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LOW TEMPERATURE ASPECTS OF A CRYOGENIC ACCELERATOR*

H.A. Schwettman,** J.P. Turneaure, W.M. Fairbank, T.I. Smith
M.S. McAshan, P.B. Wilson, and E.E. Chambers

Department of Physics
and
High Energy Physics Laboratory
Stanford University
Stanford, California

Summary

The object of this paper is to identify the many low temperature aspects of a cryogenic accelerator and to indicate how these are related to the operating characteristics which might ultimately be achieved.

I. Introduction

By exploiting the unique properties of matter at low temperature it is possible to construct an electron linear accelerator (electron linac) with characteristics far superior to those of present conventional linacs. The near-zero RF surface resistance of superconductors and the near-perfect heat transport properties of superfluid helium, in combination, offer an elegant solution to the technical problems of constructing a linac with near-ideal characteristics. The cryogenic linac would make important new areas of physics accessible to experimentation.

A. Conventional Linac Characteristics

The design of an electron linac is dominated by the RF surface resistance of the metal from which the accelerator structure is fabricated. For a given energy gradient the surface resistance determines the microwave power requirement. In turn, this power requirement directly affects the duty cycle and the energy gradient of an electron linac and indirectly influences the energy resolution and the average current.

The power required to generate the accelerating fields in the RF structure is given by

$$\frac{P}{L} = \frac{(V/L)^2}{r} \tag{1a}$$

where V/L is the energy per unit length gained by the electrons and r is the shunt impedance per unit length. The shunt impedance per unit length can be expressed as

$$r = \frac{G}{R} \tag{1b}$$

where G is a geometrical factor which increases linearly with frequency and R is the surface resistance of the metal. For a copper structure operating at room temperature and 1 GHz, $G = 2.25 \times 10^3$ ohm^2 cm^{-1}, $R = 7.5 \times 10^{-3}$ ohm and $r = 0.3 \times 10^6$ ohm cm^{-1}.*** An energy gradient of 3 Mev per foot thus implies a power dissipation in the copper accelerator structure of 1.0×10^6 watts per foot.

RF power of 10^6 watts per foot can be provided by high power klystrons, but only for short pulses at low repetition rates. The fraction of the total time that the RF power is available and therefore that electrons can be accelerated is typically 10^{-3}. This time factor is called the duty cycle of the accelerator.

The very large amount of RF power required to generate the accelerating fields in a conventional electron linac also limits the energy gradients which are obtained. Typically, electron linacs are designed for maximum energy gradients of 2-4 Mev per foot. Although it would be possible to achieve significantly higher gradients, these are not practical since the RF power requirement increases as the square of the energy gradient.

The pulsed operation of conventional electron linacs limits the energy resolution of the emerging electron beam. Typically, the electron beam pulse is one or at most a few microseconds in duration. As a result, transients are important and are, in fact, the principle limitation on energy resolution.

Finally, the low duty cycle operation ($\approx 10^{-3}$) implies that very large peak currents must be accelerated if substantial average currents are to be obtained. Ultimately, the peak current is limited by beam-break-up or by the peak microwave power requirements.*

B. Cryogenic Linac Characteristics

As described in the preceding paragraphs, the magnitude of the surface resistance exerts a major influence on the design and operating characteristic of an electron linac. Unfortunately, apart from drastic revisions such as those contemplated in the cryogenic accelerator, little can be done to improve the surface resistance. It is impossible to decrease substantially the surface resistance of normal metals because of the anomalous skin effect. Even if the dc conductivity of copper were to approach infinity, as it should as the temperature approaches zero, the surface resistance at 1 GHz would only decrease by a factor of 6.7 below its observed value at room temperature. Although this limitation applies to all normal metals, it does not apply to superconductors. The RF surface resistance of a superconductor decreases rapidly as

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***This value of the shunt impedance is appropriate to the standing wave bi-periodic π/2-mode structure.

* Usually the energy gradient is given primary consideration in the design of electron linacs. At present time, however, MIT is constructing an electron linac in which the duty cycle, the energy resolution, and the average current are improved at the expense of the energy gradient.

the temperature is reduced below the transition temperature and, in principle, it vanishes exponentially in the limit $T \to 0$.

The potential gains to be achieved in a cryogenic accelerator are enormous. First, the near-zero surface resistance of superconductors makes it possible to generate the desired acclerating fields at RF power levels on the order of watts per foot. The RF power requirements are, in fact, set by the beam power desired, since the power dissipation in the structure walls is small by comparison. Thus the RF power can be provided by a low power continuous wave (CW) klystron and the cryogenic accelerator can be operated at unity duty cycle. Second, in principle, the energy gradient can be increased appreciably. Due to the near-zero surface resistance of superconductors the energy gradient is no longer limited by power requirements, but only by magnetic or electric breakdown. In principle the breakdown phenomena might allow energy gradients as high as 16 Mev per foot in a standing wave structure. Third, it should be possible to improve the energy resolution substantially, perhaps to one part in 10^4. In a linac which operates continuously transients will no longer determine the energy resolution. Further, with CW operation, it is relatively easy to use feedback to regulate the energy. Fourth, the cryogenic accelerator could be an attractive way to achieve high average beam current. Most of the RF power is converted to beam power and even for average currents of many milliamps the power could be provided by CW klystrons with no modulators. Thus the RF power is relatively inexpensive.

The operating characteristics of a cryogenic accelerator mentioned above are, of course, optimistic. Achieving the ultimate in any one of these operating characteristics involves a number of problems of significant proportions. These problems are considered in the later sections of this paper from the point of view of low temperatures. Fortunately, the properties of matter at low temperature conspire largely in our favor.

C. Low Temperature Aspects

The implications of operating an accelerator in a cryogenic environment must be considered from many points of view. Of primary importance, however, is the quantity of RF power that is dissipated at helium temperature since the largest contributor to the initial cost and the operating expense of a cryogenic accelerator is the helium temperature refrigerator. Let us assume that a cryogenic electron linac should be constructed and operated at costs comparable to a conventional linac. Since the operating characteristics of a cryogenic accelerator can be distinctly superior to those of a conventional linac, this criterion is rather strict. However, achieving this objective is clearly desirable and is, in principle, possible.

Consider first the operating expense of a linear accelerator. As noted earlier, the peak power dissipation in a conventional linac is approximately 10^6 watts per foot. The average power dissipated at a duty cycle of 10^{-3} is then 10^3 watts per foot. The efficiency of the klystron and the associated modulators might increase the power consumption to 4×10^3 watts per foot. In addition, one must provide the RF power that is absorbed by the beam, but since this is common to both the conventional linac and the cryogenic linac, it will be neglected.* If the cryogenic linac is to operate with the same total power consumption, the RF power dissipated at helium temperature must not exceed a few watts per foot. According to the second law of thermodynamics, one watt of power removed at the temperature T requires a power input of approximately $300/T$ watts at room temperature. Present refrigerators operate at roughly 10% of this Carnot efficiency and therefore ≈ 1.5 kilowatts is required to remove one watt at the expected operating temperature. A power dissipation of a few watts per foot requires a surface resistance of $\approx 2 \times 10^{-8}$ ohms. It is shown in Section II that this value of the surface resistance can, in principle, be achieved in superconducting lead or niobium at temperatures just below 2°K. At 4.2°K by comparison the surface resistance is 50 to 100 times larger.

Power dissipation at helium temperature of a few watts per foot is also consistent with the objective of maintaining the initial cost of a cryogenic linac at levels comparable to the conventional linac. The initial cost of a refrigerator that will remove 100-1000 watts at a temperature just below 2°K is \$1000-\$1500 per watt. This is a substantial, but also a reasonable contribution to the total cost of the cryogenic accelerator.

For successful operation of a cryogenic accelerator it is essential that the temperature be less than 2°K. We have already noted the "economics" involved in the choice of an operating temperature. The low temperature is equally important in achieving thermal and mechanical stability. As discussed in Section III, this stability is extremely important in achieving energy resolution approaching 10^{-4}. Of greatest importance here is the fact that below 2°K liquid helium is a superfluid. The accelerator structure can be totally immersed in a thermal reservoir that is near ideal. The specific heat per gram of superfluid helium near 2°K is the same as that of water at room temperature. The large specific heat coupled with the near-perfect heat transport properties of the superfluid makes it possible to distribute the power dissipated in the accelerator rapidly throughout a large thermal reservoir. Further, the thermal conductivity of commercially available metals is as great at helium temperature as at room temperature. Thus one can produce an environment at helium temperature that is far more stable than possible at room temperature.

There is another advantage that follows simply from the large thermal capacity of the superfluid helium reservoir. If a cryogenic accelerator is designed to operate with unit duty cycle at an energy gradient of 3 Mev per foot, it would be possible to increase the gradient if the duty cycle is decreased.** The condition to be satisfied is that the

*In estimating operating costs we have also neglected replacement costs of klystrons which is important. We assume that this cost is also common to both linacs although the low power CW klystrons used in the cryogenic linac are less expensive and might reasonably have longer operating lives than high-power pulsed klystrons.

**Alternatively, if the surface resistance is, say, an order of magnitude greater than calculated, one could compensate by reducing the duty cycle.

average power dissipation must not exceed the power that can be removed by the refrigerator. The superfluid reservoir acts as a buffer between the power dissipated in the accelerator and the refrigerator, matching the fractional duty cycle of the former to the unity duty cycle of the latter. The thermal capacity of the superfluid reservoir is sufficiently large that for, say, a 10% duty cycle, the beam pulse could be many seconds in duration. Thus the basic operation of the cryogenic accelerator would not change markedly for such pulsed operation.

In discussing thermal stability above it was instructive to consider the role of the superfluid as that of providing a reservoir for thermal energy. At the same time, one can look at the role of superconductivity as that of providing a reservoir for RF energy. A very sizeable amount of RF energy is stored in an accelerator structure at 1 GHz and 3 Mev per foot. Since the cryogenic linac can operate at unity duty cycle, large peak currents are not required and thus energy is extracted from the structure quite slowly. As described in Section III, the accelerating fields cannot change appreciably in times much less than milliseconds. This long time simplifies considerably the requirements on the RF system.

Pulsed operation, of a different sort than mentioned above, is required in physics experiments employing time of flight techniques. For time of flight experiments a subnanosecond burst of electrons followed by a dead time of perhaps 10-10^3 nanoseconds is required. This mode of operation would require a special injector, but could be accomodated in a cryogenic accelerator with no further modifications. The current from the injector could easily be increased by a factor of 10^3 during the beam pulse and thus even for a 10^3 nanosecond dead time the average current could remain unchanged at a few hundred microamps. Again, the superconducting structure provides a reservoir for RF energy. This reservoir acts as a buffer between the pulsed beam and the CW klystron, matching the fractional duty cycle of the former to the unit duty cycle of the latter. In fact, for the sort of pulsed operation required in time of flight experiments, the cryogenic accelerator is an ideal pulsed machine.

We have attempted in this introduction to indicate how the various properties of matter at low temperature can be utilized in combination to achieve significant improvement in the operating characteristics of an electron linac. By designing the first cryogenic accelerator with the ultimate objectives in mind, one reaps a two-fold benefit. First, this approach provides a substantial margin of safety if in the early development one must settle for less dramatic improvements. Second, one has prepared the way to utilize improved techniques as they develop.

II. RF Power Dissipation

The RF power dissipation, as indicated in the Introduction, affects all of the important operating characterstics of an electron linac. Among these characteristics the RF dissipation influences the duty cycle most directly. The RF surface resistance is the primary factor in determining the power dissipation. In this section the surface resistance of superconductors and the influence of this resistance on the choice of the operating temperature and on the duty cycle are discussed.

A. Theoretical Surface Resistance of Superconductors

According to the theory of Mattis and Bardeen[1] the properties of the superconductor that determine its surface resistance are the energy gap, ϵ, the London penetration depth, λ_L, the Fermi velocity, v_F, the coherence length, ξ, and the electron mean free path, ℓ. In the limit that $\ell, \xi \gg \lambda_L$ the surface resistance can be expressed in a relatively simple form:[2]

$$\frac{R_s}{R_\infty} \approx \frac{2}{3}\left(\frac{2}{\pi}\right)^{4/3}\left[\ln\left(\frac{4kT/\epsilon}{\hbar\omega/\epsilon}\right)-0.5772\right]\left(\frac{\hbar\omega}{\epsilon}\right)^{\frac{4}{3}}\frac{1}{kT/\epsilon}\cdot\exp\left(\frac{-\frac{1}{2}}{kT/\epsilon}\right) \quad (2)$$

R_∞ is the normal state surface resistance in the anomalous limit and is given by[3]

$$R_\infty = \left[\frac{\sqrt{3}}{16\pi}\mu_o^2\omega^2\frac{\ell}{\sigma}\right]^{1/3} \text{ ohms} \quad (3)$$

The dc conductivity σ is proportional to the electron mean free path; therefore the normal state surface resistance is independent of temperature in the anomalous limit. Equation (2) indicates that the reduced surface resistance of a superconductor, R_s/R_∞, is a universal function of a reduced temperature, kT/ϵ, and a reduced frequency, $\hbar\omega/\epsilon$. Further, the temperature dependence and the frequency dependence are nearly separable.

The simple expression for the surface resistance is quite instructive. From Eq. (1) it can be seen that for a given energy gradient the RF power dissipated per unit length in a cryogenic accelerator is proportional to R_s/ω. Using Eqs. (1) and (2) above we have approximately

$$\frac{P}{L} \propto \frac{\omega}{T}\exp\left(\frac{-\epsilon}{2kT}\right) \quad . \quad (4)$$

It is clearly desirable to choose a superconductor with a large energy gap and to operate at low frequency and low temperature. Note that for a normal metal at room temperature the power dissipated is proportional to $\omega^{-\frac{1}{2}}$ (R in this case is proportional to $\omega^{\frac{1}{2}}$) and thus higher frequencies are favored. In contrast, for a superconductor the power dissipated is proportional to ω and thus lower frequencies are favored.

Among pure metals the superconductors with the largest energy gaps are lead and niobium. To achieve unity duty cycle operation in a cryogenic accelerator even for these superconductors, it is desirable to operate at temperatures below 2^oK. Calculated values of the surface resistance at 1 GHz for copper, lead and niobium are given in Table I for several temperatures. As indicated in the table, the surface resistance of lead and niobium is 50 to 100 times larger at 4.2^oK than at 1.85^oK. It should be noted that the values of R_s given in the table were not calculated from Eq. (2), but rather were numerically calculated[4] from the theory of Mattis and Bardeen including the effects of a finite coherence length. The finite coherence length changes the temperature and frequency dependence relatively

TABLE I

Theoretical Surface Resistance at 1 GHz

Material	Temperature (°K)	State	R (ohms)
Copper	300	N	7.5×10^{-3}
Lead	0	N	1.35×10^{-3}
	4.2	SC	6.3×10^{-7}
	1.85	SC	9.8×10^{-9}
Niobium	0	N	6.2×10^{-4}
	4.2	SC	7.7×10^{-7}
	1.85	SC	5.0×10^{-9}

little; its effect is largely to increase the surface resistance by a multiplicative factor. Values of the parameters used in the calculations for lead and niobium are given in Table II together with parameters appropriate to aluminum and tin. The ratios ξ/λ_L and R_s/R_s $(\xi = \infty)$ are also given in Table II. The finite coherence length increases R_s substantially when ξ becomes of order λ_L.

however, we have consistently produced TE_{011} mode cavities electroplated with lead for which Q's of a few times 10^9 are observed.[6] This is an improvement of a factor of $\simeq 10^5$ compared to room temperature copper cavities and an improvement of ≈ 100 compared to previous work. Recently Q's as high as 6×10^8 have been measured in niobium cavities.[7] Although niobium is not yet competitive with lead, this is a substantial improvement over our earlier experience and is promising for the future.

To achieve the extremely low losses essential to the cryogenic accelerator requires very careful consideration of experimental technique. There are many extraneous sources of loss which can occur. Some of the losses observed in particular experiments are not related to the superconducting surface. For example one can have losses in the coupling network or in joint where the cavity is assembled. Our interest here is in losses associated with the superconducting surface itself. We will be content to identify and to comment briefly on the sources of residual loss that we have clearly observed or that we suspect. It is important to

TABLE II

Physical Parameters for Several Superconductors

Material	T_c (°K)	$\epsilon(0)/kT_c$	$v_F \times 10^{-8}$ (cm/sec)	$\lambda_L \times 10^6$ (cm)	$\xi_o \times 10^4$ (cm)	ξ_o/λ_L	$R_s/R_s(\xi = \infty)$
Aluminum	1.18	3.4	1.32	1.57	1.6	102	1.5
Tin	3.73	3.5	0.65	3.55	0.23	6.5	3.2
Lead	7.19	4.1	0.60	3.1	0.092	3.0	4.9
Niobium	9.25	3.7	0.29	3.5	0.039	1.1	12

One can also calculate the effect of a finite electron mean free path on the surface resistance of a superconductor. The mean free path produces an appreciable change in R_s when the magnitude of ℓ becomes comparable to ξ. Interestingly enough, as the mean free path decreases, the surface resistance initially decreases. For lead at 4.2°K it would appear that the surface resistance could be improved by a factor of two in this way. This is probably not the way to improve operation of a cryogenic accelerator, but it does indicate a tolerance for this form of degradation.

Using Eq. (1) and Table I we can calculate the theoretical power dissipation for a cryogenic linac operating at 1.85°K. If the accelerator structure is plated with lead, the power dissipation is 1.3 watts per foot. This is entirely consistent with the objectives outlined in the Introduction.

B. Residual Losses

The preceding discussion neglected the unpleasant fact that in practice one always observes residual losses. The residual losses observed in previous work[5] have been many times, even 100 times, the values required for successful operation of a cryogenic accelerator. During the past few years,

note that the very high Q's reported for superconducting lead cavities have been obtained with only modest precautions. More detailed investigation into the sources of residual losses is proceeding along the lines discussed below and should result in still better results.

The source of residual loss that can be most clearly identified is trapped magnetic flux. Magnetic flux trapped in the superconducting surface can arise from the ambient field present when the cavity is cooled below the transition temperature, from ferromagnetic impurities in the surface (there are a large number of potential candidates, since we only require that the impurity have a Curie temperature greater than 2°K), or from thermoelectric currents present when the cavity goes superconducting. Experiments have been performed[8] in which the ambient field during cool down was varied from 10^{-4} gauss to 10 gauss. The enhanced losses from trapped flux are not yet understood in detail but in any case it would appear that ambient fields of a few milligauss are tolerable in a cryogenic accelerator, whereas the earth's magnetic field contributes a significant loss. Experiments are in progress to improve our understanding of the mechanism for losses associated with trapped flux and to determine if magnetic impurities or thermoelectric currents lead to flux trapping in our cavities.

The properties of a superconducting cavity can be degraded if the superconducting layer is too thin. In the proximity of the normal metal substrate the superconducting state is altered appreciably. Providing the superconducting layer is much thicker than the coherence length and the penetration depth this does not affect the interior superconducting surface of the cavity.

Surface roughness or surface contamination could also be important factors. The negative effects of surface roughness are particularly noticeable in tin. For electroplated surfaces roughness is minimized if the plated layer is thin. Roughness can be reduced further by several methods and these are being studied. In addition, surface contamination can lead to either resistive or dielectric losses. The lead surfaces presently used are dried with ethyl alchol immediately after electroplating to minimize oxidation; no high vacuum outgassing has so far been attempted.

Crystal lattice defects might also contribute to losses. As noted previously, a simple reduction in the electron mean free path leads to reduced losses. It is possible, however, that major disturbances in the lattice could result in a local variation in the superconducting state. In our present experiments lead is electroplated on a copper substrate. There is some evidence that the differential thermal contraction that occurs during cool down generates defects and thereby enhances the losses.

As a diagnostic tool in the investigation of residual losses it would be useful to measure the frequency dependence of the residual losses. To eliminate ambiguity it is desirable to make all measurements on the same sample and with the same current distribution over the sample's surface. At present we are preparing an experimental system which approximates this situation.

III. Cryogenic Stability

Achieving an energy resolution approaching 10^{-4} in a cryogenic accelerator places strict requirements on the frequency and amplitude stability of the RF source and on the thermal and mechanical stability of the accelerator structure.

Although the primary concern of this section is the stability of the cryogenic system it is of value to comment briefly on the stability required of the RF source. The fields in a microwave resonant structure depend on frequency as

$$|E(\omega)|^2 = \frac{\omega_o E_o^2/2Q_L}{(\omega-\omega_o)^2 + \omega_o/2Q_L}$$

where ω_o is the resonant frequency and Q_L is the loaded Q. The bandwidth is ω_o/Q_L and thus the higher Q_L the more strict the demands on frequency stability. But the decay time for the fields is $\tau = Q_L/\omega_o$ and thus amplitude stability is more easily achieved for large Q_L. The beam loaded Q in a cryogenic accelerator for an energy gradient of a few Mev per foot and an average current of a few hundred microamps is roughly 10^7. Fortunately, this is a reasonable compromise between the competing factors noted above.

Let us inquire then what frequency and amplitude stability is required of the RF source if the fields are to be held constant to one part in 10^4. In practice it is reasonable to tune the source frequency to within 1% of the bandwidth. If we evaluate the derivative $dE(\omega)/d\omega$ at a distance from resonance that is 1% of the bandwidth, it follows that the frequency must be held constant to 1/4% of the bandwidth. For $Q_L \simeq 10^7$ the fractional frequency stability is 2.5×10^{-10}. For the same Q_L the amplitude must be stable to one part in 10^4 averaged over a time $\tau = Q_L/\omega_o \simeq 1.5 \times 10^{-3}$ seconds. These stability requirements on the RF source are strict but not unreasonable.

The fractional frequency stability of 2.5×10^{-1} also applies to the resonant frequency of the accelerator structure. In turn this implies demands on the thermal and mechanical stability of the structure

A. Thermal and Mechanical Stability Requirement

Thermal and mechanical stability are necessary to guarantee that the resonant frequency of the structure will remain constant. This section of the paper defines the extent of the thermal and mechanical stability required while the following section demonstrates the practicability of achieving that stability.

Before treating the superconducting case, let us consider a copper structure operating at room temperature. For the copper structure $Q_L \approx 2 \times 10^4$ and, if the fields are to be held constant to one part in 10^4, the fractional frequency stability is 1.2×10^{-7}. The resonant frequency of the structure can change with temperature due to thermal expansion. For copper at room temperature the thermal expansion coefficient $\alpha = 1.7 \times 10^{-5}{}^oK^{-1}$ and thus the temperature must be held constant to 0.007°K. In the presence of peak RF dissipation of 10^6 watts per foot, this is nontrivial.

Fortunately, the problem of thermal stability is less difficult at 1.85°K. The expansion coeffieient, according to the Grueneisen relation, is proportional to the specific heat of the metal and therefore is extremely small at low temperatures. For copper at the operating temperature $\alpha \approx 6 \times 10^{-10}$c Even though the frequency shift allowed is 500 times smaller for a cryogenic accelerator the temperature must only be held constant to 0.4°K. In a cryogenic accelerator we must also consider the temperature dependence of the surface reactance. The surface reactance, X, can be expressed as a skin depth, δ, using the expression $\delta = X/\mu_o\omega$ (mks). The quantity of interest is $d\delta/dT$ which vanishes exponentially with temperature. For superconducting lead at 1.85°K the theory of Mattis and Bardeen gives $d\delta/dT \approx 4 \times 10^{-9}$ cm/°K. When consideration is given to the exponential temperature variation of $d\delta/dT$ the temperature difference implied is $\Delta T = 0.3$°K. It is worth noting that $d\delta/dT \approx 4 \times 10^{-7}$ cm/°K at 4.2°K and thus the temperature would have to be held constant to 0.01°K. Coupled with the much larger power dissipation at 4.2°K, this requirement on temperature stability provides an additional reason for operating a cryogenic accelerator below 2°K.

Another potential source of frequency shift in a cavity structure is mechanical deformations caused by changing pressure. The structure is immersed in liquid helium and even modest pressure changes result in significant frequency changes. A frequency change of $2.5 \times 10^{-10}\omega_0$ implies that the maximum change in diameter that can be permitted is approximately one angstrom unit. It is reasonable to construct an accelerator structure of sufficient strength to tolerate pressure changes of 1 mm Hg. However, to proceed along this line much further is difficult; the first structure we fabricated shifted in frequency $0.8 \times 10^{-8}\ \omega_0$ with a 1 mm Hg pressure change. Since the pressure acting on the structure is the vapor pressure of the superfluid helium, to maintain constant pressure implies restrictions on the temperature stability of the reservoir. The vapor pressure of helium can very crudely be represented by the expression

$$P = P_0 \exp\left(-\frac{\Delta}{T}\right) . \qquad (5)$$

The pressure and thus the derivative dP/dT vanish exponentially with temperature. At 1.85°K, P = 15 mm Hg and a 1 mm Hg pressure change implies a 0.020°K temperature change. With a superfluid helium reservoir this stability is within reason.

Together the effects of thermal expansion and changing skin depth restrict the temperature gradients that can be tolerated in a cryogenic accelerator to 0.15-0.20°K. In addition, to prevent mechanical deformations of the structure, pressure and temperature changes in the superfluid reservoir of less than 1 mm Hg and 0.02°K respectively are required. In the following section it is shown that these demands can be satisfied.

B. The Superfluid Reservoir and Conduction to the Reservoir

To achieve the desired frequency stability the microwave power dissipated at the interior surface of the accelerator structure must be conducted to the superfluid reservoir with a temperature difference ΔT = 0.15-0.20°K. Since the surface resistance varies exponentially with temperature still smaller temperature differences are desirable; large differences result in a significant increase in the power dissipation and thus in reduced efficiency. The calculations below assume the theoretical power dissipation for an energy gradient of 3 Mev per foot (1.3 watts per foot). For operation at 10 Mev per foot or in the event that the losses in practice exceed the theoretical losses by an order of magnitude, the thermal differences given below must be increased by a factor of ten. Even for these conditions the cryogenic stability requirements can be satisfied.

There are three major contributions to the thermal resistance which must be considered in conducting heat to the superfluid reservoir. First, there is the boundary resistance between the superconducting surface and the normal metal from which the accelerator structure is fabricated. Second, there is conduction through the walls of the structure. And, finally, there is the boundary resistance between the accelerator structure and the superfluid reservoir. The calculations which follow assume that the accelerator structure is fabricated from O.F.H.C. copper.*

* We are also considering the use of commercially available high purity aluminum in the accelerator structure. An aluminum structure is lighter, is less expensive, has a larger thermal conductivity, but is more difficult to electroplate than O.F.H.C. copper.

The temperature difference associated with heat conduction through the structure walls can be estimated from the expression

$$\Delta T \approx \frac{\ell_n(r_o/r_i)}{2\pi \bar{K}} \cdot \left(\frac{P}{L}\right) \qquad (6)$$

where P/L is the power dissipated per unit length (watts/cm), r_i is the inner and r_o the outer radius, and $\bar{K}$ is the average thermal conductivity. The theoretically calculated power dissipation is 1.3 watts per foot. The thermal conductivity of O.F.H.C. copper[10] at 1.85°K is 1 watt/cm°K, nearly as great as the conductivity of copper at room temperature, and thus $\Delta T < .001$°K. Heat conduction out through the accelerating structure disks, however, is less favorable. Rough estimates suggest the temperature difference might be .010°K in the disks.

The boundary resistance which occurs both at the lead-copper interface and the copper-helium interface also contributes a temperature difference. In a normal metal the heat current is carried by the conduction electrons while in a superconductor (at temperatures well below T_c) or in the superfluid the heat current is carried by the phonons. The heat conduction across the boundary then is determined by the coupling between the electron thermal current in the normal metal and the phonon current in the superconductor or the superfluid. For the lead-copper boundary[11] the temperature difference is

$$\Delta T \approx \frac{11}{T^4} \cdot \frac{\dot{Q}}{A} \qquad (7)$$

The calculated maximum power that must be transported across unit area of the interface is $< 10^{-3}$ watts per cm^2. At 1.85°K this implies $\Delta T < 10^{-3}$°K. For the copper-superfluid boundary[12]

$$\Delta T \approx \frac{10}{T^3} \cdot \frac{\dot{Q}}{A} \qquad (8)$$

and again the temperature difference is $< 10^{-3}$°K.

In transferring heat across the copper-superfluid boundary it is possible to initiate film boiling. The thermal contact between the accelerator structure and the superfluid reservoir decreases significantly if this occurs. The power density required for film boiling[13] is approximately 0.1 watt/cm^2 compared to a value of 10^{-3} watts/cm^2 calculated using the theoretical surface resistance.

Of the problems discussed above power dissipation in the disks causes the greatest concern. This problem is accentuated by the fact that field emission electrons can also contribute to power dissipation in the disks. Although the stability requirement is satisfied, even for losses enhanced by a factor of ten, it may still be desirable to provide direct superfluid access to this region.

The superfluid reservoir plays a major role in providing a stable thermal environment for the cryogenic accelerator. At the operating temperature, which is not far below the lambda point, the specific heat of liquid helium is still very large. The specific heat at 1.85°K is ≈ 3 joules/gm°K, nearly as large as that of water at room temperature.

The quantity of superfluid helium that surrounds the accelerator might be 30 liters per foot. Since the density is 0.146 gm/cc, the total heat capacity is 1.3×10^4 joules/°K per foot. This capacity is very large compared to the RF energy stored in the accelerator structure which is 4.2 joules per foot at 3 Mev per foot. As noted in the Introduction, this capacity is also sufficient to permit long beam pulses if the accelerator is operated at, say, 10% duty cycle. Even if the losses are ten times the theoretical losses, a pulse of 20 seconds duration increases the temperature of the reservoir by only 0.020°K.

In addition to the large specific heat, superfluid helium exhibits remarkable heat transport properties. In a metal, heat transport is a random process. For superfluid helium the mechanism of heat transport is quite different. Very crudely the superfluid can be considered as two interpenetrating fluids; one component is a superfluid which has zero entropy and flows without viscosity, the other component is a normal fluid which carries the full entropy of the liquid and exhibits normal viscosity. If heat is produced at one end of a superfluid helium "rod", the superfluid component flows to the source and there absorbs the heat in the process of being converted to normal fluid. To maintain constant density in the liquid there must be a counterflow of the normal fluid component.

For large heat currents along the superfluid "rod" a small temperature difference occurs. This temperature difference is associated with the viscosity of the counterflowing normal fluid and with vorticity generated in the superfluid component. Heat transport in superfluid helium is greatest when the density of the superfluid component and the density of the normal fluid component are approximately equal. The selected operating temperature of 1.85°K lies just to the low temperature side of the heat transport maximum. As an example, consider 100 watts transported 100 feet in a superfluid "rod" one foot in diameter. The expected temperature difference is .020°K. To maintain the same temperature difference using high conductivity copper requires a rod exceeding 100 feet in diameter. Although not perfect the superfluid is an extremely good conductor of heat.

The power dissipated in a cryogenic accelerator does not have to be transported the length of the machine; it must only be transported to the surface of the liquid where it is removed by evaporation. In any case it is clear that the demand on temperature equilibrium for the superfluid reservoir (within 0.020°K) is readily achieved.

C. Refrigeration

The remarkable heat transport properties of superfluid helium guarantee adequate thermal equilibrium throughout the helium reservoir. It is still required, however, that the pressure and thus the temperature of the reservoir remain constant for long times. Long term stability is accomplished by proper regulation of the superfluid helium refrigerator.

The very low operating temperature for a cryogenic accelerator is achieved by pumping the vapor over the liquid. At 1.85°K the vapor pressure is 15 mm Hg and changes in this pressure of 1 mm Hg can be tolerated. A simple form of pressure regulation can be provided by controlling the flow in a bleed-back line which feeds high pressure gas from the output of the pumping system to the low pressure stream at the pump input. Additional pressure regulation can be provided by means of a thermal load in the superfluid helium reservoir which can be varied to hold the reservoir temperature constant.

Although we do not intend to present a detailed discussion of the refrigerator, it is interesting to consider briefly two problems of the refrigerator that are created by the low operating temperature. First, the pumping speed $\dot{V}$ (vol./sec.) required to maintain the low temperature is given by the rate at which gas is generated (mass/sec.) divided by the density. For a fixed energy gradient in the accelerator structure, gas generation is proportional to the power dissipated at helium temperature which, according to Eq. (4), vanishes exponentially. However, the density is proportional to the pressure which, according to Eq. (5), also vanishes exponentially. Therefore, we have approximately

$$\dot{V} \propto \frac{e^{-\epsilon/2kT}}{e^{-\Delta/T}} \qquad (9)$$

where both numerator and denominator vary by a factor of 50 to 100 between 4.2°K and 1.85°K. It is a fortunate coincidence that the two factors nearly cancel leaving the pumping rate essentially constant, independent of temperature. If a substantially increased pumping speed were required at 1.85°K, the pumping system would become prohibitively large. Second, the low vapor pressure at 1.85°K makes more difficult the problem of heat exchanging the cold escaping vapor with the warm input stream. Nevertheless, at 1.85°K the heat exchange problem is still manageble.*

IV. Limitations on the Energy Gradient

The RF power dissipated in the accelerator structure increases as the square of the energy gradient. For a conventional electron linac this rapidly increasing power requirement limits the energy gradient, in practice, to 2-4 Mev per foot. For a cryogenic accelerator the RF power dissipation is not the principal limitation; the duty cycle of the cryogenic accelerator can easily be reduced so that the average power dissipation does not exceed the helium refrigerator capacity. The factors that determine the energy gradient in the cryogenic accelerator are the magnetic critical field of the superconductor and loading due to electric field emission from the surface. The fundamental limitations on energy gradient are discussed in this section.

* The first superfluid helium refrigerator is presently being constructed by Arthur D. Little, Inc. The system will remove 300 watts at 1.85°K. A prototype of this system has already been operated.

A. Critical Magnetic Field

It is to be expected that for some energy gradient the RF magnetic field at the surface of the accelerator structure will exceed a critical value and the superconducting state will be quenched. This critical magnetic field represents a fundamental limitation on the energy gradient that can be achieved in a cryogenic accelerator.

The behavior of superconductors in the presence of large RF magnetic fields is not as well understood as the dc case. In the present discussion we will assume that the known dc behavior of superconductors can be extrapolated to RF. Experimental information obtained thus far is consistent with this assumption.

At dc the superconducting state is quenched at a magnetic field level which is determined by the Gibbs free energies of the superconducting and the normal states. This field level is called the critical magnetic field, H_c, and depends on temperature as

$$H_c = H_o \left(\frac{T^2 - T_c^2}{T_c^2}\right) \qquad (10)$$

where T_c is the superconducting transition temperature. The critical magnetic field is zero at T_c and increases monotonically to H_o at $T = 0$. Values of H_o and H_c (1.85°K) for lead and niobium are given in Table III.

TABLE III

Magnetic Field Limitations on Energy Gradient

Metal	H_o gauss	H_c(1.85°K) gauss	$R_s(H_c)/R_s(0)$	$\bar{E}$(SW) Mev/ft	$\bar{E}$(TW) Mev/ft
Pb	800	750	1.8	6.5	10.5
Nb	1940	1860	3.3	16	26

For large magnetic fields the superconducting energy gap decreases; thus we might expect an increase in the surface resistance at increasing RF field levels. This increase can be estimated using the Ginzburg-Landau theory of superconductivity.* The magnetic field dependence of the Ginzburg-Landau superconducting order parameter, ψ, is[14]

$$\frac{\psi(H)}{\psi(0)} = 1 - \frac{\kappa}{4} \frac{H^2/H_c^2}{(\kappa + \sqrt{2})}$$

where κ is approximately λ/ξ. The penetration depth λ and the energy gap ϵ are related to the order parameter; we have approximately: $\lambda \propto \psi^{-1}$ and $\epsilon \propto \psi$. Since at low temperatures $R_s \propto \lambda^2 \exp(-\epsilon/2kT)$, the factor by which the surface resistance increases with increasing field level can be calculated. The ratio $R_s(H_c)/R_s(0)$ for lead and niobium at 1.85°K is given in Table III.

To determine the actual behavior of superconductors in the presence of large RF magnetic fields we have made measurements[15] on tin, lead, and niobium. Of these measurements the most definitive are those for tin. At 2856 MHz the surface resistance of tin is observed to be constant within 20% for RF field levels smaller than the dc critical magnetic field. This behavior is consistent with the factor $R_s(H_c)/R_s(0) \approx 1.2$ calculated for tin on the basis of the Ginzburg-Landau theory. When the RF field level exceeds the dc critical field the surface resistance is observed to make a sharp transition to the normal state value. As a result of heating effects, measurements on lead and niobium extend only to field levels of 0.5 H_c and 0.1 H_c respectively. Experiments extending to higher field levels are in progress.

We assume that the behavior of superconducting lead and niobium in the presence of large RF magnetic fields will be similar to that observed for tin. The implied limitation on the energy gradient that can be achieved in a cryogenic accelerator is given in Table III. Values are given for both a standing wave bi-periodic $\pi/2$-mode structure and a traveling wave simple periodic $\pi/2$-mode structure.

B. Loading Due to Field Emission

For large energy gradients the electric field at the walls of an accelerator structure can be sufficient to cause an appreciable field emission current. The emitted electrons absorb energy from the RF electric field and upon striking the structure walls convert this energy to heat. Since the heat is generated at helium temperatures, the field emission current must be kept small.

Preliminary measurements[16] on superconducting lead cavities operating in the TM_{010} mode indicate that peak electric fields of 1.5×10^5 volts/cm* can be obtained before electron loading becomes important. This value of the field is achieved by moderate processing; the cavity is overcoupled ($Q_L \approx 10^7$) and the incident power is increased. Observation of the power reflected from the cavity reveals electrical noise but no sparking in the sense that the energy stored in the cavity is lost in a discharge. At the present time an extensive investigation of the behavior of superconducting surfaces in the presence of large electric fields is being planned. The discussion below is suggestive of what could hopefully be achieved in these investigations.

For a point-to-plane geometry and for optimal experimental conditions the dc electrical breakdown field is observed[17] to be $\approx 7 \times 10^7$ volts/cm in metals such as tungsten. For smaller values of the electric field the field emission current follows the Fowler-Nordheim theory. Enhanced field emission currents and much smaller dc breakdown fields are observed for broad area electrodes. The degraded performance of broad area electrodes might be attributed to any one of several mechanisms. The most clearly defined of these mechanisms is field enhancement at sharp projections on the surface of the electrodes. It has been shown[18] that the electric field at such projects can be enhanced

* The Ginzburg-Landau theory is strictly correct only for temperatures near T_c.

* The electric field at the surface is probably 50% higher than the value quoted above; the field is enhanced in the vicinity of a cutoff hole in the cavity.

by factors as large as 200. Although an enhancement of 200 for our superconducting cavities gives a reasonable breakdown field it must be emphasized that the mechanism by which field emission is enhanced in our cavities is not established. Enhancement due to sharp projections, surface contamination etc. might all contribute.

It is likely that the maximum electric field that has been achieved in room temperature cavities provides the best indication of what can be expected ultimately in a superconducting cavity. Although the very low operating temperature may influence such things as whisker growth and surface contamination, the field emission current for a given surface is nearly independent of temperature and nearly independent of whether the metal is in the superconducting state or the normal state.[19] The highest electric field level achieved in a copper cavity, to the best of our knowledge, is $\approx 10^6$ volts/cm.[20] If fields of this magnitude can be obtained in a cryogenic accelerator, extremely high energy gradients are possible. In the standing wave bi-periodic $\pi/2$-mode structure fields of 10^6 volts/cm correspond to an energy gradient of 15 Mev per foot.

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