DESIGN OF A TRANSPORT SYSTEM FOR HIGH INTENSITY BEAMS

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Summary

A system has been designed which is calculated to be capable of transporting up to 50 mA of 3 MeV protons a distance of seventy-five feet. The beam is contained within a radius of one inch and its phase plot is optimally matched to the acceptance of the synchrotron. This phase matching is accomplished over a wide range of beam currents by varying the excitation of the final quadrupole doublet. A computer program written to calculate the dynamics of the beam is described and some of its characteristics are discussed.

Introduction

The present transport system for the 3 MeV injected beam at P. P. A. was designed without consideration of space charge effects. The major components of this system are shown in Figure 1. Its focusing elements, exclusive of the Van de Graaff, are a pair of electrostatic quadrupoles and an electrostatic deflector and inflector. The length of the system is seventy-five feet and it has to date transported a maximum of 3 mA to the point of injection.

In transporting substantially larger currents, for example 20 mA, it is necessary to provide additional quadrupole focusing elements in the system. This is due to larger space charge forces and also to an increase in ion source emittance at higher beam currents.[1] To determine the location and excitation of the quadrupoles a computer program was written to solve the equations of motion for a beam of protons under the influence of emittance, space charge and linear focusing elements. The program was then used to design a system for the P. P. A. calculated to be capable of transporting up to 50 mA of beam current with an emittance of 1.6 mrad-cm.

Computer Program

A computer program called "BEAM" was written to solve the differential equations governing the beam envelope. These equations, which have been derived by Kapchinskij and Vladimirskij[2] are applicable to a beam that is elliptical in cross section. If R_x and R_y are defined as the horizontal and vertical half-widths of the beam envelope and z is the coordinate along the beam axis, the differential equations relating these quantities are:

$$\frac{d^2R_x}{dz^2} = \frac{F^2}{R_x^3} + \frac{4Ie(1-v^2/c^2)}{mv^3(R_x + R_y)} + K_x^2R_x$$

$$\frac{d^2R_y}{dz^2} = \frac{F^2}{R_y^3} + \frac{4Ie(1-v^2/c^2)}{mv^3(R_x + R_y)} + K_y^2R_y$$

The emittance (transverse phase area/π) is designated by F and is assumed to be equal for both x and y phase plots. The quantities K_x and K_y describe the effect of linear focusing elements in the horizontal and vertical planes respectively. They are positive for diverging elements, negative for those which are converging, and zero for free space. The particle's mass, charge and velocity are given by m, e, and v, while I is the beam current and c is the speed of light. The charge density is assumed to be uniform over the beam cross-section and the phase distribution such as to maintain this uniformity. It is also assumed that the beam does not undergo acceleration. The second term on the right side of the equation describes the effect of space charge. It contains an approximation for the field of a uniformly charged elliptical cylinder. If the beam is circular R_x equals R_y and a single equation describes its dynamics. In the case of a beam which is elliptical in cross-section the space charge term couples the two equations and they must be solved simultaneously. Their solution is accomplished in "BEAM" by the Runge-Kutta Method.

In the output of the program tables of R_x, R_y, and $\dot{R}_x$, $\dot{R}_y$ (derivatives of R_x and R_y with respect to z) are printed as a function of z and graphs of R_x and R_y vs z are plotted. The programmer has three types of data to specify - the initial conditions, characteristics of the focusing elements and information concerning the form of output data. The initial conditions required are R and $\dot{R}$, the beam energy, current and emittance. One has to specify the position, length and excitation of each quadrupole. The program requires information as to how often it should print data and how large a graph should be produced. In addition to receiving the output in the form of a graph, the computer also indicates on each graph the position of every focusing element in the beam line. (See Figure 2) The program stores the input data it receives. If a new value of one of the parameters is inserted after the word "RESTART" the program will replace only this value in the stored data. The program will then run with the rest of the stored data unchanged. This convenience enables one to very readily study the dependence of the beam envelope on a particular parameter.

Design of Transport System

A transport system begins at some beam emitting device such as an ion source or a pre-accelerator. The boundary conditions on the beam, initial values of R and $\dot{R}$ and values of beam current and emittance, are determined by this device. These quantities are input data for the computer program and if they have not been measured, their values must be estimated in some way. For a given current the beam emittance may be estimated from measurements by A. van Steenbergen.[1] If the beam source produces a converging beam one may obtain initial values for R and $\dot{R}$ in the following manner: Since the beam converges it will pass through a minimum radius R_m at some distance z_m down the transport system. Values of R_m and z_m may be assigned, as long as they are within the capability of the beam source. At the point of minimum radius $R=R_m$ and $\dot{R}=o$. Since the differential equations are symmetric in z one may use these values and those for current and emittance as boundary conditions for "BEAM" and propagate the beam backward a distance z_m to its source. The quantities R and $\dot{R}$ calculated by the program at this point may then be used as the required boundary values at the beam source. The sign of $\dot{R}$ must be changed, however, since it is not symmetric in z.

Quadrupoles must be located at intervals along the transport system to compensate for the divergent effects of space charge. As mentioned earlier both the position and excitation of these quadrupoles must be given to the program as input data. If there are no other constraints on the location of the quadrupoles one may use the following method to determine their position. Using the initial conditions described above the program is run without quadrupoles to a large enough value of z so that the beam radius has increased beyond acceptable limits. A quadrupole doublet is then entered in the program at a value of z where the beam radius has reached whatever upper limit has been set. If no other limitations exist the beam diameter entering a quadrupole should not exceed half its aperture. If the beam is required to focus (a focus is defined as a point where R has a minimum) at some distance f past the doublet its excitation may be roughly estimated from the condition:

$$m_{21} = \frac{1}{f}$$

Where m_{21} is the 2, 1 element of the doublet transfer matrix. This assumes m_{11} to be unity and neglects space charge effects. For this estimate, both elements of the doublet may be taken to have equal excitations. If the doublet serves only to contain the beam one may choose f to be the distance from the focus preceeding the quadrupoles. The values of R and $\dot{R}$ for the matrix equation are obtained directly from the program output. This process of locating quadrupoles is continued until the beam line is completed. (Figure 3) In the beam line design the initial quadrupoles contain the beam within the desired radius, whereas the final elements are used to match the phase plot of the beam to the acceptance of the synchrotron.

Matching of Phase Areas

Once the beam line has been designed to contain the particles within the desired radius, it then becomes necessary to match the emittance area of the beam to the acceptance area of the synchrotron at the entrance of the machine. One can calculate the phase ellipse of the beam knowing R and $\dot{R}$ and the emittance. Assuming the synchrotron acceptance is limited by the maximum tolerable betatron oscillation, the phase area of the acceptance is an ellipse. With this assumption the horizontal acceptance of the machine has been calculated to be 0.55 mrad-cm. It was found that the final quadrupoles were sufficient to control the shape of the phase ellipse of the beam at the synchrotron entrance.

As a final section of the design, it was necessary to learn how critical the design of the transport system was to changes in current and emittance. The program was run for fixed currents and varied emittances, then for a fixed emittance and varied currents. The currents were changed from 10 to 25 mA and the emittance from 0.3 to 1.3 mrad-cm. Parameters of the transport system were held constant throughout these variations. The maximum beam diameter in the transport system was essentially unchanged for the cases tried. The effect of these variations was to change orientation of the phase ellipses at the synchrotron entrance. Figure 4 shows the phase plots at the synchrotron entrance for beams of 10 and 25 mA with an emittance of 0.8 mrad-cm. On the same graph is a plot of the synchrotron acceptance. In all cases reorientation of the phase plots for optimal phase matching could be accomplished by changes in excitation of the final quadrupole doublet.

References

1. A. van Steenbergen, Brookhaven National Laboratory, Accelerator Department Internal Report AADD-29, March, 1964.

2. I. M. Kapchinskij and V. V. Vladimirskij Conference on High-Energy Accelerators and Instrumentation, Session 3, 274, C. E. R. N. Geneva, 1959.

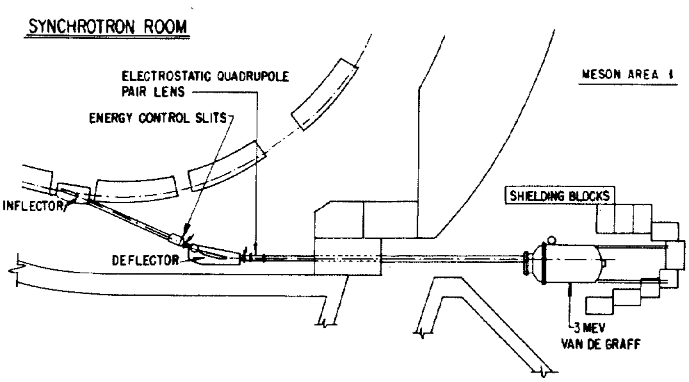

Fig. 1. Schematic of present injection beam line.

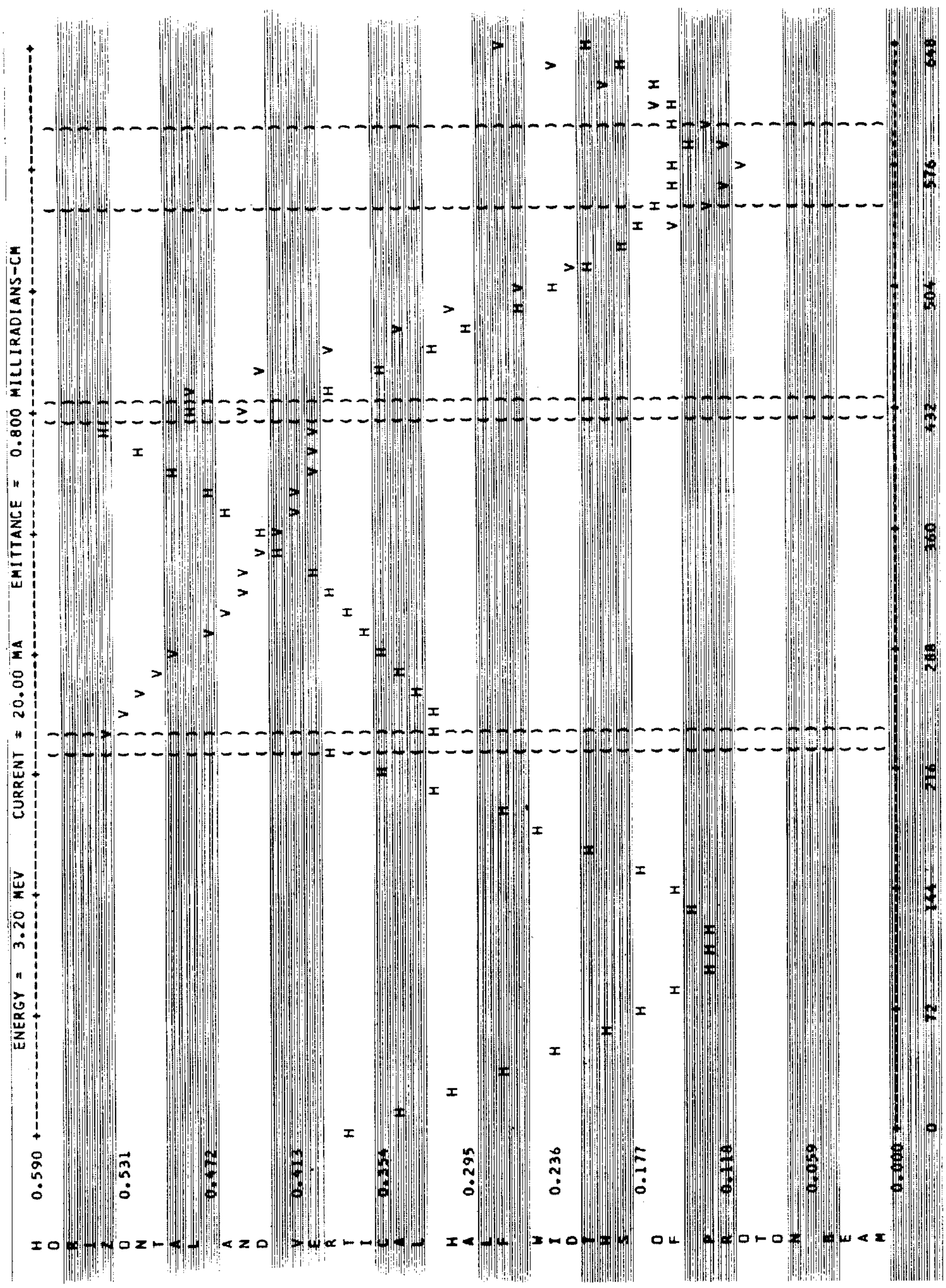

Fig. 2. Computer output with horizontal axis greatly compressed. ( ) symbol represents focusing element.

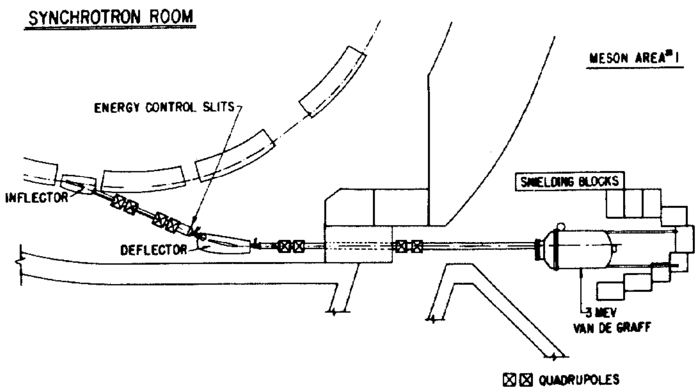

Fig. 3. Schematic of designed beam line showing position of quadrupoles.

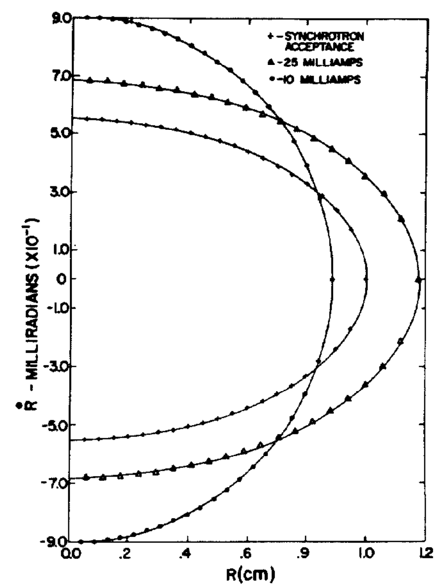

Fig. 4. Graph of the phase plots at synchrotron entrance, for 10 mA., 25 mA at .8 mrad.-cm and synchrotron acceptance of .55 mrad.-cm.