THE RF TRAVELLING WAVE PARTICLE DEFLECTOR

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Summary. As a result of interest in mass separators stimulated by Blewett, Panofsky and others, the writer suggested use of a travelling wave interaction as an improvement on existing cavity deflectors then under study at Stanford by Philips. The specific structure proposed was the use of the TM11-like mode in disc-loaded waveguide. This configuration was subsequently adopted by CERN, BNL, ENS and SLAC, where projects to demonstrate particle deflection are in progress. The principle on which the separator functions is the production of a deflecting force, resulting from field components of a wave travelling synchronously with the particle beam. A beam transport system, appropriate to the experimental arrangement, permits observation of the differentiated particles. It can be shown that the motion of a particle interacting with a propagating wave at the velocity of light is characterized by uniform, aberrationless deflection over the beam aperture. The experimentally determined properties of such a waveguide (transverse shunt impedance, group velocity and solutions of the determinantal equation) are presented for the π/2-mode of propagation. A measurement technique for determining the shunt impedance, based on a perturbation method, is also discussed.

Theory of Deflector

The rf particle deflector is a result of the interest in rf particle separators stimulated by Blewett, Panofsky and others, the writer suggested use of a travelling wave interaction as an improvement on the existing cavity deflector then under study at Stanford HEPL by P. R. Philips[1,2]. The specific structure proposed was the use of the TM_{11}-like mode in the disc-loaded circular waveguide. This configuration was subsequently adopted by CERN, BNL, ENS and SLAC, where projects to demonstrate particle deflection are in progress. A rather complete bibliography of reports covering this work is given in the reference[3].

The principle on which the separator functions is the production of a deflecting force resulting from the field components of a wave travelling synchronously with the particle beam. A beam transport system appropriate to the experimental arrangement permits observation of differentiated particles. In the following discussion we are only concerned with the particle deflection structure.

The deflecting structure specifically to be considered is the disc-loaded circular waveguide supporting the TM_{11}-like mode (also designated HEM_{11} mode), for which the field components in the aperture ($r<a$) of the fundamental space harmonic at the phase velocity of light are, the neglecting attenuation,[4]

$$
\begin{aligned}
E_z &= E_0 kr \cos\varphi \cos(\omega t-\beta z)\\
E_r &= E_0\left[\left(\tfrac{kr}{2}\right)^2+\left(\tfrac{ka}{2}\right)^2\right]\cos\varphi \sin(\omega t-\beta z)\\
E_\varphi &= E_0\left[\left(\tfrac{kr}{2}\right)^2-\left(\tfrac{ka}{2}\right)^2\right]\sin\varphi \sin(\omega t-\beta z)\\
\eta H_z &= -E_0 kr \sin\varphi \cos(\omega t-\beta z)\\
\eta H_r &= -E_0\left[\left(\tfrac{kr}{2}\right)^2-\left(\tfrac{ka}{2}\right)^2+1\right]\sin\varphi \sin(\omega t-\beta z)\\
\eta H_\varphi &= E_0\left[\left(\tfrac{kr}{2}\right)^2+\left(\tfrac{ka}{2}\right)^2-1\right]\cos\varphi \sin(\omega t-\beta z)
\end{aligned}
\tag{1}
$$

The equations of motion in cylindrical coordinates for a charged particle interacting with an electromagnetic field are

$$
\begin{aligned}
&\frac{d}{dt}(\gamma\dot z) = \frac{e}{m_0}\left(E_z+\dot r B_\varphi - r\dot\varphi B_r\right)\\
&\frac{d}{dt}(\gamma\dot r)-\gamma r\dot\varphi^2 = \frac{e}{m_0}\left(E_r + r\dot\varphi B_z - \dot z B_\varphi\right)\\
&\frac{1}{r}\frac{d}{dt}(\gamma r^2\dot\varphi) = \frac{e}{m_0}\left(E_\varphi + \dot z B_r - \dot r B_z\right)
\end{aligned}
\tag{2}
$$

Thus, a highly relativistic particle ($\dot z = c$) synchronized with the transverse electric field ($E_z = H_z = 0$) is governed by the equations of motion,

$$\frac{d}{dt}(\gamma\dot z) = 0 \tag{3a}$$

$$\frac{d}{dt}(\gamma\dot r) - \gamma r\dot\varphi^2 = \frac{eE_0}{m_0}\cos\varphi \tag{3b}$$

$$\frac{d}{dt}(\gamma r^2\dot\varphi) = -\frac{eE_0}{m_0}\sin\varphi \tag{3c}$$

Several generalities of the motion are at once apparent: (1) the transverse force is constant over the aperture and in the direction of the axis of symmetry of the radial component of the field ($\varphi=0$); (2) to the approximation assumed there is no longitudinal energy gain (and therefore the deflector is free of aberrations over the aperture).

The solution of Eqs (3b) and (3c) is

$$r\cos\varphi = \frac{eE_0 t^2}{\gamma m_0 2} + r_0\cos\varphi_0$$

where r_0 and φ_0 are initial conditions ($t = 0$). Obviously the solution to Eq (3a) is $z = ct$, within the degree of approximation undertaken, although for this equation the result is actually inconsistent with the field equations.

Experimental Results

The experimental results reported here are based upon cold-test measurements obtained on a resonant length of the transmission line under study. The general appearance of the HEM_{11} mode (or TM_{11}-like mode) in the $\pi/2$ mode is as shown in Figure 1 for two instances $\pi/4$ radians apart. It is apparent that shorting planes may be placed in reflection planes of symmetry to obtain resonances on the dispersion diagram. By this method the dimensions necessary to obtain velocity of light propagation at the specified frequency (in these experiments, 2856 mcs) were determined, Figure 2. The group velocity was determined from the slope of the dispersion diagram at the velocity of light, Figure 3.

Of interest in a deflector is the transverse force which can be developed for a given amount of input power. Ordinarily, in a travelling wave structure a shunt impedance is defined as the square of the electric line integral per unit length for an observer moving synchronously with the wave crest divided by the power spent per unit length to maintain the field, ie

$$r = \frac{\left(\int E\, e^{j(\omega t-\beta z)} dz\right)^2}{dP/dz} = \frac{E^2}{2IP} \qquad \left(z = \frac{\omega t}{\beta}\right)$$

Introducing the concept of the Q of a transmission line, that is, the ratio of stored energy per unit length (W) to the energy lost per unit length per radian $(\omega = 2I v_g Q)$, it can be seen that we may put

$$\frac{r}{Q} = \frac{E^2}{\omega W}$$

This turns out to be particularly convenient since the r/Q can be determined by simple perturbation techniques, and the series impedance is calculable in the form,

$$\frac{E}{\sqrt{P}} = \sqrt{2Ir} = \sqrt{\frac{\omega}{v_g}\frac{r}{Q}}$$

In the case of the HEM_{11}-mode these concepts can readily be extended with certain appropriate modifications to the radial electric field, if we define the radial r/Q in terms of the radial electric field on the axis as seen by a synchronous observer travelling with the wave crest,

$$\left(\frac{r}{Q}\right)_T = \frac{E_r^2}{\omega W} \tag{4}$$

This parameter can conveniently be measured by means of a dielectric rod inserted through the cavity in the plane of the disc (when the radial field standing wave is maximum at the disc).

The perturbation produced on the resonant frequency $\frac{\Delta f}{f} = -\frac{\Delta W}{W} = \frac{\frac{1}{2}\int_0^a E_{rc}^2\, dr\, (\epsilon-\epsilon_0)\pi r_0^2}{W_c}$

Note that the time average of the energy perturbation produces the frequency perturbation, and that the fields of the resonant cavity must be used. Evidently the energy stored in the cavity $W_c = 2WL$ where L is the length of the cavity. Eliminating the stored energy density between this expression and Eq (4) we have

$$\left(\frac{r}{Q}\right)_T = \frac{120\,\lambda L}{(\epsilon-1)\pi r_0^2 4}\,\frac{\Delta f}{f}\left(\frac{E_r^2}{\Sigma E_{rn}^2}\right)\frac{1}{\int E_r^2/E_{r0}^2\, dr}$$

where we have put $\pi\epsilon_0 = 1/120c$ and have noted that the fraction of the power associated with the synchronous space harmonic is given by $E_r^2/\Sigma E_{rn}^2$

The integral $\int_0^a E_r^2/E_{r0}^2\, dr$ is easily evaluated by drawing a dielectric bead radially across the disc aperture, noting the perturbation. The field intensity in the aperture is remarkably constant across most of the diameter, increas- rapidly near the edge. The experimental value of the integral is closely given by the theoretical value, (14/15)a.

The fraction $E_r^2/\Sigma E_{rn}^2$ is obtained by drawing a dielectric bead axially along the cavity, noting the perturbation. The plot obtained is a measure of the (time-average) square of the radial electric field intensity. A spatial Fourier analysis of the field intensity, using propagation constants permitted by the periodicity of the cavity, assigns relative amplitude coefficients to the space harmonics. The value of this fraction is necessarily, of course, less than unity and has, except near the case of zero group velocity, been determined experimentally to be nearly unity in most cases. The cause of zero group velocity within the pass-band is evidently because alternate space harmonics have equal amplitudes; hence very near the propagation stop this fraction should approach a value of about one-half.

One observes now, however, that the transverse force on a particle is not entirely due to the radial electric field in the guide. From the solution of the equations of motion given above we concluded that the transverse force is $F_T = eE_0$, where E_0 is defined by the power flow in the waveguide.

Thus the transverse force due to an "effective transverse electric field" is to be calculated from the r/Q obtained from radial electric field measurements, according to the relation

$$\left(\frac{r}{Q}\right)_e = \left(\frac{r}{Q}\right)_T \left(\frac{2}{ka}\right)^4$$

It can be shown in a straightforward calculation (using the above field components) that the time-average value of the Poynting vector integrated over the disc aperture indicates that the power flow and field strength are given by the relation

$$P_z = \frac{E_0^2\pi}{7}\left(\frac{ka}{2}\right)^2 \frac{a^2}{2}\left[\frac{4}{3}\left(\frac{ka}{2}\right)^2 - 1\right]$$

Parenthetically, one notes that we have here a theoretical expression for the "effective transverse electric field". One also observes that a zero of the group velocity can be expected when $ka = \sqrt{3}$.

In Figure 4 we present the experimental results of r/Q measurements made as indicated above.

The writer gratefully acknowledges the assistance of Mr. H. De Ruyter in obtaining the experimental data reported herein.

References

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2. Linear Electron Accelerator Studies, Status Report 1 Oct-31 Dec, 1958. Microwave Lab. Report No. 581, Stanford University (1959)
3. O. A. Altenmuller, et al., Investigations of Travelling-Wave Separators for the Stanford Two-Mile Linear Accelerator, SLAC Report No. 17 August (1963)
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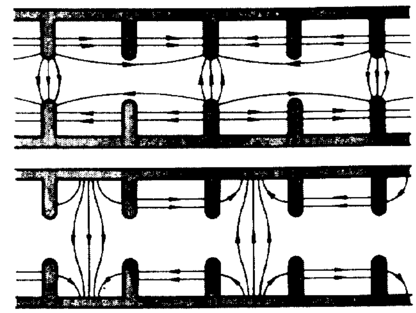

Figure 1.

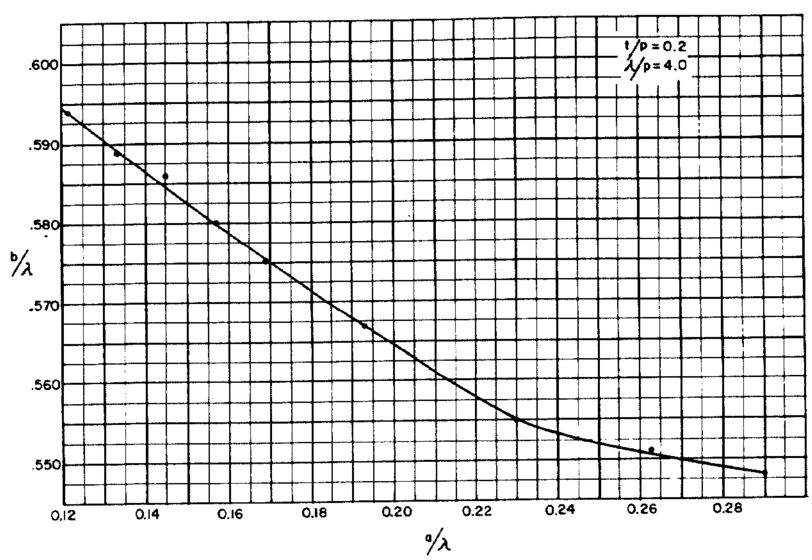

Figure 2.

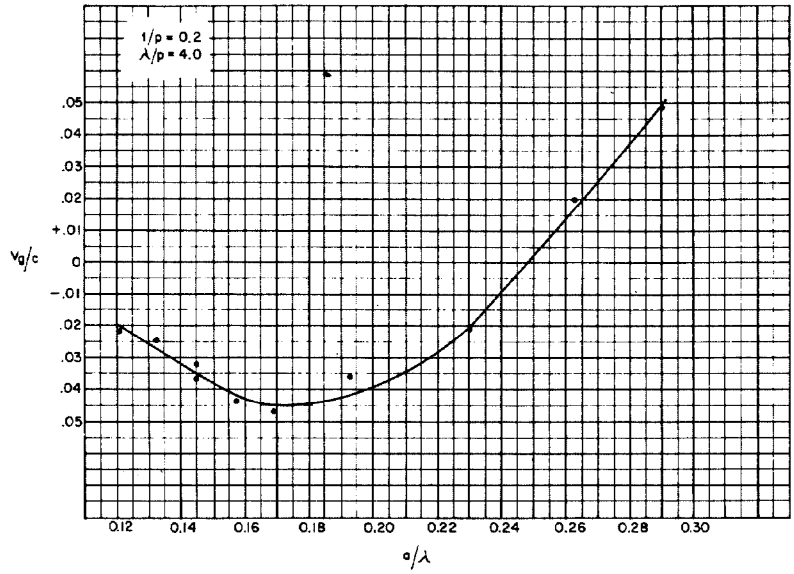

Figure 3.

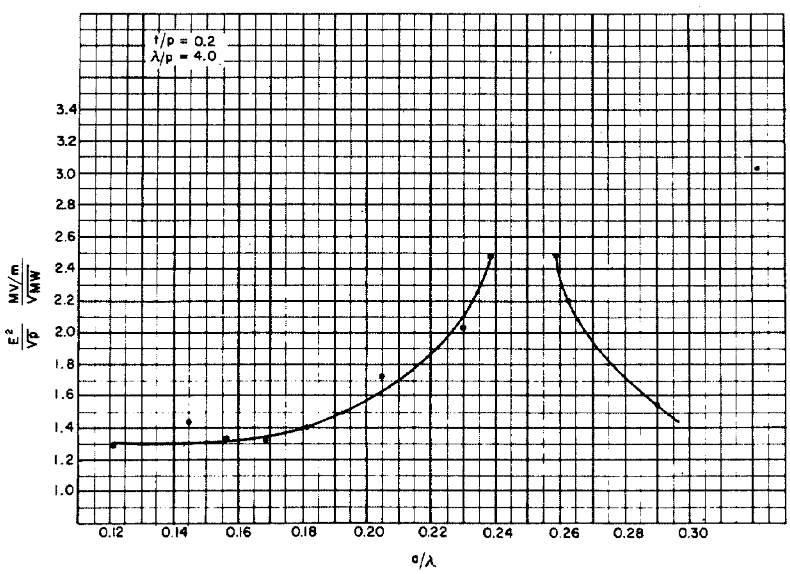

Figure 4.