DESIGN OF TRAVELLING WAVE ELECTRON LINEAR ACCELERATORS

W. J. Gallagher
Applied Radiation Corporation
Walnut Creek, California 94596

Introduction

It is well-known that the shunt impedance per unit length, r, the attenuation coefficient per unit length, I, and the energy velocity (or group velocity), V_g, are sufficient to completely describe the travelling wave interaction between a beam and the wave in an acceleration waveguide. The choice of these parameters is usually made assuming a steady-state, synchronous interaction where a specified input power, P_o, is available to accelerate a current i to an energy V. The parameters of the waveguide may, in general, vary along the length of the structure. In particular, two such structures are widely used, the constant impedance (or uniform) structure and the constant gradient waveguide. Only these two waveguides are considered in the following discussion.

The energy gain equation, characteristic of the interaction which occurs in a linac, is a quadratic function in terms of dimensionless parameters of the waveguide. In the constant impedance structure, for example,

$$\frac{V}{E_oL} = \frac{1-e^{-IL}}{IL} - \frac{\eta}{2IL}\,\frac{E_oL}{V}\left(1-\frac{1-e^{-IL}}{IL}\right) \tag{1}$$

where $E_o^2 = 2IP_or$ and $\eta = iV/P_o$ is the beam power conversion efficiency. Similarly, for the constant gradient structure,

$$\frac{V}{E_oL} = 1 - \frac{E_oL}{V}\,\eta\left(\frac{2I_oL-(1-2I_oL)\ln(1-2I_oL)}{4I_oL}\right) \tag{2}$$

where the subscript indicates initial values of the parameter. These expressions are illustrated in Figures 1 and 2.

The use of these universal energy gain diagrams can be illustrated by an exemplary problem. Suppose that it is required to find a constant gradient disc-loaded waveguide at 1300 mcs which will produce 12 MeV at 200 ma loading with 4 MW input power. The beam power conversion efficiency is, therefore, $\eta = 0.60$. Since the disc-loaded waveguide will exhibit a shunt impedance of about 37 megohms per meter at 1300 mcs, we note that $E_o = 17.2\sqrt{I_o}$ MV/m and $V/E_oL = 0.697/\sqrt{I_o}L$. Thus, we may make a tabulation of the intersection of values of V/E_oL, η and $2I_oL$. For a specific value of V/E_oL values of $\sqrt{I_o}L$ and I_oL are obtained, and hence I_o and L. One can then scan the tabulation eliminating unrealizable or undesirable cases. In the example above $V/E_oL = 0.74$, $2I_oL = 0.60$, $I_o = 0.102$, $L = 2.94$ m appears particularly appropriate.

Constant Impedance Waveguide

It is possible to describe certain optimum accelerator designs which may be included on the above diagram. In the case of constant impedance waveguide, the energy gain may be written for the synchronous, steady state condition, in the form

$$V = E_o\frac{1-e^{-IL}}{I} - irL\left(1-\frac{1-e^{-IL}}{IL}\right) \tag{3}$$

Maximum Energy Gain

If we maximize the energy gain with respect to the choice of attenuation constant we find the condition,

$$\frac{i^2r}{P_o} = \frac{I}{2}\left[\frac{(e^{IL}-1)-2IL}{IL-(e^{IL}-1)}\right]^2 \tag{4}$$

Re-inserting this condition in the energy gain equation we obtain the beam power conversion efficiency,

$$\frac{iV_m}{P_o} = (1-e^{-IL})\frac{(e^{IL}-1)-2IL}{IL-(e^{IL}-1)} - \frac{IL}{2}\left[\frac{(e^{IL}-1)-2IL}{IL-(e^{IL}-1)}\right]^2\left(1-\frac{1-e^{-IL}}{IL}\right) \tag{5}$$

If we re-insert the expression for the beam current given by Eq. (4) into the energy gain equation and factor out the no-load energy gain, we obtain,

$$\frac{V_m}{V_o} = 1 + \frac{1-e^{-IL}-IL}{2(1-e^{-IL})}\left[\frac{(e^{IL}-1)-2IL}{IL-(e^{IL}-1)}\right] \tag{6}$$

These conditions describe a <u>maximum energy gain</u> accelerator; there is no waveguide providing the specified efficiency that will produce a higher beam energy than the one having the attenuation length indicated. The case of vanishing beam current ($IL = 1.255$) was pointed out by R.B.S. Harvie.[1] By means of either Eq. (5) or Eq. (6) this function may be shown on the universal diagram, Figure 1.

Maximum Beam Power

Suppose now that we maximize the energy gain with respect tothe shunt impedance of the structure. The condition is obtained that

$$\frac{i^2r}{P_o} = \frac{I}{2}\left[\frac{1-e^{-IL}}{IL-(1-e^{-IL})}\right] \tag{7}$$

Re-inserting this expression in the energy gain equation, we find

$$\frac{V_p}{V_o} = \frac{1}{2} \tag{8}$$

and

$$\frac{i V_p}{P_o} = \frac{1}{2} \frac{(1-e^{-IL})^2}{IL-(1-e^{-IL})} \tag{9}$$

These conditions describe a maximum beam power (or maximum efficiency) accelerator; there is no waveguide which will provide the specified conversion efficiency and produce a greater beam power than at the indicated value of attenuation length.

This design has been described by Saxon[2] and Leiss[3] but their method is the maximuzation of the beam power, iV, with respect the value of beam current. The condition is precisely the same as that given above. By means of either Eq. (8) or Eq. (9) this function may be shown on the diagram of Figure 1.

Maximum X-Ray Output

It is an experimental observation that below 10 MeV the forward x-ray intensity per unit beam current varies nearly as the cube of the beam energy. (While above 10 MeV it is generally observed that the dependency upon energy decreases, there is some evidence to suspect that this is due to the use of thin targets and, in fact, x-ray production can be considerably larger than is commonly obtained.) If we, nevertheless, maximize the product iV^3 with respect to the beam current to obtain the great x-ray output we will obtain the condition

$$\frac{i^2 r}{P_o} = \frac{I}{8}\left[\frac{1-e^{-IL}}{IL-(1-e^{-IL})}\right]^2 \tag{10}$$

Re-inserting this condition into the energy gain equation we find

$$\frac{V_x}{V_o} = \frac{3}{4} \tag{11}$$

and a machine efficiency

$$\frac{i V_x}{P_o} = \frac{3}{8} \frac{(1-e^{-IL})^2}{IL-(1-e^{-IL})} \tag{12}$$

which is also shown in Figure 1.

It is curious to note that N.C.Chang discovered the intersection of the maximum energy gain curve and the maximum x-ray production curve intuitively.[4]

Zero Power Condition

It is, further, of some interest to determine the point at which zero power occurs in the waveguide. This condition,

$$\frac{i^2 r}{2IP_o} = \frac{e^{-2IL}}{(1-e^{-IL})^2} \tag{13}$$

is also shown on Figure 1.

Constant Gradient Waveguide

It would be repetitious to reproduce a similar discussion for the constant gradient waveguide; the results are therefore somewhat summarized below.

The energy gain equation for this case may be written

$$V = E_o L - \frac{ir}{4I_o}\left[2I_oL-(1-2I_oL)\ln(1-2I_oL)\right] \tag{14}$$

Maximum Energy Gain

Maximizing this expression with respect to the attenuation coefficient I_o,

$$\frac{i^2 r}{P_o} = 2I_o\left[\frac{2I_oL}{2I_oL+\ln(1-2I_oL)}\right]^2 \tag{15}$$

Re-inserting this condition into Eq. (14) we have for the conversion efficiency

$$\frac{iV_m}{P_o} = -\frac{2I_oL}{2}\,\frac{3(2I_oL)+(3-2I_oL)\ln(1-2I_oL)}{[2I_oL+\ln(1-2I_oL)]^2} \tag{16}$$

Maximum Beam Power

Maximizing the energy gain with respect to the shunt impedance,

$$\frac{i^2 r}{P_o} = 2I_o\left[\frac{2I_oL}{2I_oL+(1-2I_oL)\ln(1-2I_oL)}\right]^2 \tag{17}$$

Re-inserting this condition in Eq. (14), and solving for the beam power conversion efficiency,

$$\frac{iV_p}{P_o} = \frac{(2I_oL)^2}{2[2I_oL+(1-2I_oL)\ln(1-2I_oL)]} \tag{18}$$

When the condition of Eq. (17) is inserted into the energy gain equation in the form Eq. (2) one finds

$$\frac{V_p}{E_oL} = \frac{1}{2} \tag{19}$$

Maximum X-Ray Output

Maximizing the expression for iV^3 with respect to the beam current i,

$$\frac{i^2 r}{P_o} = \frac{I_o}{2}\left[\frac{2I_oL}{2I_oL+(1-2I_oL)\ln(1-2I_oL)}\right]^2 \tag{20}$$

Re-inserting this condition into Eq. (14), the conversion efficiency

$$\frac{iV_x}{P_0} = \frac{3}{8}\,\frac{(2I_0L)^2}{2I_0L + (1-2I_0L)\ln(1-2I_0L)} \tag{21}$$

When the condition Eq. (19) is inserted into the energy gain equation in the form Eq. (2) one finds

$$\frac{V_x}{E_0L} = \frac{3}{4} \tag{22}$$

Zero Power Condition

In the constant gradient waveguide the input power will vanish, due to attenuation and beam loading, when

$$\frac{i^2r}{P_0} = \frac{8I_0}{[\ln(1-2I_0L)]^2} \tag{23}$$

so that the beam power conversion efficiency will become at that condition,

$$\frac{iV}{P_0} = 2\,\frac{(4I_0L-1)\ln(1-2I_0L) - 2I_0L}{[\ln(1-2I_0L)]^2} \tag{24}$$

The above expressions, Eqs. (16), (18), (21) and (24) are shown on the dimensionless energy gain diagram, Figure 2.

The writer would like to acknowledge the assistance of Jane Benson Silldorff in preparing these calculations.

Literature Cited

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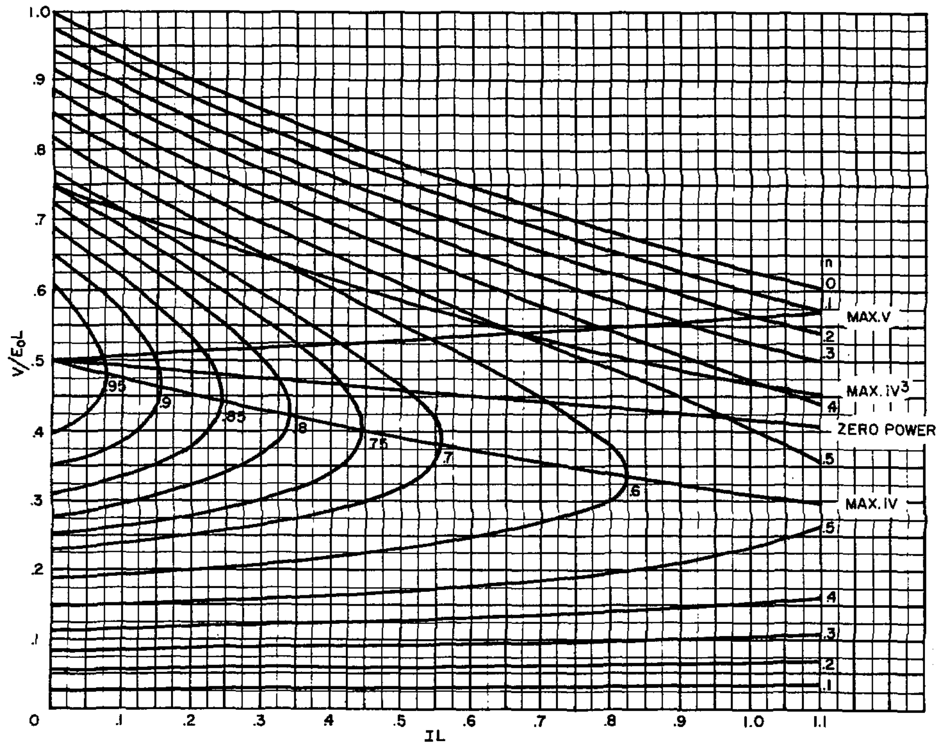
MAX.V
MAX.IV^3
ZERO POWER
MAX.IV
V/E_0L
IL

Fig. 1.

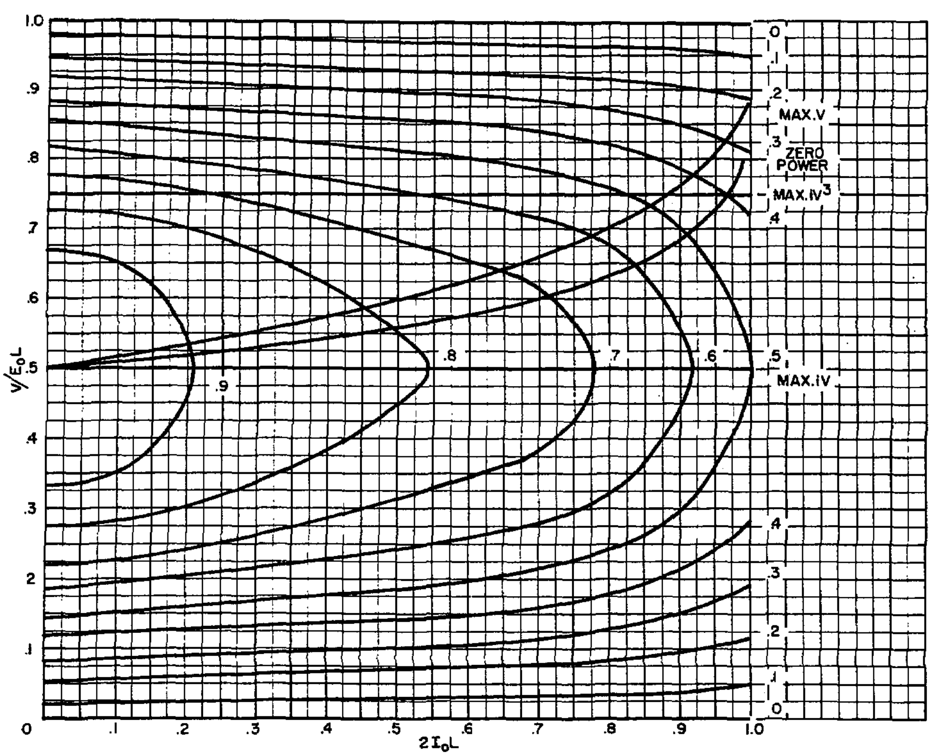
MAX.V
ZERO POWER
MAX.IV^3
MAX.IV
V/E_0L
2I_0L

Fig. 2.