DESIGN STUDY OF A SUPERCONDUCTING RADIOFREQUENCY BEAM SEPARATOR*

H. Hahn and H.J. Halama
Brookhaven National Laboratory
Upton, New York

Summary

A design study of a superconducting RF beam separator suitable for counter beams is presented. A simple two-deflector separation scheme permits one-contaminant rejection in momentum ranges from 8.5 to 19 GeV/c for P and π beams and from 4.5 to 10 GeV/c for K beams. The operating wavelength λ = 24 cm was obtained by optimizing the transmission of wanted particles. An interdeflector spacing of L = 33 m follows. The geometrical configuration of the lead-coated iris-loaded structure is selected for highest bandwidth which is obtained when the ratio of aperture radius and wavelength is close to 0.2. The maximum deflecting field in the 3 m long resonant ring deflector is limited to about 7 MV/m. The superconducting structure is expected to have a shunt impedance of $R \approx 10^{12}$ Ω/m and an unloaded quality factor of $Q_0 \approx 1.5 \times 10^9$. The ensuing RF power requirements are discussed.

I. Introduction

The present RF beam separator at the Brookhaven Alternating-Gradient Synchrotron is well matched to bubble chamber experiments with kaons, pions, and protons at selected momenta between 7 and 18 GeV/c.[1-4] The application of this device to counter experiments is, however, prevented by the demands for (1) a typical operational pulse length of 100 ms, (2) high fluxes resulting from an accepted solid angle at the target of the order of several 100 μsr, and (3) operation at continuously variable momenta. On the other hand, considerable simplification stems from the fact that no high beam purity is required and rejection of the predominant contaminant suffices.

The most difficult requirement is the pulse length, because the operation desired necessitates the use of CW power tubes. The state-of-the-art in S band amplifiers is represented by Raytheon's 425 kW (CW) Super Power Amplitron. In combination with the BNL deflector,[5] one can in principle expect a transverse momentum of about p_T = 3.4 MeV/c. Since, typically, particle deflections of 1 mrad are considered necessary, the useful momentum region for conventional, long pulse RF beam separators lies below 3 or 4 GeV/c. In view of the tremendous engineering efforts involved in constructing an RF separator of this type, it must be concluded that dc separators are preferable in this range. In the momentum range where electrostatic separators become impractical, say above 5 GeV/c in the case of kaons, the natural and unique approach is to use superconducting (SC) deflectors.[6]

In SC cavities it is, in principle, possible to achieve the same deflecting fields as in normal copper structures with considerably less RF power. The reduction in power is conveniently expressed by the improvement factor, which is defined as $I = Q_{SC}/Q_{Cu}(273^{\circ}K)$. Theoretical improvement factors for lead-plated structures at 2.856 GHz were calculated[7] from the formulas for the surface impedance, given by Khalatnikov et al.,[8] to be about 1.3×10^4 at 4.2°K and 6.8×10^5 at 2°K. Improvement factors close to the theoretical values were obtained experimentally with TE_{011} cavities by groups at Stanford,[9] Paris[10] and Karlsruhe.[11] For the purpose of this paper we will take the optimistic attitude that the technological problems encountered in fabricating complicated SC structures such as iris-loaded deflectors can be overcome, and that improvement factors in the order of 10^5 are attainable. It follows from $ estimates made by Wilson et al., that SC RF beam separators are then also economical.[12]

Estimates of particle fluxes show that separated beams will be highly desirable for counter experiments, using the increased proton intensity expected after the AGS conversion. The aim of this paper is to present a design study of a SC RF beam separator which will demonstrate the adequacy of such a device. We concentrate our investigations on the deflector design and the choice of a frequency optimized for maximum particle transmission. Problems of beam optics are only marginally considered. Questions related to the microwave system are treated in a separate paper at this conference.

II. Operating Momenta

Counter beams require only rejection of the predominant contaminant, that is, pions in the case of K and P beams, and protons in π^+ beams. π^- beams are possible without separation. One-contaminant rejection can be achieved over relatively wide momentum regions by means of a linearly polarized two-deflector system. The momentum analyzed beam of wanted (W) and unwanted (U) particles is focused into the first deflector and receives there a sinusoidal deflection with peak value $\hat{\theta}$. Due to their different rest masses, W and U particles arrive at the identical second deflector with a phase difference $\Delta\psi$ relative to the circular frequency ω of the deflecting fields. It is now always possible to adjust the relative phase between deflectors so that the deflection of U particles cancel exactly, whereas W particles receive a net deflection $2\hat{\theta}\epsilon$, with the deflection efficiency $\epsilon = \sin \frac{1}{2} \Delta\psi$. Operation will be possible around $\Delta\psi = m\pi$, with m = odd integer. All U particles are intercepted (or degraded) in the beam stopper, whereas the fraction of W particles passing the beam stopper depends on the intrinsic angular spread, δ_V , of the beam, the beam stopper size, the peak deflection and the deflection efficiency. Typically, the beam stopper size is made equal to δ_V in order to

---

*Work performed under the auspices of the U.S. Atomic Energy Commission.

intercept all U particles (muon background will be disregarded) and δ_V is made not larger than the net deflection

$$\delta_V \leq 2\,\hat{\theta}\,\epsilon \quad . \tag{1}$$

The losses of W particles in the beam stopper are then ≤ 36%. The separation scheme discussed has the inherent disadvantage of some beam stopper losses, but its simplicity makes it preferable to a lossless circularly polarized separator. Circular polarization requires in practice two additional deflectors rotated by 90° which would essentially double costs.

To simplify the considerations, we assume a monochromatic beam of particles with rest masses m_W and m_U and momentum p . The transit time t for a particle to traverse the deflector distance L is in relativistic approximation

$$t \approx \frac{L}{c}\left\{ 1 + \frac{1}{2}\left(\frac{mc^2}{pc} \right)^2 \right\} . \tag{2}$$

The deflection efficiency of W particles may then be expressed by $\epsilon = \sin \frac{1}{2}\,\omega\,(t_W - t_U)$. It is convenient to introduce a nominal design momentum p_o by

$$p_o c = m_p c^2 \, (L/\lambda)^{\frac{1}{2}} \tag{3}$$

where m_P = proton rest mass and λ the wavelength in free space. Operation with maximum deflection efficiency ($\epsilon = 1$) is possible at the momenta

$$\left(\frac{p}{p_o} \right)^2 = \frac{1}{m}\left\{ \left(\frac{m_W}{m_P} \right)^2 - \left(\frac{m_U}{m_P} \right)^2 \right\} \tag{4}$$

whence follows that P and π beams are possible, for example, at $p \approx p_o$ and $0.58\ p_o$ and K beams at $p \approx 0.53\ p_o$ and $0.30\ p_o$. Operation at neighboring momenta reduces ϵ . The useful momentum bands can be found from the somewhat arbitrary condition that $\epsilon \geq 0.5$. The resulting values based on a design momentum p_o = 11 GeV/c are summarized in Table I. It is seen that this particular choice can provide useful beams (m = 1) at momenta just above the region presently served by dc separators. Operation with m = 3 or higher is probably undesirable because of chromatic aberrations.

Table I. Useful momentum bands of two-deflector system with p_o = 11 GeV/c.

Beam	Momentum (GeV/c)	
	m = 1	m = 3
P, π	8.5 ÷ 19.1	5.7 ÷ 7.2
K	4.5 ÷ 10.0	3.0 ÷ 3.8

III. Deflector Design

The design of a SC deflector differs in many respects from the conventional approach,[13] and because beam loading can be disregarded, it is also distinct from the design of SC accelerators. However, an iris-loaded structure still seems to be the adequate solution. To make full use of the reduced losses in a traveling wave (TW) structure, that is, to operate with attenuation parameters close to $\alpha\ell = 1$, it would be necessary to select an unacceptably small group velocity. It is more natural to operate the deflector as a standing wave (SW) cavity. However, SW operation is comparable in efficiency to optimum TW operation only for structures with $v_g = 0$, in other words for π or 0 modes. These modes, on the other hand, are lacking the easy adjustability in phase velocity or in resonant frequency (which will be essential for synchronizing the two deflector cavities) and they are more sensitive to machining errors. Consequently operation with $v_g \neq 0$ is considered mandatory. The disadvantage of doubled peak fields and RF losses (at given equivalent deflecting field, E_o) inherent in SW cavities with $v_g \neq 0$ suggests the choice of resonant rings (RR), in which the versatility of TW guides and the efficiency of SW operation are combined. The rest of this paper is based on the assumption that the enormous mechanical engineering problems expected in the construction of SC resonant rings can be overcome. Should this assumption turn out to be wrong, then the performance of the RF beam separator, especially the acceptance figures, will be reduced. The deflector parameters, on the other hand, remain unaltered.

The power gain of a RR is at resonance given by the approximation valid for small losses ($\alpha\ell \ll 1$) :[14]

$$G = \frac{P_{guide}}{P} \approx \frac{4\varkappa^2}{(\varkappa^2 + 2\alpha\ell)^2} \tag{5}$$

where $\varkappa$ is the coupling coefficient of the directional coupler (coupling in dB = 20 lg $\varkappa$) and P is the power delivered by the source. The maximum gain $G_{max} = (2\alpha\ell)^{-1}$ is achieved at critical coupling $\varkappa_{cr}^2 = 2\alpha\ell$. Because the unloaded $Q_o = \omega/(2\alpha\, v_g)$, one obtains for the maximum gain

$$G_{max} = \frac{Q_o v_g}{\omega\ell} \quad . \tag{6}$$

The transverse momentum, p_T , of the particle after traversal of the RR is $p_T = q\ell E_o = q\ell(ZGP)^{\frac{1}{2}}$. The series impedance, Z, is related to the shunt impedance, R, by $Z = \omega R/Q_o v_g$. At critical coupling, one finds for the RR the expected expression

$$p_T = q\,(R\,P\,\ell)^{\frac{1}{2}} \quad . \tag{7}$$

This should be compared to a SW cavity with $v_g \neq 0$, where $p_T = q\,(\frac{1}{2}\,RP\ell)^{\frac{1}{2}}$. The power required by a SW cavity is twice that of a RR.

The choice of the deflector length ℓ depends on particle dynamics and acceptance considerations and it will be discussed later on. From (7) follows that, contrary to the TW case, p_T is independent of v_g . The choice of v_g can thus be made without knowledge of the operating frequency or deflector length. The group velocity is determined by the ratio C_a = beam hole radius, a , divided by wavelength, λ . In a previous paper the shunt impedance and group velocity of various structures as function of C_a are listed.[5] Backward wave operation gives the highest shunt impedance together with nondegenerate operation. A large $|v_g|$ reduces the requirements on mechanical tolerances. It is suggested to use C_a yielding the largest $|v_g|$, that is typically $C_a \approx 0.2$. This results in higher values of shunt impedance, but the aperture is smaller than for conventional deflectors. Adequate acceptance figures will be obtained by lowering the operating frequency (Fig. 1).

Details of the deflector design, such as choice of the number of irises per wavelength ($N = 3$ or 4) or shape of irises will depend on the method of fabrication. It is anticipated to use copper structures coated with a SC material. The deposition of the surface sheet (about 10 μm thick) could be carried out by electroplating, chemical and vapor deposition, or by any other procedure yet to be developed. If the complete structure can be coated in one step, then the larger slot width and higher shunt impedance of a $2\pi/3$ mode seem preferable. If the structure must be assembled from smaller pieces, then a $\pi/2$ mode (simple or biperiodic) with zero currents in alternate cells should be preferred.[15]

In selecting the SC material one must consider its properties and the technological difficulties in producing good surfaces. SC deflectors are never power limited, but the critical magnetic field, H_c, entails an upper limit for E_o and materials with large H_c are desirable. Because the structures are cooled by evaporation of liquid helium, the transition temperature, T_c, of the SC material must be $> 4.2^oK$. Furthermore, operation with $T_c \gg$ operating temperature, T, results in large improvement factors and small sensitivity of the surface reactance (hence the resonance frequency) to temperature variations. A search in the list of 23 metallic elements known to exhibit superconductivity reveals two candidates, niobium and lead, with $T_c = 8.8$ and 7.22^oK and $H_c = 1960$ and 800 G respectively. The present-day technology favors Pb, although Nb would have the more desirable properties. Due to lack of sufficient information on their properties, SC alloys or compounds are presently not considered.

The equivalent deflecting field strength, E_o, will be limited by the critical magnetic field or by field emission.[16] The dc critical field is temperature dependent according to $H_c = H_c(0)\{1-(T/T_c)^2\}$, whence follows that the peak RF magnetic field anywhere in the lead cavity $c\hat{B} < c \times 465$ G $= 14$ MV/m. However, the highest magnetic fields reported[9] so far are only $c\hat{B} = c \times 300$ G $= 9$ MV/m. The peak of the electric field strength is limited to $\hat{E} < 15$ MV/m because of field emission. The approximation of the RR deflector by a loosely coupled chain of TM_{110} cavities permits to estimate the maximum equivalent deflecting field:

$$E_o < \frac{1}{2}\,\frac{d}{h}\,\frac{C_{oo}}{J_1(j'_{11})}\,\hat{E} \tag{8}$$

and

$$E_o < \frac{1}{2}\,\frac{d}{h}\,\frac{C_{oo}}{J'_1(0)}\,c\hat{B} \tag{9}$$

where $J_1(j'_{11}) \approx 0.582$, $J'_1(0) = 0.5$, h = pitch, d = slot width, and the transit time factor $C_{oo} = (\sin \frac{1}{2} \beta_o d) / (\frac{1}{2} \beta_o d)$, with β_o being the wave number in the guide. It may be seen that a $\pi/2$ mode has a small advantage (about 8%) over the $2\pi/3$ mode with respect to peak fields. For the BNL structure, a theoretical ratio $\hat{E}/E_o = 1.55$ compared to the experimental value of about 2.0 is found.[13] The theoretical ratio $c\hat{B}/E_o = 1.35$ was not verified. From these discussions follows that $E_o \approx 7$ MV/m seems to be a practical limit. Note that operation as a SW cavity would halve this number.

The deflector may be designed without considering the operating frequency, which will be optimized for maximum transmission of particles. It should, however, be kept in mind that, in contrast to conventional structures, the shunt impedance of SC devices decreases with frequency according to the scaling law

$$R \propto \frac{T}{\omega\, e^{-\Delta/k_B T}\, \ln\,(4\, k_B\, T/\gamma\, \hbar\, \omega)} \tag{10}$$

where Δ = half energy gap of superconductor, k_B = Boltzmann's constant and $\gamma = e^C \approx 1.78$, C being the Euler constant. Lower frequencies are more economical in RF power and refrigeration requirements, but the physical dimensions impose a practical limit.

IV. Optimization

The operating frequency must be chosen to maximize the number of wanted particles, n_W, available at the counters. The transmission of particles with momentum p, rest mass m and lifetime τ, through the separator stage, depends on the accepted solid angle in the deflector (or the product of vertical times horizontal angular divergence) $\Delta\Omega' = \delta_V \delta_H$, and on the decay factor. Hence, n_W is related to the wavelength λ according to

$$n_W \propto \delta_V \delta_H\, e^{-L/\Lambda} \tag{11}$$

where the decay length $\Lambda = \tau\, p/m$ and L is given by (3). The optimization can be carried out analytically under the following assumptions:[17]

1. The solid angle accepted at the target, $\Delta\Omega = M_V M_H \delta_V \delta_H$, is determined by the deflectors only and limitations of the angular acceptance due to the first set of quadrupoles or the interdeflector system are neglected.

2. The maximum magnifications, M_V and M_H, from target to deflector are limited by chromatic aberrations to about 4 and 1 respectively.[4] Since the spot size of the slow external beam,[18] in which the SC RF beam separator will be used, is foreseen to be about 1 mm × 1 mm, it can be assumed that the size of the target image in the deflector, $S_H \times S_V$, is much smaller than the deflector aperture (inscribed square, $\sqrt{2}\,a \times \sqrt{2}\,a$).

3. The vertical angular divergence of the beam in the deflectors is limited, as discussed in Section II, by the maximum deflecting field:

$$\delta_V = 2\hat{\theta}\epsilon = 2\epsilon\, q\, E_o \ell\, /\, pc\ . \tag{12}$$

4. The horizontal angular divergence of the beam is limited by the deflector aperture only:

$$\delta_H = 2\sqrt{2}\,\frac{a}{\ell}\left(1 - \frac{S_H}{\sqrt{2}\,a}\right) \approx 2\sqrt{2}\,C_a\,\frac{\lambda}{\ell}\ . \tag{13}$$

Under the assumptions listed, (11) takes the form

$$n_W \propto 4\sqrt{2}\,\frac{q\,E_o\,\varepsilon}{pc}\,C_a\,\lambda\,\exp\left(-\frac{\lambda}{\tau c}\,\frac{p_o^2\,m}{p\,m_p^2\,c}\right). \tag{14}$$

The transmission of particles will be maximum for $\partial n_W/\partial\lambda = 0$, whence the optimum wavelength

$$\lambda_{opt} = \tau c\,\frac{m_p\,c^2}{p_o c}\left\{\frac{p\,m_p}{p_o\,m}\right\}. \tag{15}$$

To avoid unnecessary decay losses, it is desirable to optimize at the lowest operating momentum for particles with the shortest lifetime, that is, kaons. From Table I follows in the interesting case m = 1 that $\{\ \} = 0.78$. The optimum wavelength now takes the value $\lambda_{opt} \approx 24$ cm. The operating frequency follows as $f \approx 1.25$ GHz, the interdeflector spacing $L \approx 33$ m and the beam hole diameter $2a \approx 9.6$ cm. The transmission is insensitive to small changes in frequency and the precise operating frequency will depend on the availability of microwave equipment.

It is interesting to note that n_W in (14) is independent of the deflector length, and it would appear that the choice of ℓ is arbitrary. The price for the construction and operation of the separator increases with deflector length, and economical as well as technological arguments favor a short structure. However, a small δ_V must be compensated by a larger δ_H and the beam optics will establish a lower bound on ℓ. Furthermore, secondary particles are produced in the target with a pronounced forward peak, and an increase in δ_H may no longer be accompanied by a rise of the particle number. The conventional *specification that* $\hat{\theta} \geq 1$ *mrad at all operating* momenta satisfies grosso modo both conditions. A deflector length of about 3 m, which is expected to impart a transverse momentum of 21 MeV/c to a passing particle, is felt to be an adequate choice.

The solid angle $\Delta\Omega'$ follows to be $\geq 45\ \mu$sr over the full momentum range. This result must be compared with the "intrinsic" value[3] $\Delta\Omega' = (2/3)(a/\ell)^2$, which is obtained from the condition that $3\delta_V = \delta_H = \sqrt{2}\,a/\ell$. It is achieved with a deflector length of

$$\ell'^2 = \frac{\sqrt{2}}{6}\,C_a\,\frac{pc}{q\,E_o\,\varepsilon}\,\lambda_{opt}. \tag{17}$$

We require a complete filling of the deflector under the most favorable conditions only, i.e., when $p/(\epsilon p_o) \approx m_K/m_P$. ℓ' is then given by

$$\ell'^2 = \{0.78\}\,\frac{\sqrt{2}}{6}\,C_a\,\frac{\tau_K c\ m_K c^2}{q\,E_o} \tag{18}$$

whence $\ell' \approx 3.1$ m in good agreement with the chosen value. Note that ℓ' from (18) is independent of p_o.

The RF power requirements of a critically coupled RR deflector are found from (7) to be $P = E_o^2\,\ell/R$. By applying well known scaling laws,[5] the transverse shunt impedance of normal copper structures at room temperature and at the design frequency is typically $R \approx 10$ MΩ/m, whence follows P = 15 MW. The theoretical improvement factors for superconducting lead surfaces at 1.25 GHz were calculated and are listed in Table II. Improvement factors attainable in practice are *hoped to be about* 10^5 *when the structure is operated* at 2°K. This brings the RF power dissipated in each deflector to a 150 W level, if CW operation is assumed. The entire power is dissipated in the structure and must be carried away by evaporation of liquid helium. Pulsed operation with a duty factor of about 1/3 will somewhat alleviate the refrigeration problem, but questions of filling time must then be considered.

Table II. Theoretical improvement factors with superconducting lead at 1.25 GHz.

T(°K)	I
2.0°	1.9×10^6
4.2°	3.7×10^4

V. Conclusion

In the preceding sections a design study for a superconducting RF beam separator was presented, and the principal parameters established are summarized in Table III. It may be concluded that a simple two-cavity system would be able to provide beams of kaons, pions and protons suitable for counter experiments. The high particle fluxes expected after completion of the AGS Conversion will make purified beams a necessity and it appears that superconductivity provides the unique answer to this demand. However, *it must be realized that*, despite great progress, the problem of fabricating superconducting deflectors is at present not yet mastered, and considerable development work is expected.

Table III. Principal parameters of superconducting RF beam separator.

Design momentum, p_o	11 GeV/c
P, π beam	8.5 to 19 GeV/c
K beam	4.5 to 10 GeV/c
Design wavelength, λ	24 cm
Interdeflector spacing, L	33 m
Solid angle at target, ΔΩ	≥ 180 μsr
Beam hole diameter, 2a	9.6 cm
Deflector length	3 m
Deflecting field, E_o	7 MV/m
Shunt impedance, R (Cu, 273°K)	10 MΩ/m
Circulating power, P_{guide}	15 MW
Improvement factor, I	10^5
Duty factor	1/3
Dissipated power at 2°K	2 × 50 W
Refrigerator input power	150 kW
Unloaded quality factor, Q_o	1.5×10^9

References

1. H. Hahn, H.J. Halama, and H.W.J. Foelsche, Proc. V International Conference on High Energy Accelerators, Frascati, 1965 (CNEN, Rome, 1966) p. 548.

2. Anonymous, Physics Today 19, No. 5, 87 (1966).

3. H. Hahn, Proc. International Conference on Instrumentation for High Energy Physics, Stanford, 1966 (CFSTI, Springfield, Virginia, 1966), p. 245.

4. H.W.J. Foelsche, H. Hahn, H.J. Halama, J. Lach, T. Ludlam, and J. Sandweiss, submitted for publication in the Rev. Sci. Instr.

5. H. Hahn and H.J. Halama, Nucl. Instr. & Methods 45, 141 (1966).

6. B.W. Montague, Report AR/Int.PSep/63-1 (CERN 1963).

7. H. Hahn and H.J. Halama, Accelerator Dept. Internal Report AADD-129 (Brookhaven National Laboratory, 1967).

8. I.M. Khalatnikov and A.A. Abrikosov, Advances in Physics, Vol. 8 (Taylor & Francis, London, 1959), p. 48.

9. H.A. Schwettman, P.B. Wilson, J.M. Pierce, and W.M. Fairbank, Proc. 1964 Cryogenic Engineering Conference (Plenum Press, New York, 1965), p. 88.

10. Nguyen Tuong Viet and F. Biquard, C.R. Acad. Sc. Paris 262, B590 (1966).

11. P. Flécher, J. Halbritter, R. Hietschold, and K. Hofmann, Proc. Linear Accelerator Conference, Los Alamos, 1966 (CFSTI, Springfield, Virginia, 1966), p. 499.

12. P.B. Wilson and H.A. Schwettman, IEEE Transactions NS-12, No. 3, 1045 (1965).

13. H. Hahn and H.J. Halama, Rev. Sci. Instr. 36, 1788 (1965).

14. H. Golde, IEEE Transactions MTT-8, 560 (1960).

15. T.I. Smith, H.A. Schwettman, W.M. Fairbank, and P.B. Wilson, ref. 11, p. 491.

16. P.B. Wilson, W.M. Fairbank, H.A. Schwettman, T.I. Smith, and J.P. Turneaure, ref. 3, p. 236.

17. W. Jüngst, ref. 3, p. 260.

18. M.Q. Barton, paper presented at this conference.

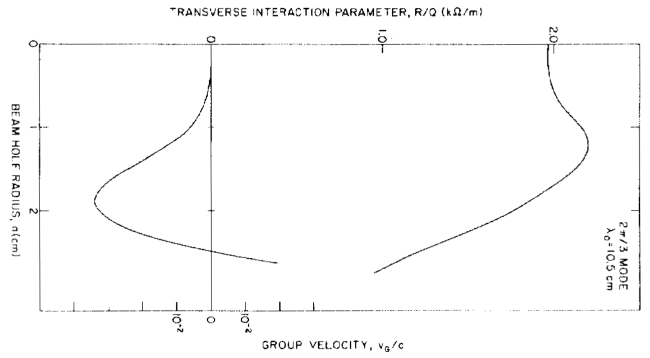

Fig. 1. Transverse interaction parameter and group velocity as fraction of beam hole radius. The results were obtained at λ = 10.5 cm and corresponding values at λ = 24 cm may be obtained by using well-known scaling laws ($R/Q \propto \lambda^{-1}$; $v_g/c \propto \lambda^0$).