaking, it would be a difficult one to machine as indicated in Fig. 4A, so it was decided to modify the pole profile as in Fig. 4B for ease in machining. With this modification the $6\theta/2\theta$ term increased by 0.78%; again based on the information obtained from the iron shim perturbations, increasing the iron shim width by 0.045-in. compensates for the increase in $6\theta/2\theta$ due to shaping the steel shim to fit the pole contour.

The results of this final perturbation are given in Table II.

## Discussion and Conclusions

The rapid convergence on an optically pure profile verifies the basic symmetry argument as well as the assumption that the aberration multipolarities can be separately perturbed. Referring to Table I, column IX gives the sum of the perturbations B and C as measured individually and added, while Column X gives the measured perturbation D (i.e., B and C physically combined). The very close agreement for all three relevant aberration terms verifies indeed the assumption of linear superposition of the perturbations (and the accuracy of the experimental data). Similarly, columns XI and XII show the results of 0.120-in. iron shims on each side of the coil slots applied to configuration B and D respectively. Again, linear superposition and experimental accuracy is confirmed by the near identity of the terms. In addition either column XI or XII shows that indeed the $6\theta$ aberration term varies strongly with coil slot width, while the higher multipolarities are only modestly effected.

Table II gives the amplitudes and phases for both the two-dimensional and azimuthally integrated aberration terms present in the final quadrupole model, measured at a tip field of 5 kG. The two-dimensional $6\theta$ aberration term is quite small, <0.1%, and can be made arbitrarily zero by adjusting the width of the coil slot. The ~1% $14\theta$ term varies as $r^{13}$ so that inside a vacuum pipe it is already negligible. The variation of harmonic content with excitation is quite small. Furthermore, the $18\theta$ term is also quite small (0.07%) and is only of academic interest because of its high radial dependence. Additional perturbation seemed fruitless and the matter has not been pursued further. Reference to the Table shows that both the two dimensional and integrated octupole aberrations are quite small. Independent measurements made on a production N8Q32 magnet show also the octupole aberration term to be negligible (0.07%) and furthermore, the aberration is essentially constant (to 0.01%) over the whole range of excitation even though there is appreciable yoke saturation (7% nonlinearity) at 15 kG pole tip field. Such a small octupole term represents only mechanical imperfection.

Provided an adequate yoke thickness is used, the fringing field at the side of the magnet is very low (of the order of 1 gauss at a distance of 1-in. from the side supporting plate). Either ferrous or non-ferrous plates will behave identically with a low reluctance yoke, but iron supporting plates are better when yoke saturation is significant.

Column V in Table II gives the aberrations including octupole, due to end effects for the final model. Both past experience[1] and measurements on a production N8Q32 magnet indicate that the aberrations are quite independent of excita-

tion. The method of perturbing the coil slot width to control to two dimensional $6\theta$ aberration can be applied to making the azimuthal integral of the $6\theta$ term zero to arbitrary accuracy. This, in fact, has already been achieved for a quadrupole which is only one diameter long, i.e., one in which the major portion of the field is due to end effects. This method is superior to end shimming because variation over any reasonable range of excitation is quite small.

Another feature of this design is that the coil slot is sufficiently wide that high pole tip fields can be obtained which require quite modest power consumption in the coils. Since the length of the coils and poles in the radial direction are irrelevant to the optical properties, the parameters of overall size versus current density and power consumption are varied for different applications.

The profile can be linearly scaled to any size and can be made without special equipment, since only plane surfaces are involved. For small quadrupoles or for any application where only about 80% of the aperture need be used, case A with a flat pole is quite excellent with a slight modification of the coil slot width to make the $6\theta$ term vanish.

Reference

1.  G.T. Danby and J.W. Jackson, Magnetic Measurements of AGS Experimental Beam Quadrupoles, BNL Int. Rep. (1963)

Table I Coefficients of Nonlinear Terms Relative to Quadrupole Term in Narrow Quadrupole Model.

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Iron shim thickness | Ratio of co-effs. | A | B | C | D | (B-A) | (C-A) | (B-A)+(C-A) | (D-A) | B (.120-.000) | D (.120-.000) |
| 0.000 | $6\theta/2\theta$ | +0.75 | +2.60 | +2.85 | +4.83 | +1.85 | +2.10 | +3.95 | +4.08 | | |
| | $10\theta/2\theta$ | -4.40 | -2.44 | -2.06 | -0.24 | +1.96 | +2.34 | +4.30 | +4.16 | | |
| | $14\theta/2\theta$ | -1.83 | -1.42 | -0.57 | -0.17 | +0.41 | +1.26 | +1.67 | +1.66 | | |
| 0.120 | $6\theta/2\theta$ | | +0.71 | | +3.00 | | | | | -1.89 | -1.83 |
| | $10\theta/20$ | | -2.49 | | -0.29 | | | | | -0.05 | -0.05 |
| | $14\theta/20$ | | -0.91 | | +0.31 | | | | | +0.51 | +0.48 |
| 0.250 | $6\theta/20$ | | | | +0.20 | | | | | | |
| | $10\theta/20$ | | | | -0.35 | | | | | | |
| | $14\theta/20$ | | | | +1.14 | | | | | | |

Explanation of Column Headings

I.  Thickness of iron shim (in.) which was placed on both sides of the coil block against the pole.

II.  Ratio of the coefficients of the nonlinear terms to the coefficient of the quadrupole term at the maximum radius of the harmonic coil (approximately 1/4 in. from the pole tip). All ratios are expressed in percent.

III-VI.  Ratio of the coefficients for cases A,B,C and D respectively.

VII-VIII. Ratio of the coefficients for perturbations B and C respectively.

IX.  Ratio of the coefficients for the sum of perturbations B and C.

X.  Ratio of the coefficients for perturbation D.

XI-XII.  Ratio of the coefficients for two 0.120" iron shims in coil slot with perturbations B and D respectively.

Table II Two-Dimensional and Azimuthally Integrated Coefficients for Final Quadrupole Model.

| I | II | III | IV | V | VI | VII | VIII |
|---|---|---|---|---|---|---|---|
| Iron shim thickness | Ratio of co-effs. | Point coil | Long coil | Ends | Theoretical phase angle | Meas'd phase angle | Difference |
| 0.295 | 4θ/2θ | +0.025 | +0.063 | +0.038 | | $-37°07'$ | |
| | 6θ/2θ | +0.093 | +0.607 | +0.514 | $-15°00'$ | $-14°35'$ | $+0°25'$ |
| | 10θ/2θ | -0.245 | -0.113 | +0.132 | $- 9°00'$ | $-8° 53'$ | $+0°07'$ |
| | 14θ/2θ | +1.117 | +0.877 | -0.240 | $- 6°26'$ | $-6° 30'$ | $+0°04'$ |

Explanation of Column Headings

I.   Thickness of iron shim (in.) which was placed on both sides of the core block against the pole.

II.  Ratio of the coefficients of the nonlinear terms to the coefficient of the quadrupole term at the maximum radius of the harmonic coil (approximately 1/4 in. from the pole tip). All ratios are expressed in percent.

III. Ratio of interior two-dimensional coefficients.

IV.  Ratio of azimuthally integrated coefficients.

V.   Ratio of contributions due to end effects, i.e., IV-III.

VI.  Theoretical phase angles which can be expected from the four-pole symmetry.

VII. Measured phase angles.

VIII. Deviation of measured phase angles from those predicted by four-pole symmetry.
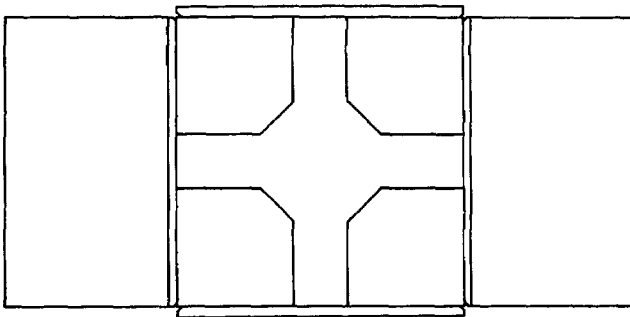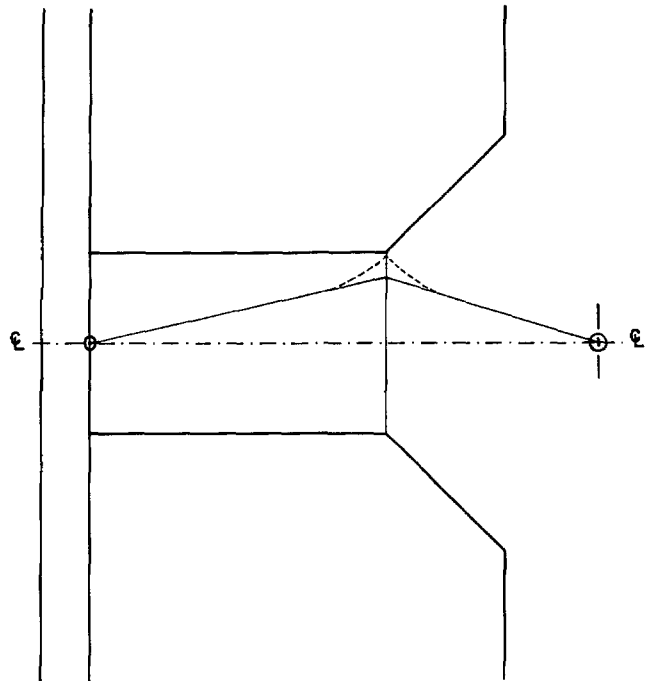


Fig. 1. Developmental Model Magnet.



Fig. 2. Illustration of Median Plane Projections at Copper-Aperture Interface for Coil Slot Width Equal to Aperture Radius.
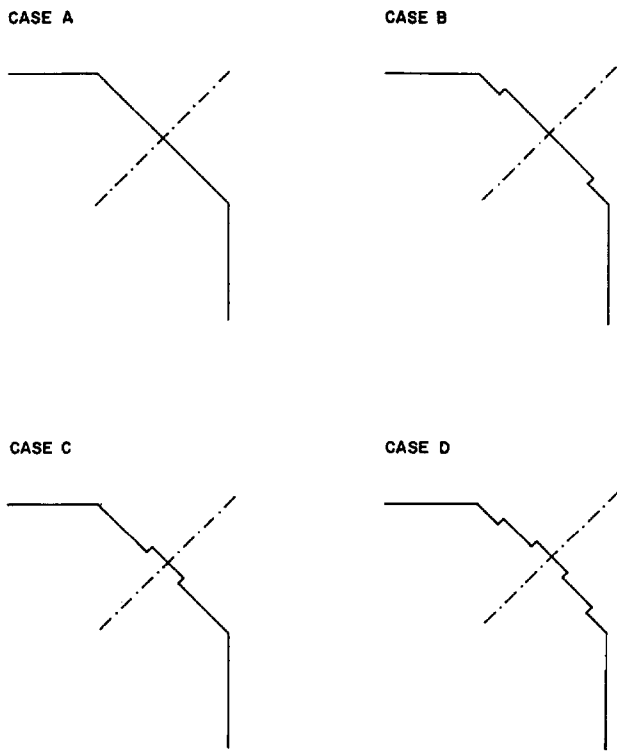
CASE A

CASE B

CASE C

CASE D

A

B

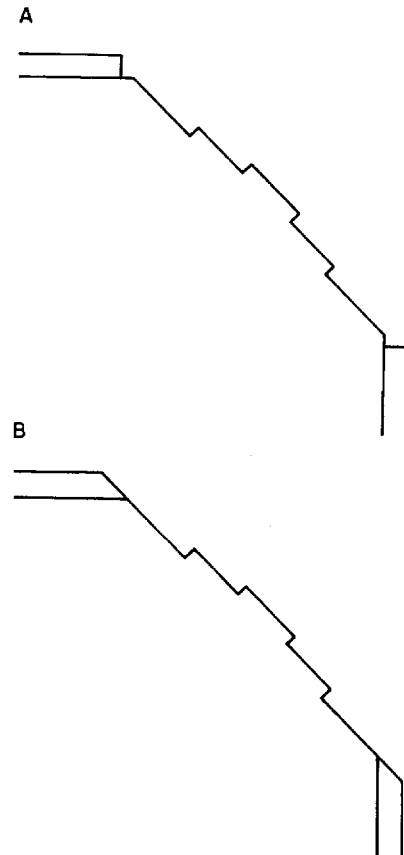Fig. 3. Pole Perturbations of Quadrupole Profile.

Fig. 4. Modification to Iron Pole Shim of Model to Produce Mechanically Simple Pole Profile.