

## THE CAMBRIDGE ELECTRON ACCELERATOR BEAM POSITION MONITORS

L.A. Law and L. Holcomb  
Cambridge Electron Accelerator, Harvard University  
Cambridge, Mass.

### Summary

Electromagnetic beam monitors have been developed and used to measure the position of the center of charge of the electron beam in the C.E.A. Wideband (10 MHz) and narrow band (18 KHz) monitors are used, having sensitivities of 1.75  $\mu\text{a}/\text{ma}\cdot\text{inch}$  and .472  $\text{ma}/\text{ma}\cdot\text{inch}$ , and minimum detectable signals of 0.29  $\text{ma}\cdot\text{inch}$  and 0.1  $\mu\text{a}\cdot\text{inch}$  respectively.

### Introduction

When the accelerator was in the design stage, it was considered necessary to have a considerable number (16) of beam position monitors around the orbit. It was demonstrated conclusively that during injection, an electrostatic pick-up plate system would suffer very badly from collection of stray charge, and it was thought that the same thing would happen later in the acceleration cycle due to photo-emission caused by synchrotron radiation. This has recently been confirmed.<sup>1</sup> Thus a low impedance electromagnetic system was conceived.

Bandwidth requirements are set by the distribution of electrons around the accelerator, which is determined by the inflector. A highly relativistic electron requires 750 nanosec. to complete one revolution in the synchrotron and the inflector is designed such that electrons are accepted into the machine at injection for only 650 nsec. Hence bandwidth is determined by the accuracy with which it is required to measure the 100 nsec pulse corresponding to the gap. The waveform to be detected is thus ideally an 8 msec. train of pulses each of duration 650 nsec. and the repetition period of 750 nsec. The output signal of the monitors is proportional to the product of beam current and the displacement of center of charge from the axis of the monitor, each measured at the same instant. Beam intensity or current is measured in milliamperes, i.e.  $N_e q_e / 0.75 \mu\text{secs}$ , where  $N_e$  is the number of electrons per pulse and  $q_e = 1.6 \times 10^{-19}$  coulomb.

A narrow band monitor tuned to the orbital frequency (1.3 MHz) could be used to measure the position of the beam averaged over many orbital periods. The val-

ue of beam intensity or current used to derive the beam position from the monitor signal must then be measured as a function of the 1.3 MHz component. This component, for a given number of electrons circulating in the accelerator, varies considerably with the electron distribution around the orbit, which fluctuates widely from day to day.

### The Synchrotron Beam Position Monitor

This monitor is dealt with briefly as details have already been published.<sup>2</sup> See Figure 1. It comprises four sets of coils and has an aperture of 4" x 1.5". The coils for measurement of horizontal displacement are compact, above and below the beam, but those for vertical displacement are distributed to achieve reasonable linearity. Due to the large number of turns in the coils, the windings were divided into sections and each section individually damped to prevent ringing. In order to achieve maximum bandwidth, the stray capacity to ground was reduced to a minimum, resulting in large electrostatic pick-up. Electrostatic shielding reduced this effect, with some loss in sensitivity, but a differential amplifier having high commonmode rejection was necessary. The time to construct a monitor, after all parts had been machined, was about two man weeks. Sixteen such monitors were used in the synchrotron.

### The Shorted Turn Beam Position Monitor

When the synchrotron was in operation position monitors were needed for a momentum matching system to be installed between the linac and the synchrotron. An aperture of 1.5" x 1.5" and a minimum detectable signal of 0.1  $\text{ma}\cdot\text{ins}$ . were required. It was suggested that a shorted turn could be used to replace the conventional coils of the synchrotron monitor, and the induced current then measured with a current transformer. This proved perfectly feasible.

A simplified illustration of the geometry of the new position monitor is given in Fig.2. It is in the form of a cage-like structure made from heavy cop-

per bars. When the electron beam passes exactly through the center of this structure, the induced loop currents are zero because of the symmetry. If the electron beam is given a displacement in the positive x direction, then loop currents  $i_1$  and  $i_2$  will flow in the sense indicated. Giving the beam such a displacement, x, is equivalent to introducing a very long current loop, i of width x as shown in Fig.2. The sensitivity of the position monitor is found by calculating the currents  $i_1$  and  $i_2$  induced by the long current loop I.

If  $y=0$ , then no loop currents flow around the vertical faces and one has:

$$i_1 R_{loop} + M(x) \frac{di}{dt} + L \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} = 0 \quad (1)$$

$R_{loop}$  = the resistance in loop  $i_1$ .  
 $M(x)$  = the mutual inductance between loop  $i_1$  and loop I.  
 $L$  = the self inductance of loop  $i_1$ .  
 $M_{12}$  = the mutual inductance between loop  $i_1$  and  $i_2$ .

Making use of the fact that for  $y=0$ ,  $i_1=i_2$  and taking the Laplace transform of Eq. (1) the result is:

$$I_1(s) = \frac{M(x)s}{R_{loop} + (L + M_{12})s} \cdot I(s) \quad (2)$$

$$I(s) = \mathcal{L}\{i(t)\}, \quad i(0) = i_1(0) = i_2(0) = 0$$

In the actual monitor the sum of the two loop currents,  $i_1 + i_2$ , is measured by placing a ferrite toroid around one of the legs in each loop. The ferrites have secondary windings consisting of n turns of fine wire. The secondary windings of the two transformers are connected in parallel and the output current is fed into a current amplifier with a low input resistance. The equivalent circuit is given in Fig.3a. Taking the sum of the two loop currents,  $i_1 + i_2$ , minimizes the dependence of the signal on the y displacement of the beam.

Eq. (2) will be correct if  $R_{loop}$  is made to include the impedance reflected into the loop by the transformer. Knowing  $I_1(s)$  it is straight-forward to calculate the input current to the amplifier,  $I_{in}(s)$ . The correct result may also be obtained from the equivalent circuit referred to the amplifier input (Fig.3b). It is:

$$I_{in}(s) = \frac{nM(x)s}{Z_1(s) + \left[1 + \frac{Z_1(s)}{Z_2(s)}\right] \left[Z_3(s) + R_{in}\right]} \cdot I(s) \quad (3)$$

$R_2$  represents the losses in the two ferrite cores and  $L_2$  is one half the trans-

former secondary inductance. Eq.(3) is the general expression for the response of the position monitor to an x-displacement when the beam current is an arbitrary function of time.

In order to see how the physical dimensions affect the response of the monitor, the important parameters, L,  $M_{12}$ , and  $M(x)$ , were calculated for the simple geometry of Fig.2. To simplify the calculation further, it was assumed that the fields produced around the cylindrical conductors were the same as those produced by equivalent line currents along the axes of the cylinders.  $M(x)$  was found by using the magnetic vector potential and evaluating the two line integrals:

$$\vec{A} = \frac{i\mu}{4\pi} \oint_{loop} \frac{d\vec{\ell}}{r} \quad (4)$$

$$M(x) = \frac{1}{i} \oint_{loop} \vec{A} \cdot d\vec{\ell}$$

The result of this was:

$$M(x) = \frac{\mu \ell}{4\pi} \ln \frac{\left(\frac{w}{2} - x\right)^2 + \left(\frac{h}{2} - y\right)^2}{\left(\frac{w}{2} + x\right)^2 + \left(\frac{h}{2} + y\right)^2} \quad (5)$$

$M_{12}$  was evaluated in the same way with Eq.(4) using the vector potential produced by loop  $i_2$ . The result was:

$$M = \frac{\mu}{4\pi} \sum_{k=1}^4 (-1)^{k+1} \left[ a_k \sinh^{-1} \left( \frac{a_k}{b_k} \right) + b_k - \sqrt{a_k^2 + b_k^2} \right] \quad (6)$$

$$\begin{aligned} a_1 &= a_2 = \ell & b_1 &= b_3 = h & b_4 &= \sqrt{h^2 + \ell^2} \\ a_3 &= a_4 = w & b_2 &= \sqrt{h^2 + w^2} \end{aligned}$$

To find L, the mutual inductance between the loop  $i_1$  on the axis of the conductor forming the loop, and a loop on the inner surface of the cylindrical conductors was computed. Again Eq.(4) was used. Because at high frequencies the current in the conductors is concentrated at the surface, the internal inductance of the conductors was neglected. It was assumed that the diameter of the conductors, d, was small compared to the other dimensions l, h, and w. The expression obtained was:

$$\begin{aligned} L = \frac{\mu}{\pi} \left\{ \left( \ell - \frac{d}{2} \right) \left[ \sinh^{-1} \frac{2\ell}{d} - \sinh^{-1} \frac{\ell}{w - \frac{d}{2}} \right] \right. \\ \left. + \left( w - \frac{d}{2} \right) \left[ \sinh^{-1} \frac{2w}{d} - \sinh^{-1} \frac{w}{\ell - \frac{d}{2}} \right] \right. \\ \left. - w - \ell + 2d - \sqrt{\ell^2 + \left(\frac{d}{2}\right)^2} - \sqrt{w^2 + \left(\frac{d}{2}\right)^2} \right. \\ \left. + \sqrt{\ell^2 + \left(w - \frac{d}{2}\right)^2} + \sqrt{w^2 + \left(\ell - \frac{d}{2}\right)^2} \right\} \quad (7) \end{aligned}$$

The actual C.E.A. position monitor is more complicated than the simplified geometry of Fig.2. Fig.4 is a photograph of the monitor, which is mounted in a relatively small stainless steel chamber. So in this case, the values of  $L$ ,  $M_{12}$ , and  $M(x)$  given above are only approximate.

Fig.4 shows that two additional ferrite transformers have been placed around the side legs to measure the vertical displacement of the beam. In this way with one monitor it is possible to obtain simultaneously two output signals, one proportional to horizontal and one proportional to vertical beam displacement. It was found necessary to shield carefully the transformers and leads from stray electric and magnetic fields.

To achieve a wide band system,  $C$ , (see Fig.3b) is chosen so large that its impedance is negligible at all frequencies of interest. In this case, making the approximation  $|Z_2| \gg |Z_3 + R_{in}|$  Eq.(3) becomes:

$$I_{in}(s) = \frac{nM(x)s}{R_1 + R_3 + R_{in} + L_1 s} \cdot I(s) \quad (8)$$

For a unit step function of beam current,  $I(S)=1/S$ :

$$I_{in}(t) = \frac{n}{L_1} M(x) e^{-\frac{t}{\tau}} \quad (9)$$

So the decay time constant is:

$$\tau = \frac{L_1}{R_1 + R_3 + R_{in}} \quad (10)$$

and the high frequency sensitivity,  $A$ , is:

$$A = \frac{I_{in}}{I \cdot x} = \frac{n}{L_1} \frac{M(x)}{x} \quad (11)$$

Although  $C$  has been neglected in the above, it is necessary that it be larger than a certain minimum value if low frequency ringing is to be avoided.

In the analysis above, there is nothing that limits the rise time of the signal from the position monitor. A more exact analysis would have to take into account stray inductance and capacitance at the amplifier input. In the case of the position monitors in the C.E.A. injection

system the rise time is less than fifteen nsecs with  $R_{in}=50\text{ohms}$ , which is faster than the amplifiers that are used there.

### Tuned Monitor

To obtain greater sensitivity it is possible to tune the monitor to the fundamental frequency (1.3 MHz) of the current waveform and hence use a narrow band system. Noise power in the amplifier is also reduced in proportion to the bandwidth. The impedance in Fig.3b seen by the voltage generator is:

$$Z = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \quad (12)$$

The system is resonant when  $Z$  is resistive, and this results in a resonant frequency

$$\omega_0^2 = \frac{1 + \frac{L_1}{L_2} + \frac{R_1}{R_2} + \frac{R_1 R_3 C}{L_2}}{L_1 C \left(1 + \frac{R_3}{R_2}\right)} \quad (13)$$

It is obvious that maximum current flows through  $R_{in}$  when  $R_2$  and  $L_2$  tend to infinity, i.e., a perfect transformer. The  $Q$  of the system is approximately:

$$Q = \frac{\omega_0 L_1 \left(1 + \frac{R_3}{R_2}\right)}{R_1 \left(1 - \frac{1}{\omega_0^2 L_2 C}\right) + R_3 \left(1 + \frac{R_1}{R_2}\right) + \frac{L_1}{C R_2}} \quad (14)$$

Tuned monitors are used for correcting the position of the external electron beam where the peak current is small, of the order of  $1\mu\text{amp.}$ , and absolute measurement of position is not required. The system could be made absolute by employing a tuned intensity monitor.

### Linearity of Monitors

The non-linearity of a beam position monitor, defined as the departure from true proportionality between displacement and output signal, is important and should be minimal if the true position of the center of charge of a beam of large cross-section is to be measured accurately. The linearity of the shorted turn monitor is determined by Eq.(5). If this is expanded in powers of  $x$  and the  $y$  displacement considered zero, then

$$\frac{M(x)}{x} = -\frac{2\mu w \ell}{\pi(w^2 + h^2)} \left[ 1 - \frac{4(3h^2 - w^2)}{3(w^2 + h^2)} x^2 + \dots \right] \quad (15)$$

The second degree term gives the first order deviation from linearity for an  $x$  displacement from the axis. Fig. 5 shows plots of Eq.(5) with  $y$  displacements as a parameter, calculated for a 1.5" by 1.5" nominal aperture monitor. The measured output signal in arbitrary units from a position monitor for various "beam" displacements  $x$  and  $y$  is also shown, and agrees quite well with the calculated values. To obtain these results the beam was simulated by a taut length of wire carrying a suitable current.

#### Sensitivity and Minimum Detectable Signal

The sensitivity of the monitor is a function of the mechanical dimensions and of the output transformers. The sensitivity can be set by the following amplifier to a convenient value, and thus the turns ratio  $n$  of the transformers is chosen to minimize the noise factor of the amplifier in the case of the tuned system, and to maximize the low frequency time constant in the wideband system. This subject is discussed fully in a report by Holcomb.<sup>3</sup>

The parameters and sensitivities of the four types of monitors constructed to date are given in Table 1. The wideband system has not been optimized for minimum noise.

#### General Description of the New Monitor

The shorted turn beam monitors presently in use have a framework consisting of copper bars soldered together which form the shorted turns. This assembly is insulated from the stainless steel chamber. The ferrite cored transformers mounted on the framework are each shielded by a copper cup which is isolated from the framework, but grounded to the cham-

ber through the braid used to shield the transformer leads to the output connector. It was found convenient to connect the associated transformers in parallel at the connector. The output connectors are mounted so that the connections from the transformers are symmetrical.

The amplifiers used are transistorized and are mounted directly on the monitor output connectors, one each for horizontal and vertical. Power is fed to the amplifiers from the accelerator control room along the same coaxial cable that is used to carry the monitor output signal.

#### Advantages of the New Monitor

The new monitor has the prime advantage of simplicity of construction. Incidental advantages are 1) faster response time than the synchrotron beam monitor, as stray capacity is considerably lower; 2) the monitor can be positioned more accurately and is more stable mechanically; and 3) it should be possible to have a wide or narrow band system solely by changing the amplifier.

#### References

1. G.E. Fischer and R.A. Mack, "On Synchrotron Radiation Produced Photoelectrons from Copper", C.E.A. Report CEAL 1017.
2. L. Holcomb, D.I. Porat and K. Robinson. "Measuring Position and Current of an Accelerated Particle Beam", Nuclear Instruments and Methods 24. (1963), pp. 399-407.
3. L. Holcomb, "Measurement of Electron Beam Position in the C.E.A.", C.E.A. Report CEAL-TM-135.

Table 1. Parameters of monitors constructed to date.

	Synchrotron Beam Monitor		Shorted Turn Beam Monitor.		
	Wideband		Wideband	Tuned	Tuned
Useful aperture, inches.	4.5	1.5	1.5 x 1.5	1.5 x 1.5	2.5 x 2.5
Length, $l$ , inches.	10	8.5	2.6	2.6	8
Width, $w$ , inches.	3.88	4.5	2.5	2.5	3.13
Height, $h$ , inches.	3.63	2.2	2.5	2.5	3.13
Inductance, $L_1$ , microhenries.	1900	690	10.7	10.7	19.6
No. of turns, $n$ , on coil or transf.	120	80	101	15	15
Sensitivity, $A$ , amps/amp.in.	$0.58 \times 10^{-3}$	$3.8 \times 10^{-3}$	$1.75 \times 10^{-3}$	0.472	0.395
Risetime, $\tau_r$ , nanoseconds.	43	16	< 10	-	-
Droop timeconstant, microseconds.	5.5	3.5	6	-	-
Amplifier input resistance, ohms.	345	195	50	0.36	0.36
Minimum detectable signal, $(I_x)_{\min}$ microamp. ins.	130	35	290	0.1	0.1

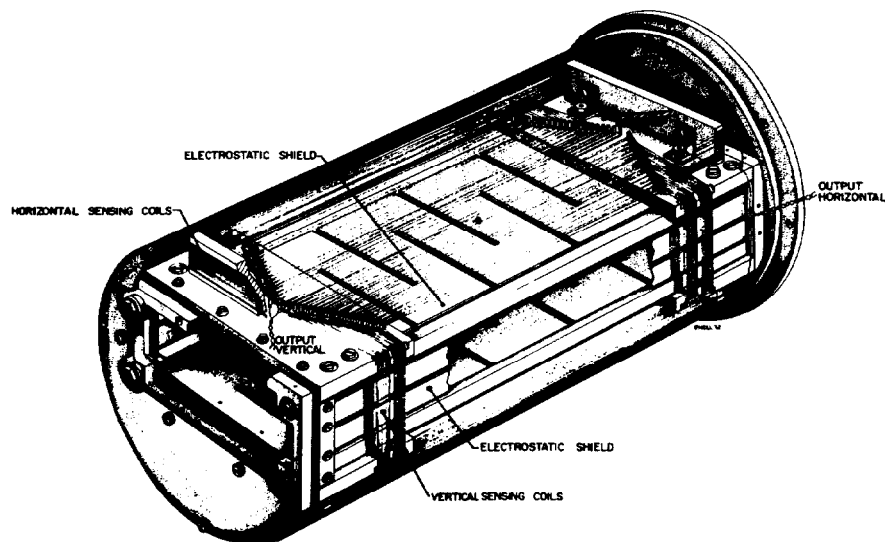


Fig. 1. Photograph of synchrotron beam position monitor.

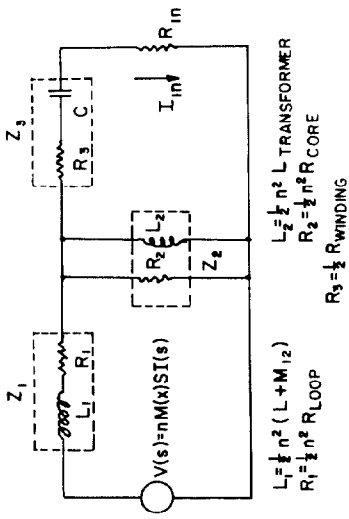


Fig. 3. (b) Equivalent circuit of monitor referred to the input of the amplifier.

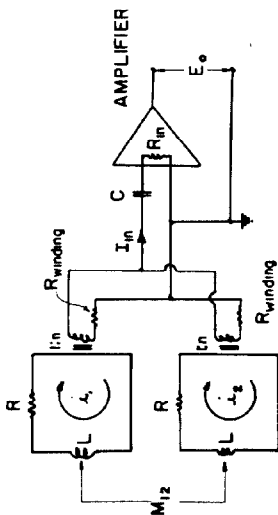


Fig. 3. (a) Equivalent circuit of monitor.

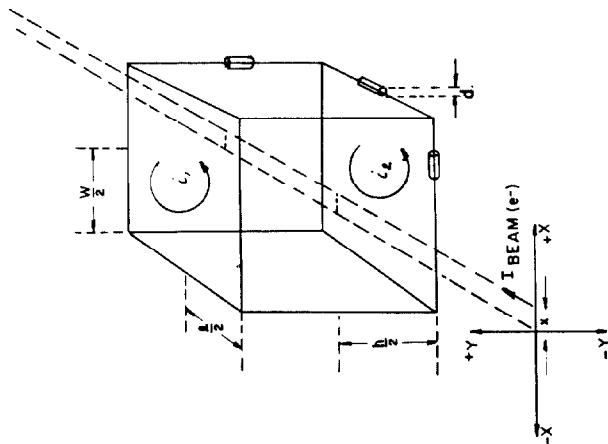


Fig. 2. Simplified geometry of the shorted turn monitor.

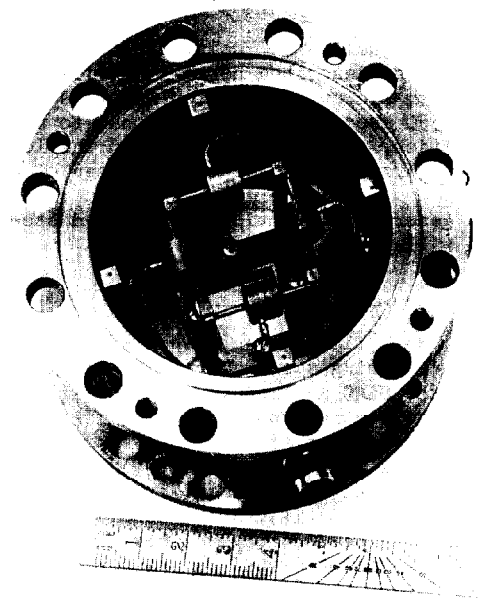


Fig. 4. Photograph of the shorted turn beam position monitor.

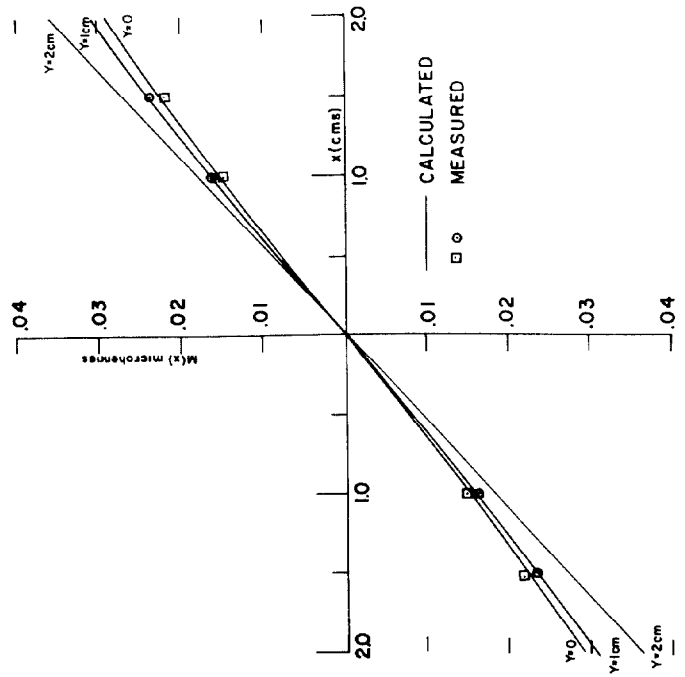


Fig. 5. Linearity curves for the monitor.