COAXIAL CAVITIES FOR SEPARATED ORBIT CYCLOTRONS*

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Summary

Coaxial cavities are recommended over TM cavities for use in separated-orbit cyclotrons (SOC's) under 100 MeV primarily because they are smaller and construction tolerances are less stringent. At some energy above 50 MeV, however, the TM cavity[1] becomes more efficient. Particles can gain energy in a coaxial cavity with arbitrary spacing between gap centers; however, maximum energy gain occurs for a spacing of $0.5\beta\lambda$. In a multi-particle SOC a spacing can be selected to provide acceleration of several different particles.

It becomes impractical to define modes in a coaxial cavity relative to the beam motion since the primary TEM mode is defined relative to the direction of wave propagation in the transmission line. Thus TE modes in this paper are likewise defined. The TE modes in a coaxial cavity produce an accelerating voltage which varies with machine radius.

Three coaxial cavities have been constructed at ORNL. RF measurements on a 1/4-scale model of a 50-MeV cavity and on the SOCE model have been completed. The 50-MeV prototype cavity is now ready for testing.

Introduction

Coaxial cavities with which we are concerned may be described as foreshortened $\lambda/4$ wide strip lines capacitively loaded along the open end. Due to the capacitive loading the center conductor, or stub, is less than $\lambda/4$, in most cases only 0.15λ to 0.2λ. Since the height of TM cavities is always greater than $\lambda/2$, a complete coax cavity, with top and tuners, is usually about half that height. The width (radial distance in SOC) of a coax cavity need be only slightly greater than the distance between injection and maximum orbit radius but the TM cavity must have a radial dimension 50% to 100% greater. In spite of its smaller size, however, the coax cavity may have greater power losses than an equivalent TM cavity. This disadvantage is accentuated as the energy of the SOC increases; at some poorly defined energy level the TM cavity becomes more economical. This crossover point appears to be between 50 and 100 MeV and is probably dependent on several machine parameters other than energy.

The ease with which a coax cavity may be tuned is, of course, another advantage over the TM type. The effective capacitance from the open end of the coax stub can be easily varied by the insertion of metal plates opposite the stub end to provide a tuning range of ±1% or more. This relatively wide range permits a relaxation of construction tolerances and reduces the cost.

Two machines with coax cavities have been studied at ORNL. The first of these, a 50-MeV multi-particle SOC, was designed to accelerate protons and alpha particles to 50 MeV and deuterons to about 25 MeV. A quarter-scale model cavity and a full-scale prototype cavity were constructed for this machine, see Figs. 1 and 2. The second machine, called the SOCE (Separated-Orbit Cyclotron Experiment), is being constructed as funds permit. The SOCE is also a multi-particle machine and is designed to accelerate protons to 4 MeV. Further characteristics of the machine are presented in another paper in these transactions.[2] A full-scale model of the SOCE cavity was constructed of plywood and copper foil, see Fig. 3. An operating frequency of 49.2 MHz was selected for both the 50- and 4-MeV machines.

Energy Gained in a Coaxial Cavity

To the SOC beam a coax cavity appears as simply two accelerating gaps and a drift space.

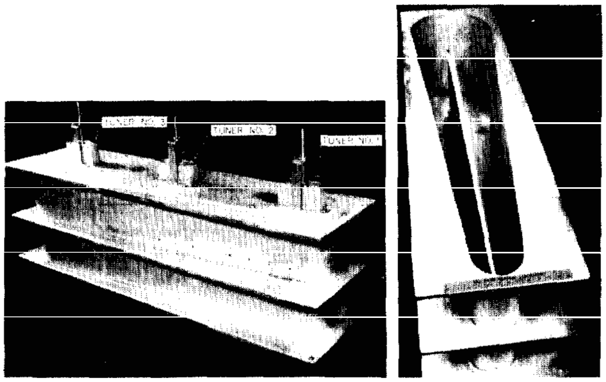

Fig. 1 The 1/4-scale model cavity for the 50-MeV SOC.

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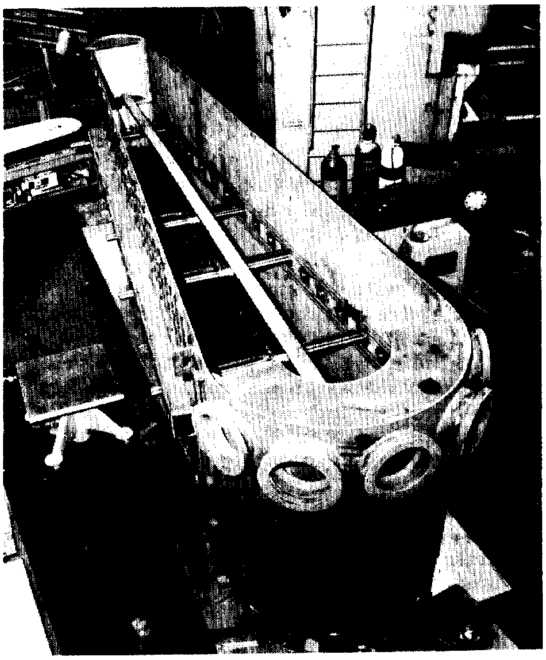

Fig. 2 Prototype cavity under construction for the 50-MeV SOC.

The center conductor forms the drift space and, ideally, is centered between the two gaps. The electric field in the gaps is then anti-symmetrical about the center of the stub. For an electric field of $E_z = E_o f(z)\sin(\omega t+\varphi)$ the energy gained by charge q in passing through the cavity is

$$\Delta T = 2qE_o\cos\varphi\int_o^a f(z)\sin 2\pi\frac{z}{\beta\lambda}\,dz, \text{ if } \Delta T \ll T.$$

Z is assumed to be zero in the center of the stub and the distance "a" is to some point in the beam pipe outside the cavity at which the field is essentially zero. If a uniform field exists in the gaps then the energy gain becomes

$$\Delta T = 2qE_o g\cos\varphi\sin(\pi\frac{g+d}{\beta\lambda})\frac{\sin\frac{\pi g}{\beta\lambda}}{\frac{\pi g}{\beta\lambda}},$$

where d = drift length, or stub thickness, and g = gap length. The transit time factor may then be defined as

$$F = \sin(\pi\frac{g+d}{\beta\lambda})\frac{\sin\pi\frac{g}{\beta\lambda}}{\frac{\pi g}{\beta\lambda}}.$$

In a multi-particle SOC it would be desirable to operate the cavities with the gap voltage approximately fixed for all particles since this condition corresponds roughly to minimum rf power loss, for equal beam losses. It is necessary that the velocities of different particles, at a point in SOC, be expressible as a rational fraction if the cavities are operated at a fixed frequency. It is possible then to determine "g" and "d" so as to satisfy the fixed voltage requirement. For example, in the 50-MeV machine it is necessary that the proton energy gain (ΔT_p) be approximately twice the deuteron energy gain (ΔT_d) so $\beta_p = 2\beta_d$. For uniform fields then

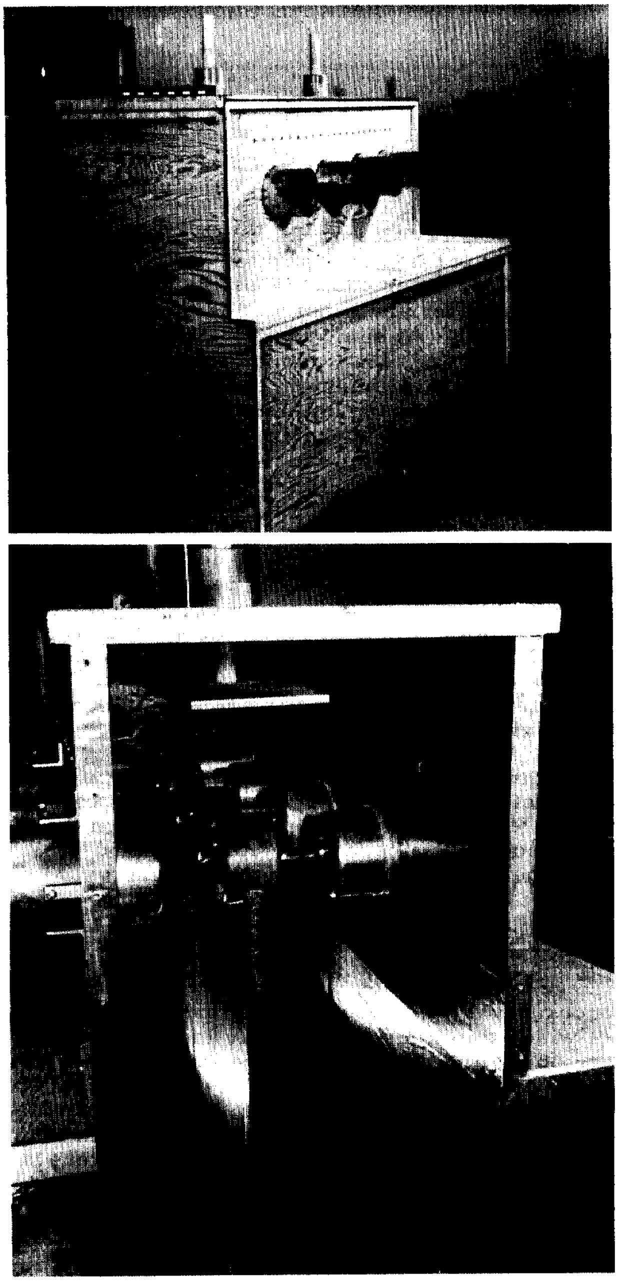

Fig. 3 Model cavity for SOCE.

$$\sin(\pi\frac{g+d}{\beta\lambda})\ \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda} = \sin(2\pi\frac{g+d}{\beta\lambda})\ \frac{\sin(2\pi g/\beta\lambda)}{2\pi g/\beta\lambda}$$

or

$$\cos(\pi\frac{g+d}{\beta\lambda}) = \frac{1}{4\cos(\pi g/\beta\lambda)} \quad \text{where } \beta = \beta_p.$$

This equation then defines "d" as an implicit function of "g". The 50-MeV model and prototype were constructed with a gap of $0.3\beta\lambda$ and a stub thickness of $0.06\beta\lambda$. The proton transit time factor for this case is then 0.776 if uniform gap fields are assumed.

TE Modes in Coaxial Cavities

Coaxial cavities are assumed, ordinarily, to support only a TEM mode. If the peripheral dimension of a cavity is comparable to, or greater than, a wavelength, however, TE modes can become important. There are, of course, many modes generated near the open end of a coax line, but these modes are normally far below cut-off and the net effect can be represented by a capacitance. The TE modes in a "large" cavity produce a voltage on the stub which is a function of peripheral distance around the stub. Consider a wide strip line of width "w" symmetrically located between two ground planes with uniform spacing. The fields between the strip and one ground plane can be written as

$$E_x = \sum_{n=0}^{\rho} A_n \cos(n\pi y/w) \cos\left(2\pi \frac{z - d_n}{\lambda} \sqrt{1 - (n\lambda/2w)^2}\right)$$

$$H_y = (j/\eta) \sum_{n=0}^{\rho} A_n \sqrt{1 - (n\lambda/2w)^2} \cos(n\pi y/w)$$

$$\cdot \sin\left(2\pi \frac{z - a_n}{\lambda} \sqrt{1 - (n\lambda/2w)^2}\right.$$

$$H_z = -(j/\eta) \sum_{n=0}^{\rho} A_n (n\lambda/2w) \sin(n\pi y/w)$$

$$\cdot \cos\left(2\pi \frac{z - a_n}{\lambda} \sqrt{1 - (n\lambda/2w)^2}\right),$$

where the "a_n" are arbitrary constants. These equations are strictly correct for parallel planes of infinite extent. If, however, the width "w" is much greater than the spacing between conductors, the equations should be good approximations, except near the two edges of the center conductor. The terms in the above series with $n > 0$ are due to TE modes and their effect on the field distribution can most easily be seen by considering a simple example. Let $w = 0.708\lambda$, $A_o = 1.0$, $A_1 = -0.2$; $a_o = \lambda/4$ and $a_1 = 1.381(\lambda/4)$. This choice of a_o and a_1 produces a zero value for E_x at $z = 0$, and a shorting plane can be inserted across the line at this point. If the strip line terminates in an open circuit at $z = 0.125\lambda$, the fields along the open end are (neglecting fringing)

$$E_x = 0.707 - 0.1056 \cos(\pi y/w)$$

$$H_y = \frac{-j}{\eta} \{0.707 - 0.1203 \cos(\pi y/w)\}$$

$$H_z = \frac{j}{\eta} (.0746) \sin(\pi y/w).$$

To approximately meet the boundary conditions along the open end an admittance may be placed there having a value of

$$Y = \frac{H_y}{-sE_x} = \frac{j}{\eta s} b_o,$$

where "s" is the spacing between strip line and ground plane and

$$b_o = \frac{0.707 - .1203 \cos \pi y/w}{0.707 - .1056 \cos \pi y/w}.$$

A plot of E_x and b_o as functions of y for this example is presented in Fig. 4. The important point to note in this figure is the "in phase" variation of E_x and b_o. This indicates that variations in the capacitive loading along the open end of a wide strip will be accompanied by similar variations in the end voltage.

The 50-MeV cavities employ inclined planes, in effect, rather than parallel planes. A similar approximation to that above can be made, however. The field equations then contain a Bessel function and sinusoidal function rather than two sinusoidal functions. The primary difference between this case and the parallel plane case is that the capacitance along the open end of the stub must vary inversely with radius (in SOC) to achieve a constant voltage with radius. An increase in local capacitance still results in a local voltage increase, however.

Models and Measurements

As mentioned previously, three coaxial cavities have been built at ORNL in connection with the SOC program. The first of these, the quarter-scale model of the 50-MeV cavity, was constructed primarily as a proof of the design. The prototype cavity was built directly from scaled dimensions of the model except for a slight increase in stub height, which should center the cavity frequency in the desired tuning range. The prototype will be used primarily to establish voltage breakdown limits, multi-

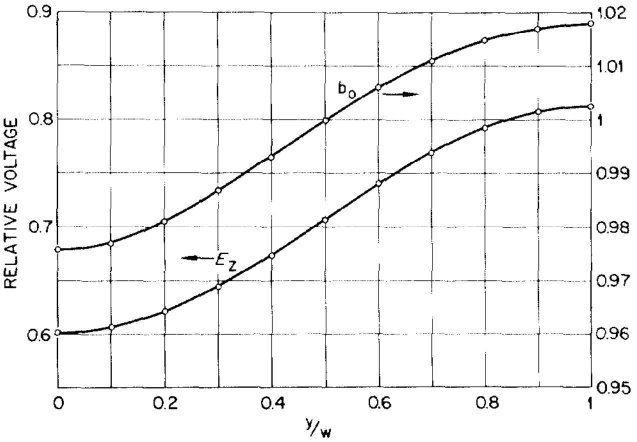

Fig. 4 Electric field and admittance along end of wide strip line.

pactoring levels, and beam loading effects. The cavity has been vacuum tested and the rf drive and tuning elements have been installed, so it is now ready for rf testing. In the prototype the stub has a height of 48 5/8" and a width of 196". The gap length varies from 10 1/4" to 22 3/4".

Model measurements indicate a Q of 25,000 and a shunt impedance of 0.53 megohm for the 50 MeV-prototype cavity. Since the voltage is not constant along the stub at beam orbit height, the shunt impedance is defined at a single point in the cavity. This point is taken as the outermost beam hole. The shunt impedance, relative voltage, and energy gain were determined from perturbation measurements on the model. The "perturber" was a 1/2-inch length of #16 wire (.051" diam) oriented with its axis parallel to the beam path so that, in effect, only the accelerating component of electric field was measured. A plot of the field across the accelerating gap at a point near maximum radius is shown in Fig. 5. In the model the holes in the cavity walls are very small so that the "beam hole effect" on the field is not measurable. From data such as that in Fig. 5 the relative voltage and energy gain along the accelerating gap was computed with the result shown in Fig. 6. The three curves are individually normalized since a common normalization would result in three widely separated curves. The relatively large difference between the proton $(\Delta T)_p$ and deuteron $(\Delta T)_d$ energy gain at small radius in Fig. 6 can be overcome by differentially adjusting the tuners.

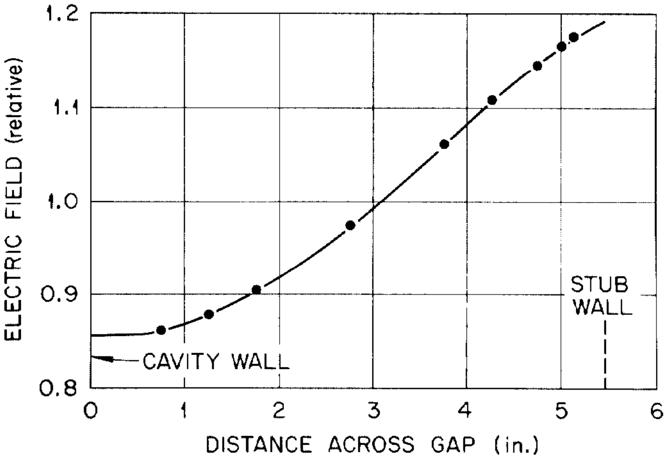

Fig. 5 Electric field in gap of 50-MeV model.

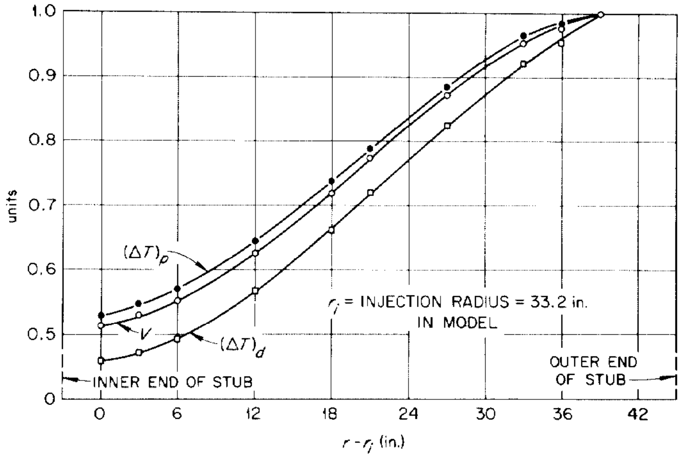

Fig. 6 Relative voltage and energy gain for 50-MeV model.

Considerable data has also been obtained from the SOCE model cavity shown in Fig. 3. Some pertinent characteristics of this model are listed in Table I.

TABLE I. SOCE Model Cavity Parameters

Stub height	34 7/8 in.
Stub width	47 in.
Drift tube diameter	6 in.
Beam hole dimensions	1 3/4 x 3 1/4 in.
Accel. gap length	0.2 $\beta\lambda$
Drift tube length	0.18 $\beta\lambda$
Q (computed)	22,000
Q (measured)	20,500
Shunt resistance (computed)	0.87 MΩ
Shunt resistance (from measurement)	0.785 MΩ

This model differs considerably from the 50-MeV cavity due to the presence of the drift tubes. Design of the model was greatly complicated by the drift tubes since there was no simple method for calculating the effective capacitance between stub end and ground. A cut-and-try procedure was adopted and the model was constructed based on a rough estimate of the capacitance. The first trial resulted in a frequency error of about 20%. From this data an equivalent end capacitance could be calculated for the model, and the stub height corrected. The second trial resulted in the desired resonant frequency being well centered in the model tuning range. The large plate above the center drift tubes in Fig. 3 provides a tuning range of about ±1.6 percent.

The electric field in the accelerating gaps of the SOCE model was again determined by perturbation techniques. To reduce errors due to frequency drift the "bead-pulling" equipment was partially automated. The cavity is operated as the tank circuit of an oscillator and a signal from the cavity is mixed with the output of a stable oscillator to produce an output signal having a frequency of a few kilohertz, which is fed into a frequency meter/discriminator. The analog output of the frequency meter provides the y-input to an x-y recorder whose x-input is proportional to the bead's position in the cavity. This arrangement permits a scan through one beam passage of the cavity in a few seconds thereby minimizing errors from drift. A typical field distribution along one beam passage is shown in Fig. 7, and the relative voltage of the four drift tubes is plotted in Fig. 8.

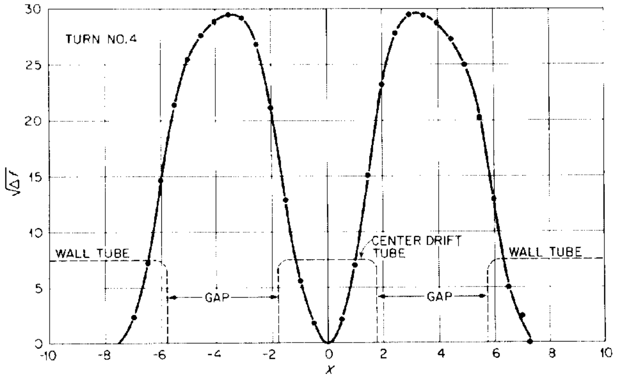

Fig. 7 Relative electric field along beam passage in SOCE model.

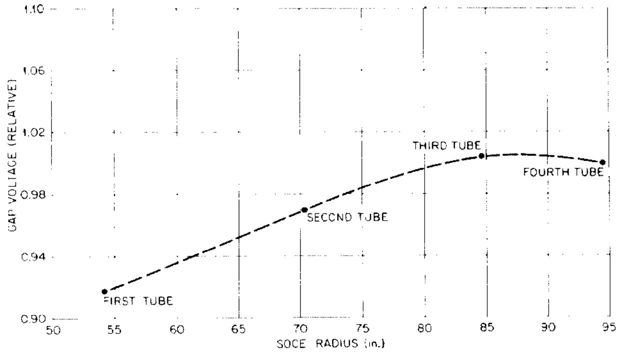

Fig. 8 Relative drift tube voltages in SOCE model.

Conclusion

The use of coaxial cavities rather than TM cavities in SOC's can result in smaller and more economical machines in the low energy range. Coaxial cavities are useful in multi-particle SOC's as well as single particle machines. In large coaxial cavities having stub widths comparable to a $\lambda/2$, TE modes may be excited to produce a variation of accelerating voltage with machine radius. If the maximum voltage occurs at maximum orbit radius the TE modes can be an advantage since the rf power loss in the cavity can be decreased from that resulting from a pure TEM mode.

Model tests have demonstrated the applicability of coaxial cavities to both the 50-MeV SOC and to the 4-MeV SOCE.

References

1. N. F. Ziegler, IEEE Trans. on Nuclear Science, NS-12, 128 (June 1965).

2. R. E. Worsham, E. D. Hudson, R. S. Livingston, J. E. Mann, J. A. Martin, S. W. Mosko, and N. F. Ziegler, "A 4-MeV Experimental SOC."