

ELECTROSTATIC INDUCTION ELECTRODE SYSTEMS FOR BEAM-POSITION DETECTION

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Abstract

Using the integral solution for the field and the reciprocity of the Green's Function, we show that the response of these devices to a line charge obeys Laplace's equation, if this operator is applied with respect to the source coordinates.

In a cylindrical system of electrodes the boundary values of the response can be determined by inspection. Knowing these values, one can easily find the response over the entire aperture. This allows the use of electrodes of unusual shapes, e. g., sections of the beam tube.

There exists a requirement for a device that will detect a transverse coordinate center of mass of a particle beam; i.e., a device that has a response

$$R = \int_V k \times \rho(v) dV. \quad (1)$$

One standard answer to this problem is the split (or two-electrode) induction electrode; an example of this device is shown in Fig. 1.

It is clear that this is essentially a capacity pickup device, and works by detecting the electric field of the beam. It's output can be taken as either the sum or difference of some measurable variable, measured at the detection point (e. g., current or voltage).

An understanding of the mechanism involved can be gained by idealizing the situation. First, assume that the beam is a line charge with no variation in z, and that the time variation can be separated in the form

$$U(x, y) e^{j\omega t}.$$

Further suppose that the device is small enough so that the quasi-static approximation applies. All components are conductors and form a cylindrical system with the exception of the separating cuts, which are imagined as being infinite-

simally thin. (The term "cylindrical system" is used in a general sense; a cylindrical surface is defined as a surface generated by the movement of a straight line, parallel to a fixed line, around any closed curve.) Measurements are to be made by grounding the detection points and measuring any charge that flows through them to the electrodes.

With these assumptions, it is seen that the field has no z dependence, or the problem has been reduced to two dimensions. The charge acts as a source for an electric field, which terminates on the metal walls of the system, where a surface charge density is induced. The charge measured as having flowed through a particular detection point is the integral of this charge density over the appropriate surface.

Although it is clear that the sum of the two charges induced on the electrodes is equal to the charge of the enclosed beam, it is not clear what their difference measures. However, there is still one degree of freedom, the shape of the cuts; and we might hope to select the cuts so that the difference is the desired response.

Examining the geometry more closely, one sees that as the beam approaches the boundary, the charge difference becomes proportional to the lengths of the electrodes in that region (see Fig. 2). It seems that the boundary values of the response are determined; we would like a differential equation.

This can be obtained by proceeding more formally, and writing the integral solution for the field, which because of the quasi-static assumption is governed by Poisson's equation,

$$\nabla^2 \phi(r) = \rho(r).$$

$$\text{Thus } \phi(r) = \int_{V(R)} G(r, R) \rho(R) dV(R). \quad (2)$$

Here,  $\Phi(r)$  = potential at any point inside the boundary, with the electric field  $\vec{E}$  given by  $E = -\nabla \Phi$ ,

and  $G(r, R)$  = The Green's function for the problem,

$\rho(R)$  = the charge density,

Here  $\mathbf{r}$  = a field point,  
and  $\mathbf{R}$  = a source point.

The Green's function is in general determined by the shape of the boundaries, and the boundary conditions. It has a simple physical interpretation in this problem; it is the field due to a line charge. This is seen by taking

$\rho(\mathbf{R}) = \delta(\mathbf{R} - \mathbf{R}_0)$ ,  $\delta(\mathbf{x})$  = Dirac delta function,  
and thus, from (2),

$$\phi(\mathbf{r}) = G(\mathbf{r}, \mathbf{R}_0).$$

Two properties of  $G$  allow us to solve this problem:

$$\nabla_{\mathbf{r}}^2 G(\mathbf{r}, \mathbf{R}) = 0 \quad \text{if } \mathbf{r} \neq \mathbf{R},$$

and  $G(\mathbf{r}, \mathbf{R}) = G(\mathbf{R}, \mathbf{r})$  (reciprocity).

Here the subscript  $\mathbf{r}$  on  $\nabla_{\mathbf{r}}^2$  means "differentiate with respect to the coordinate  $\mathbf{r}$ ."

Combining the last two equations, one has

$$\nabla_{\mathbf{R}}^2 G(\mathbf{r}, \mathbf{R}) = 0 \quad \text{if } \mathbf{r} \neq \mathbf{R}.$$

This almost solves the problem, since any linear operation with respect to  $\mathbf{r}$  doesn't change this result. In particular, writing the total charge difference due to a line source, we have

$$Q_{\Delta} = \int_{S_1} \frac{\partial \phi}{\partial n(\mathbf{r})} dS_1(\mathbf{r}) - \int_{S_2} \frac{\partial \phi}{\partial n(\mathbf{r})} dS_2(\mathbf{r}),$$

where we have used  $q_s$  = surface charge density =  $\mathbf{D} \cdot \mathbf{n}$ , and  $\mathbf{E} = \mathbf{D}$ , and

$$Q_{\Delta} = \int_s q_s dS.$$

We have

$$Q_{\Delta} = \int_{S_1} \frac{\partial G(\mathbf{r}, \mathbf{R})}{\partial n} dS_1(\mathbf{r}) - \int_{S_2} \frac{\partial G(\mathbf{r}, \mathbf{R})}{\partial n} dS_2(\mathbf{r}),$$

and still

$$\nabla_{\mathbf{R}}^2 Q_{\Delta} = 0.$$

Now find the charge induced by any source distribution:

$$Q = \int_{S_1} \frac{\partial}{\partial n} \left[ \int_{V(\mathbf{R})} G(\mathbf{r}, \mathbf{R}) \rho(\mathbf{R}) dV(\mathbf{R}) \right] dS_1(\mathbf{r}) \\ - \int_{S_2} \frac{\partial}{\partial n} \left[ \int_{V(\mathbf{R})} G(\mathbf{r}, \mathbf{R}) \rho(\mathbf{R}) dV(\mathbf{R}) \right] dS_2.$$

We find, by interchanging derivatives and integrals,

$$Q = \int_{V(\mathbf{R})} Q_{\Delta} \rho(\mathbf{R}) dV(\mathbf{R}).$$

Since knowing the boundary values of a solution to Laplace's equation determines the solution uniquely,  $Q_{\Delta}$  is thus specified for all values of  $\mathbf{R}$ . In particular, the kernel of Eq. 1,

$$Q_{\Delta}(\mathbf{R}) = kx,$$

satisfies

$$\nabla_{\mathbf{R}}^2 Q_{\Delta} = 0.$$

Examining Fig. 1, and imagining the line source very close to the wall at various positions, one sees that to obtain this kernel, the cuts should be a set of intersections of vertical planes with the electrodes (see Fig. 3).

In practice, one would not ground the electrodes; but the principle of superposition applies, and it is clear that the equivalent circuit of this system consists of two charge (or current) sources driving three capacitors in the configuration of Fig. 4.

The implications of these results of course go beyond the example used. Since nothing was said in the derivation about the shape of the boundary, the derivation can be extended to two more general classes: (a) cylindrical systems, and (b) noncylindrical systems.

In the first class it is clear that a set of cuts as in Fig. 3 will give

$$Q_{\Delta} = kx.$$

This class includes, as a practical matter, any arbitrary beam tube (see Fig. 5).

In the second case, which might be considered if an offset electrode were required, no simple solution for the cut need exist. Any complicated analysis, however, need cover only the boundary region, and again as a practical matter, capacitive tuning devices, if adjusted for the boundary values, will not cause strange results elsewhere.

The stability property of solutions to Laplace's equation is a great aid in the design of an electrode system. That is, if the values of two solutions (say the desired solution and the real solution) at a boundary point differ by  $\epsilon$ , then the difference between these solutions inside the boundary is less than  $\epsilon$ . There are many formulas in the literature that make stronger statements than this, and, in general, describe the "averaging" nature of Laplace's equation. Using these ideas, one might, for example, estimate the effects of removing the "infinitesimally thin"

requirement on the cuts, and establish manufacturing tolerances.

Experimentally, these devices are usually tested with the use of thin wires, which are more readily available than true line charges. The drawback in the use of wires is that the charge distribution on the wire is affected by voltages on the electrodes (which are usually the measured quantities) and by uneven boundaries. Both these effects became more serious as the wire is moved close to a boundary, but can be minimized if cylindrical systems of electrodes are used, the wire voltage is made as high as possible, and the electrodes are loaded to make their voltages as low as practical. In an case, more reliance can be placed on the results when the wire is near the center of the tube, and, here at least, one might have a tendency to believe the theory rather than the experiment.

Glen Lambertson of this Laboratory has suggested another form of electrode that has the same  $Q_{\Delta}$  as the electrode discussed above, but seems to  $\Delta$  have several advantages. Referring to Fig. 6, one can see the correspondence in  $Q_{\Delta}$  and note the following differences:

1.  $C_{12}$  (Fig. 4) is considerably reduced.
2.  $Q_1$  and  $Q_2$  are not the same as in Fig. 4, and in fact are functions of the shape of the beam tube. It appears that the magnitudes of  $Q_1$  and  $Q_2$  are considerably reduced near  $x = 0$ , which is of course an advantage in measuring their difference.
3. The system is symmetrical in  $z$ , which should reduce any problems due to phase difference between the two currents derived from the induced charges.
4. The possibility of normalizing  $Q_{\Delta}$  has been lost. This problem would require the use of an auxiliary electrode. This is a minor problem if a system involving many such electrodes is contemplated.

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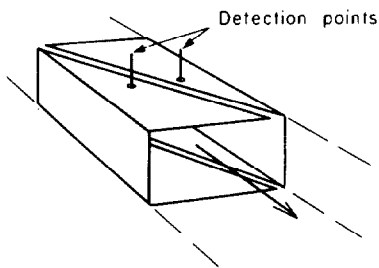


Fig. 1. Split induction electrode.

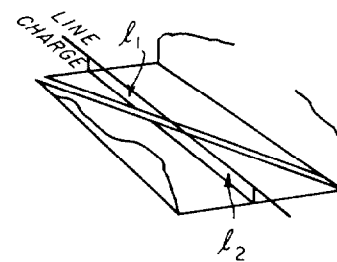


Fig. 2. Boundary conditions of response function.

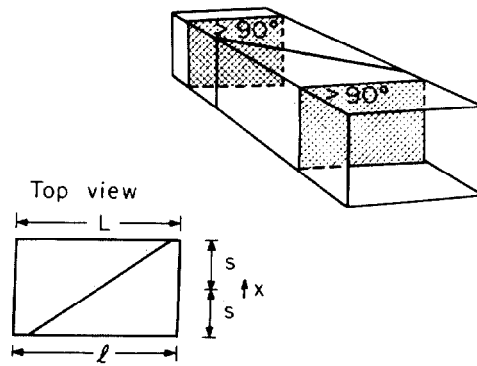


Fig. 3. Configuration to obtain  $Q_{\Delta} = kx$ .

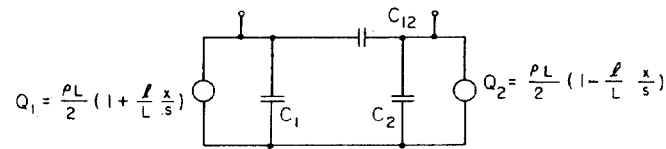


Fig. 4. Equivalent circuit of electrode system.

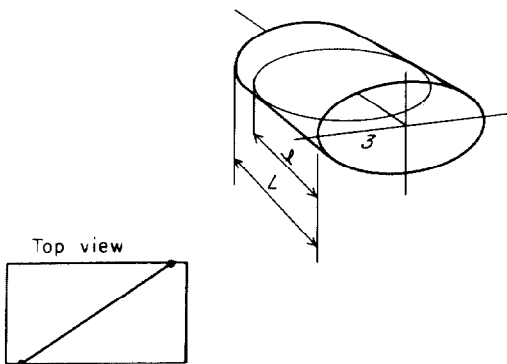


Fig. 5. A cylindrical electrode system formed from a beam tube.

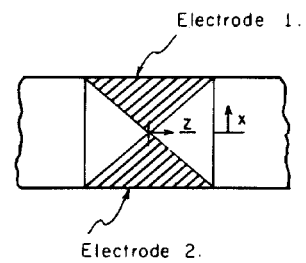


Fig. 6. Another form of a split induction electrode.