THE DESIGN OF HIGH-INTENSITY MUON CHANNELS FOR A MESON FACILITY*

H. S. Butler

University of California, Los Alamos Scientific Laboratory
Los Alamos, New Mexico

Summary

This paper describes a general computer program which calculates the pion acceptance and emittance spectra together with the muon emittance spectrum for a muon channel composed of any arbitrary arrangement of quadrupole magnets and drift spaces. The calculation procedure does not rely on the thin lens approximation nor does it make any assumptions about the length or periodicity of the structure or about the distribution of particles across the acceptance ellipse. This program was used to evaluate the effects on the pion acceptance and muon emittance spectra of varying the spacing, length and field intensity of the quadrupole magnets, of varying the initial drift distance from the target to the start of the channel and of turning off certain magnets in the channel. The results of these calculations are discussed as they relate to the design of muon channels for a high intensity meson facility.

Introduction

During the past eight years two muon channels have been designed, fabricated and brought into successful operation. Both channels consist of an extended array of quadrupole magnets arranged in a focusing-defocusing sequence as shown in Fig. 1. The channel parameters indicated in Fig. 1 were chosen at the two installations to satisfy different design criteria. At CERN[1] the initial motivation was high momentum (300 Mev/c) muon scattering experiments whereas the Chicago channel[2] was designed to produce low momentum (stopping) muons and to do so symmetrically for positive and negative muons. At both facilities the injected pions were produced at an internal target in a synchrocyclotron. An added feature of the Chicago system is a single quadrupole lens at the entrance to the channel. This magnet has twice the aperture of the other magnets in the channel and serves to match the synchrocyclotron to the channel in the vertical plane thereby enhancing the overall performance of the system.

Designing muon channels for a meson facility poses a more general problem because of the broad spectrum of experiments which will be performed and the element of uncertainty regarding the experimental apparatus and techniques which will be available five or six years hence. The keynote features of such channels must be good overall performance and a high degree of flexibility at reasonable cost.

Computer Program

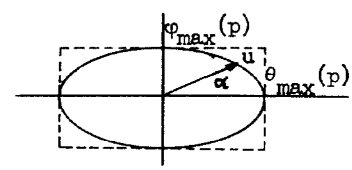
The initial design work for the CERN channel was predicated on the theory of alternating gradient synchrotrons. The lack of advanced computers prior to 1960 forced the design group to make various approximations in order to reduce the problem to one amenable to analytic methods. The channel was assumed to be infinitely long and strictly periodic; the acceptance ellipses were taken to be uniformly filled and the quadrupole magnets were approximated by their thin lens equivalent. The availability of more powerful computers has facilitated the writing of more sophisticated programs which remove various of the above restrictions. But when the present study was undertaken, there seemed to be no readily available program which was free of all simplifications including the reliance on AGS theory; hence, the author prepared one. Pertinent aspects of the program are documented in succeeding paragraphs.

The momentum-dependent solid angle acceptance of any arbitrary arrangement of quadrupole magnets and drift spaces is obtained by numerical integration of

$$\Omega(p) = 4\int_0^{\pi/2} d\alpha \int_0^{u_{max}(\alpha)} u\, du \qquad (1)$$

where $u^2 = \theta^2\varphi^2/(\theta^2 \sin^2\alpha + \varphi^2 \cos^2\alpha)$

and the maximum acceptance angles ($\theta_{max}(p)$ in the horizontal plane and $\varphi_{max}(p)$ in the vertical plane) are calculated for the system at each momentum value of interest. Equation (1) simplifies for a given momentum to $\Omega = \pi\theta\varphi$, the area of the acceptance ellipse in θ-φ space assuming a point source. The elliptical area of integration was



chosen rather than a rectangular region because the former is consistent with the concept of a circular quadrupole aperture of specified radius.

The pion emittance spectrum from a channel is calculated from

$$dN_\pi(p)/dp = 2\int_0^{\pi} d\alpha \int_0^{u_{max}(\alpha)} u\, du\, f(p,\Psi) \qquad (2)$$

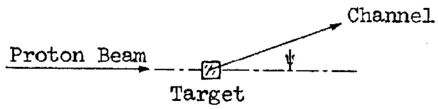
where

$$f(p,\Psi) = [d^2N_\pi(p,\Psi)/dp\, d\Omega]\, \exp[-L/\lambda(p)]\ ,$$

$$\Psi = [(\psi + \theta)^2 + \varphi^2]^{1/2}$$

and $[d^2N_\pi(p,\Psi)/dp\, d\Omega]$ is the pion production cross section, L is the channel length, $\lambda(p)$ is the decay length of a pion of momentum p and ψ is the angle of the channel to the proton beam.

*Work performed under the auspices of the U. S. Atomic Energy Commission.



The total number of pions follows from an integration over all momenta between two specified values p_1 and p_2.

$$N_\pi = \int_{p_1}^{p_2} [dN_\pi(p)/dp]\, dp \,. \qquad (3)$$

Equation (3) is implemented on the computer by means of a triple integration scheme employing an adaptive Simpson's Rule algorithm which adjusts the integration step size in each region to minimize the number of steps necessary to guarantee the specified accuracy. Acceptance spectra of the type shown in this paper require about two seconds of IBM-7094 time per channel quadrupole for 2% accuracy.

The muon emittance spectrum from a channel is calculated from

$$\frac{dN_\mu(p)}{dp} = 2 \int_{p_1}^{p_2} dp_\pi \int_0^\pi d\alpha \int_0^{u_{max}(\alpha)} u\, du\, F(\psi, p_\pi, p_\mu) \qquad (4)$$

where

$$F(\psi, p_\pi, p_\mu) = [d^2N_\pi(p,\psi)/dp\, d\Omega]\, W(p_\pi, p_\mu)\, C(p_\pi, p_\mu).$$

The quantity $W(p_\pi, p_\mu)$ is the familiar differential momentum spectrum of muons arising from the decay of pions of momentum p_π. $C(p_\pi, p_\mu)$ is the probability that a muon of momentum p_μ decaying from a pion of momentum p_π will be captured by the channel. The procedure for calculating C is straightforward but time-consuming. The running time necessary to obtain the muon emittance spectrum from a channel typical of the Chicago channel is 10-12 minutes on an IBM-7094.

Effect of Channel Parameters

A plot of the solid angle accepted by a channel at each momentum reveals at a glance the pertinent properties of the channel, namely the (low) momentum at which the acceptance becomes vanishingly small, the momentum value at which the acceptance maximizes, the value of the maximum acceptance and the momentum bandwidth of the channel. Since the abscissa of such a plot is particle independent, the channel properties pictured on the plot are equally valid for muons and pions. The $\Omega(p)$ vs p curve is thus the signature of a channel and provides a convenient method for displaying the effect on the channel properties of varying certain channel parameters.

Effect of Initial Drift, D

Figure 2 shows the acceptance spectra of a channel for various values of D. The channel parameters are those given for the Chicago channel, namely ℓ = 28 cm, s = 3 cm, a = 10 cm and B = 10 Kg. The effects of the synchrocyclotron fields and the large matching lens are not included. The jagged appearance of portions of the acceptance spectra is a reflection of the fact that the channel is not continuous, but discrete. The extreme rays which define the acceptance shift from one magnet to the next in discontinuous jumps as the momentum is varied.

Clearly there is an optimum value of D, one for which the most pions are accepted. The insert in Fig. 2 shows a plot of the solid angle integrated over momentum for a given D versus D. The integral maximizes when the value of D is approximately equal to the focal length (in the focusing plane) of the first magnet. The curves in Fig. 2 emphasize the necessity of getting as close to the target as possible. At D = 60 inches the integrated acceptance has dropped by a factor of five from its optimum value and the peak acceptance is down by a factor of 30.

Note that the low-momentum cutoff is unaffected by D; p_{cutoff} is strictly a property of the channel geometry. Note also that the location of the maximum acceptance shifts toward higher momenta as D is increased.

Effect of Pole Field, B

Figure 3 shows the effect of reducing B by a factor of 2, that is, halving the gradient. All other parameters were held constant. The reduction in field compresses the spectrum in momentum space by the factor 1/2. The maximum value of the curve and its general profile, which are characteristic of the channel geometry, are preserved except for the compression effect. Thus the maximum acceptance of a channel may be shifted to any momentum below that which corresponds to maximum pole field simply by lowering B.

Effect of Matching Elements

It is possible to shape the pion acceptance spectrum by appropriately adjusting the fields in the first few magnets of a channel. To exhibit this effect the following channel parameters were chosen: ℓ = 12", s = 2", D = 18", a = 4", and B = 10 Kg. The tall, narrow spectrum in Fig. 4 resulted from turning off magnets 4-7 and operating the first three magnets as a triplet to image 225 Mev/c pions from the target on the entrance of magnet 8. The momentum interval accepted (15% in Fig. 4) can be narrowed by turning off more magnets; conversely, the bandpass can be broadened by turning off fewer magnets.

The momentum at which the maximum acceptance occurs can be shifted by readjusting the fields in the first three magnets. The shorter curve in Fig. 4 shows the acceptance spectrum when the first three magnets are tuned to focus 450 Mev/c pions on the 8^{th} magnet. The acceptance is lower in this case because the acceptance of the rest of the channel is lower at this momentum.

It can be seen from these examples that it is possible to inject a "narrow" spectrum of pions into a channel. This would permit at least a partial separation of pions and muons at the end of the channel (by means of a bending magnet) to give a high muon-to-pion ratio.

Effect of Magnet Spacing

The magnets in the CERN and Chicago muon channels are uniformly spaced. If an additional constraint is imposed on those designs, namely that the channel be made longer but the number of magnets kept fixed, the question of optimum spacing arises. The dimensionalized channel parameter which is pertiment to this problem is s/d where s and d are defined in the picture below.

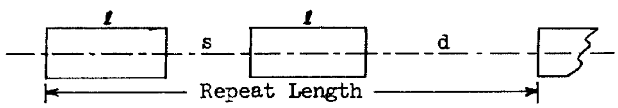


Figure 5 shows the solid angle acceptance spectra for three channels with different s/d ratios. For each channel the repeat length was 96", ℓ = 12", D = 18", a = 4", and B was adjusted to make all three channels have the same momentum cutoff. The optimum ratio is clearly s/d = 0. This result leads to the following conclusion: Given a fixed number of magnets with which to construct a long muon channel, the magnets should be arranged along the channel as close-spaced doublets rather than spaced uniformly. This effect is less obvious when the drift spaces s and d are of the same order as ℓ, but it becomes more dramatic when sparsely filled channels are considered.

Added evidence for the conclusion above was obtained from runs which compared the acceptance spectrum of a close-spaced triplet channel with that of a close-spaced doublet channel. All aspects of the two channels were equivalent except that in the triplet channel the length of magnet in a repeat unit was arranged as a symmetric triplet rather than a doublet. The doublet channel gave 25% more pions when the pole fields were equal in the two channels. Viewed another way, the pole field of the doublet channel could be reduced to make its cutoff momentum equal to that of the triplet channel. In this case the resulting compression of the doublet acceptance spectrum would make the integrated areas of the two channels approximately equal but the power requirements for the doublet channel would be appreciably lower because the power goes as the square of the pole field. One is led to conclude that the optimum magnet configuration for a muon channel is a close-spaced doublet. Of course, a channel completely packed with magnets satisfies this principle since s equals d and is as small as the coil windings permit.

The design problem involving a total length constraint is a very practical one because it is often necessary to penetrate a thick shielding wall or reach a specified experimental area with a given complement of magnets which is fixed by budget considerations or availability of magnets by a given date. This latter situation was faced by Marshall (Ref. 3) who solved the design problem by arranging the available magnets as close-spaced doublets.

Effect of Drift Between Doublets, d

Figure 6 exhibits the acceptance spectra of four channels with differing values of drift space d between doublets. For all these channels ℓ = 18", s = 6", D = 0, a = 4", and B = 10 Kg. Two major effects can be noted. The low-momentum cutoff moves to higher values when d is increased and the peak (and integrated) values of the spectrum are decreased. Comparing the cases d = 6" and d = 30" reveals that leaving out every third magnet costs over half the pions.

Figure 7 shows the muon emittance spectra for two values of d. The spectra are normalized to the same number of acceptance pions. One sees that the effect on the muon capture probability, N_μ/N_π, of increasing d is not as strong. In this case the capture probability only dropped to 0.88 of its original value.

Two other effects are less noticable in these runs but still quite real. The acceptance spectrum rises most steeply and the momentum bandpass is widest for the smallest value of d.

Effect of Magnet Length, ℓ

Figure 8 shows the effect of distributing equal amounts of iron as short magnets in one channel and as long magnets in a second channel. The short-magnet channel had the doublet configuration sketched in Fig. 6 with ℓ = 12", s = 2", and d = 16". The long-magnet channel had ℓ = 18", s = 3", and d = 24". The total length of two repeat units in the long-magnet channel was equal to three repeat units of the short-magnet channel. Both channels had equal amounts of iron and equal s/ℓ ratios and in both D = 18", a = 4" and B = 10 Kg. Comparing the acceptance spectra shows that using longer magnets moves the acceptance spectrum to higher momenta and reduces the value of the acceptance. These facts lead to the conclusion that channels designed for higher muon momenta should utilize longer magnets. This design principle was followed at CERN where ℓ = 40 cm magnets were used for 300 Mev/c muons and at Chicago where shorter (ℓ = 28 cm) mangets were used for lower (155 Mev/c) momentum muons.

There is another facet to this consideration. If the pole field of the long-magnet channel had been reduced so that its acceptance spectrum lay in the same momentum range as that for the short magnets, then in this case half as many pions would have been captured for 1/6th the power expenditure. Clearly there is a compromise between power costs, magnet costs and pions captured that can only be resolved after the requirements for a channel have been prescribed.

Application to Meson Facilities

The conclusions derived in the last sections should be incorporated in the design of muon channels for a meson facility. Several such facilities are under study in the United States and Europe. Most of the design groups contemplate a proton beam of one milliampere average current and energy variable up to 700-800 Mev. This primary proton beam is made to pass through one or more targets in series and magnetic channels form pion and muon beams with momenta ranging as high as 600 Mev/c.

The broad range of muon momenta virtually precludes the use of a single channel; there should be at least two. One channel should be optimized for muons with momenta in the range 100-250 Mev/c and should have a reasonable acceptance even for 300 Mev/c muons. The momentum at which maximum acceptance occurs can be varied by adjusting the quadrupole fields. The second channel should be optimized for 300-450 Mev/c muons with the capability of capturing 600 Mev/c muons. The low energy channel should be built with short magnets (10-15") while the high energy channel should utilize long magnets (18-24").

The total length of the channels will ultimately be fixed by economic considerations. The length of present channels is such that 45-65% of the accepted pions decay along the length. The length cannot be optimized in the usual sense because the optimum length in terms of muons/dollar cost of channel is zero; the gain in muons goes as $[1-\exp(-L/\lambda)]$ whereas the cost goes up linearly. In general as many magnets should be purchased as is financially feasible. The magnets should be placed as close together as possible but, if practical considerations preclude this, the magnets should be grouped as close-spaced doublets. Of course, the high energy channel should be longer.

One universal difficulty in any of the proposed facilities is getting the first magnet of the channel near to the target. When the proton beam is derived from a linear accelerator, it is necessary to align the channel at an angle ψ to the beam direction in order for the first quadrupole to clear the beam pipe. If $\psi = 15^{\circ}$ and an 8" I.D. quadrupole is used, it is difficult to get closer to the target than 60 inches and a glance at Fig. 2 emphasizes the resulting loss of pions. Choosing a larger ψ permits the quadrupole to be placed closer to the target but the production cross section for high momentum pions falls off as $\cos^n\psi$ where $2 < n < 3$ and there is no net gain. For low momentum pions the situation is better because for a fixed incident proton energy, the production cross section for low momentum pions maximizes at larger ψ. Hence some optimization is possible and the low energy channel probably should be oriented at a larger ψ than the high energy channel. Circular accelerators fare no better. If the target is external, then the problem is the same as for linear accelerators. If the target is internal, it is still difficult to locate the channel closer to the target than 3-4 feet because of the accelerator structure. However it is possible to accept pions from the forward direction, $\psi = 0$.

Shaping the pion acceptance spectrum of the channel can be accomplished by turning off appropriate magnets. It is possible to get nearly complete separation of pions and muons in this way. However if it is necessary to have a sharp cutoff in the pion spectrum to insure a pure muon beam, then it will be necessary to insert a bending system near the start of at least the high energy channel.

It is difficult to go beyond the general statements above until a specific set of beam requirements is derived from proposed experiments. However it is not at all unrealistic to expect muon beam intensities of 3×10^9/sec at 400 Mev/c and 4×10^8/sec at 600 Mev/c from future meson facilities.

References

1. A. Citron, M. Morpurgo and H. Øverås, CERN 63-35 (1963).
2. G. Culligan, H. Hinterberger, H. Øverås, V. L. Telegdi, and R. Winston, Conference for the Improvement of Synchrocyclotron Facilities, College of William and Mary, 1964, pp 237-263.
3. L. Marshall and T. Yamanouchi, BNL Internal Report PD-47, 1962.

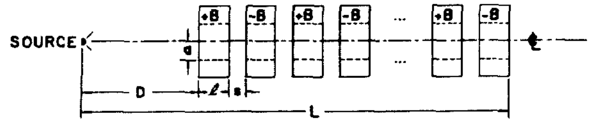


Fig. 1. Schematic drawing of a muon channel with the channel parameters labeled.

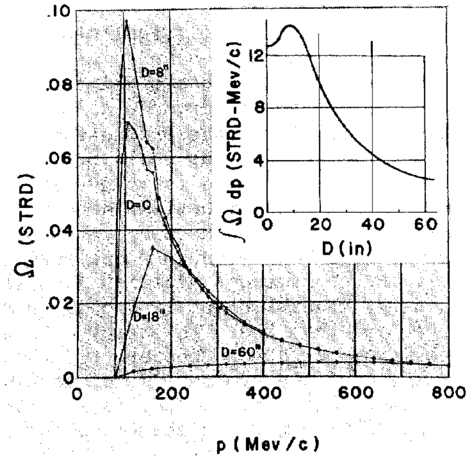


Fig. 2. Pion acceptance spectra for different D of a channel similar to the one at Chicago. Insert graph shows integrated acceptance versus D.

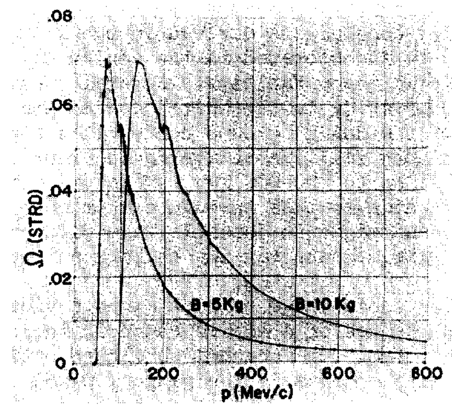


Fig. 3. Acceptance spectra of a channel for two values of pole field, B. Channel parameters: ℓ = 12", s = 2", a = 4", and D = 0.

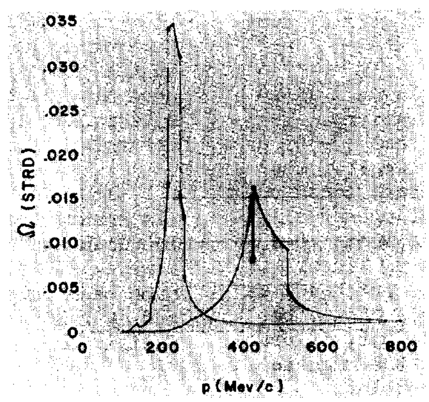


Fig. 4. Result of turning off certain magnets in a channel to shape the pion acceptance spectrum.

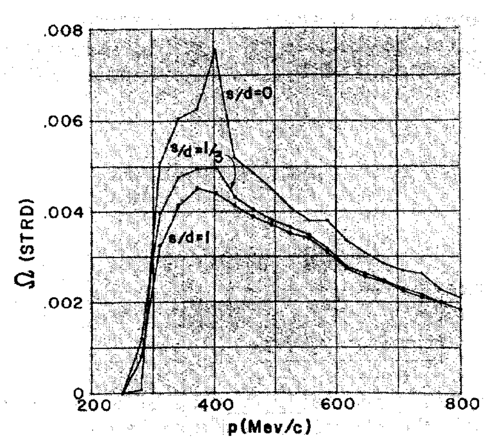


Fig. 5. Acceptance spectra of channels with three values of s/d. Channel Parameters: Repeat length = 96", ℓ = 12", D = 18", and a = 4".

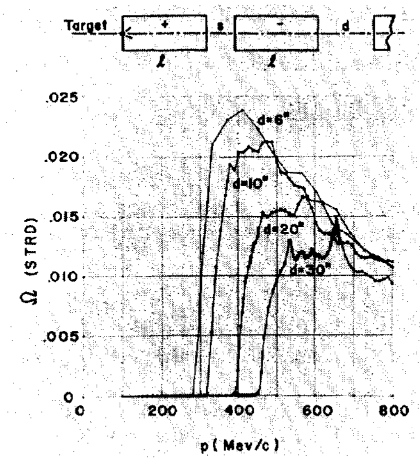


Fig. 6. Acceptance spectra for channels with different d. Channel parameters: ℓ = 18", s = 6", D = 0, a = 4" and B = 10 Kg.

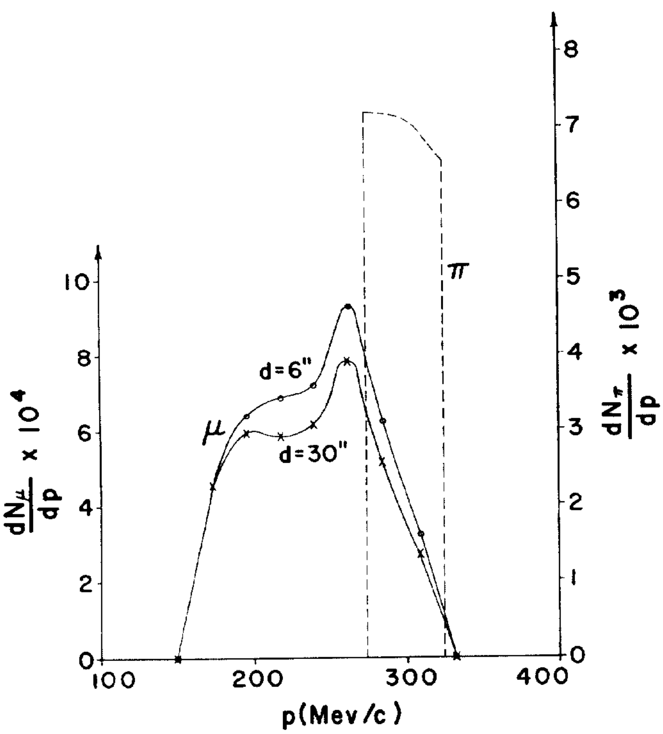


Fig. 7. Muon emittance spectra from two channels with differing d. See Fig. 6. for channel parameters.

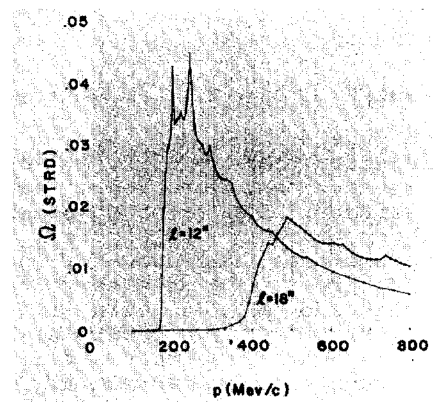


Fig. 8. Comparison of a short-magnet channel (ℓ = 12") with a long-magnet channel (ℓ = 18").