STABILITY AND TOLERANCES OF THE SEPARATED ORBIT CYCLOTRON*

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Summary

The physical arrangement of the magnets and rf cavities in a Separated Orbit Cyclotron are described, and the equations of motion and their solution are discussed. The stability limits are determined for triplet and singlet lens systems, and the choice of operating values to give maximum acceptance are examined. The treatment of random misalignment errors of the magnets is described; the six types of errors whose relative strengths are compared lead to bench alignment errors of $\sim$1 mil. The alignment of adjacent lenses and alignment over a distance long compared with a betatron oscillation wavelength are shown, however, to have relatively relaxed tolerances.

Introduction

The first concept for the Separated Orbit Cyclotron was originally described by F. M. Russell.[1] Some recent work at Oak Ridge is described in other conference papers.[2,3,4,5] The principal change from Russell's work is that construction in a single plane was found to be more economical than the original "bee-hive" configuration. In addition, the optimum frequency is in the region of 50 Mc/s, where large cavity gaps and, hence, large cavity voltages are possible.

Sector Geometry

The physical arrangement of the rf cavities and magnets in the Separated Orbit Cyclotron is shown in Fig. 1. The synchronous ion crosses a cavity perpendicular to the plane of the cavity, receives an energy gain and a focusing or defocusing impulse, passes through a drift section AB, enters the magnet BC, and so on. Because of the requirement that gradients also alternate radially[4] for minimum spacing of turns, the total number of magnets in a turn must be odd. Two of the sectors such as AD constitute the minimum magnet period. Also the gradients must alternate along the beam path for this shortest magnet period such that if QW goes FDF, or focus, defocus, focus, XS must go DFD. Therefore, SOC may consist of a series of F, D, F, D, F... lenses (alternating singlets) or FDF, DFD,... (alternating triplets). There appears to be nothing gained by a more complicated structure. There is a possibility that a practical SOC could be built without this restriction of two sectors per magnet period if the field shaping shims[4] are used. The halved magnet period would cause the acceptance to be increased by about 50%. Each sector is adjusted so that the transit time for synchronous particles is a constant in all sectors.

Deflection Sector

In the deflection sector there is a drift space slightly longer than that of a normal sector (i. e., WX > CQ), which upsets the periodicity and requires special treatment. The magnets QW and XS are treated as part of a beam-matching system[6] in that their configurations and gradients must be chosen to make the beam area in phase space match that of the sector at D, and of the following sector at T. It has been shown[7] that matching of about 99% can be obtained. The problem is identical to other matching problems.

Analysis

The properties of the normal sectors vary smoothly so that the method of analysis developed for AG synchrotrons[8] may be used. The differential equation for radial or axial motion is

$$\frac{d^2y}{ds^2} + k(s)y = 0,$$

where s is the length along the path and k(s) is the focusing function:

$$k(s) = \frac{n}{\rho^2}, \text{ for axial motion,}$$

$$= \frac{1-n}{\rho^2}, \text{ for radial motion,}$$

where

ρ = radius of curvature in a field, B.

$$n = \frac{\rho dB}{B d\rho} .$$

The matrix describing motion from one point to another is

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\frac{1+\alpha^2}{\beta}\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix},$$

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$$\begin{pmatrix} y \\ y' \end{pmatrix} = M \begin{pmatrix} y_o \\ y_o' \end{pmatrix} ,$$

where $y' = dy/ds$. The trace of M is $2 \cos \mu$. For stability, the value of μ must be real.

β and $\alpha (= -\frac{1}{2}\frac{d\beta}{ds})$ are periodic in a periodic focusing system. The solution of the equation of motion may be written:

$$\left.\begin{matrix} y_1 \\ y_2 \end{matrix}\right\} = \beta^{1/2}(s) \begin{cases} \cos \phi(s) \\ \sin \phi(s) \end{cases} ,$$

where the phase $\phi(s) = \int_0^s \frac{ds}{\beta}$. The propagation constant μ, is $\int_0^L \frac{ds}{\beta}$, where L = the period length.

The value of β throughout a period may be determined by the following differential equation:

$$\beta''' + 4\, k(s)\beta' + 2\, k'(s)\, \beta = 0.$$

It can be shown that there is a constant of the motion

$$W = 1/\beta \left[y^2 + (\alpha y + \beta y')^2 \right] .$$

This quantity, which is an ellipse of area πW in $y - y'$ space, is an adiabatic invariant if the focusing parameters vary slowly. For an aperture of Y, the maximum value of W is given by

$$W = \frac{Y^2}{\beta_{max}} .$$

The variation of β through a sector of alternating triplets is shown in Fig. 2. Since the area in phase space is conserved, W varies throughout the machine as the square root of momentum. Therefore, the beam size will vary as $\sqrt{p/\beta_{max}}$.

The stability diagram for an SOC sector at 200 MeV, Fig. 3, shows that at high energies the rf produces a negligible effect on focusing. It is of particular interest that the betatron oscillation phase shift per period can be changed over a range of ~40° with only a very small change in β_{max}. Therefore, the betatron frequency may be adjusted to prevent any resonance from occurring due to magnet spacing, and yet there will be but a small effect on the acceptance.

The results shown in Table I are typical for alternating triplet lens in a 200-800 MeV machine. To get any particular value of gradient for simplifying magnet pole tip shaping, the pole may be made with partly a flat field and partly a high gradient field. The acceptance could be increased by increasing the number of sectors (thereby reducing β), as well as by increasing the aperture, of course. β must necessarily increase with energy in a given SOC since its value depends most strongly upon the length of the magnet period for a given focusing configuration.

Table I. Separated Orbit Cyclotron.

200-800 MeV 24 Sectors 20th Harmonic	Balanced alternating triplets 1.5-in. aperture 7-kilogauss fields			
Energy (MeV)	200	400	600	800
Gradient (kg/cm)	0.97	1.33	1.73	1.53
Fraction of pole	1.0	0.7	0.5	0.5
β_{max}(M), axial	5.92	7.08	7.56	8.30
radial	5.45	6.52	7.05	7.72
ν_z	~3.2	~3.2	~3.2	~3.2
ν_x	~3.6	~3.6	~3.6	~3.6
Acceptance, axial	27.2			
(mm-mrad), radial	29.6			
Damping	1.0	0.94	0.89	0.88

Relative quantities for singlets and triplets are shown in Table II.

Table II. Relative Values of β, ν, and Acceptance.

	Sectors/period	β	ν	Acceptance
Singlets	2	1.5	1.0	0.7
Triplets	2	1.0	1.0	1.0

Random Error Analysis

The random error analyses for AG synchrotrons and linear accelerators[9,10] are almost directly applicable to the SOC. If there is an error $\Delta(s)$ in an element of the focusing system, the differential equation of motion becomes

$$\frac{d^2 y}{ds^2} + k(s)y = k(s)\,\Delta(s).$$

The adiabatic invariant will have the approximate value

$$W^{1/2} \cong W_o^{1/2} + \int ds'\Delta(s')k(s')\left[y_2(s')\cos x - y_1(s')\sin x\right],$$

where W_o and x give the amplitude and phase of the betatron oscillation without errors. Now

$$Y_F^2 = \beta_{max,\,F} W_F,$$

$$\Delta Y_{rms}^2 = \left\langle (Y_F - Y_{F_o})^2 \right\rangle = \beta_{max,\,F} \left\langle (W^{1/2} - W_o^{1/2})^2 \right\rangle ,$$

where Y_F is the final amplitude of oscillation with errors, Y_{F_o} is the corresponding final amplitude of oscillation if there are no errors, $\beta_{max, F}$ is the final value of β_{max}, and ΔY_{rms} is the final rms amplitude of oscillation caused by the errors. For example, if there is a random error of mean square value δ^2 in the radial or axial position of a triplet lens, the oscillation growth will be given by

$$\Delta Y_{rms}^2 = \beta_{max, F} \frac{\delta^2}{2} \sum_{m=1}^{n} k_m^2 \ell_m^2 (2\beta_\pm + 4\beta_\mp)_m ,$$

where n = total number of elements, ℓ = length of m^{th} element, β_+ and β_- = values of β at the focusing and defocusing lens respectively.

The values for the types of errors that can occur in the magnet of the 200-800 MeV machine and their relative sensitivity are listed in Table III. Correlated errors refer to displacement of the triplet sets of pole tips from the synchronous path. Uncorrelated errors refer to relative displacement of the individual pole tips in a triplet, relative to the mean axis of the individual lens. The worst error is non-collinearity caused by displacement of the center magnet of the triplet relative to the end magnets. The available area in which the beam may grow is determined from the difference between aperture and the damped beam size for no errors. Coupling between the radial and longitudinal motion uses some of the space. For this machine, the rms value of Δ must be near 10^{-3} inches. Therefore, the bench alignment of the poles must be completed to about 1 mil rms. The alignment of the assembled lens relative to each other can, however, be ~5-10 mils rms. A smoothly increasing radial error going from 0 at injection to 1 inch at the final energy will rpoduce only 4 mils of radial oscillation. Therefore, it is the bench alignment of the poles and the alignment from pole-to-pole that is critical.

These tolerances are based on no correction at all for the random errors. Since SOC is very nearly a linear machine and since each random error amounts to a linear transformation, John Martin[11] has recently suggested that a pair of small bending magnets could induce an oscillation to compensate for the error-induced oscillation. The magnets might appear in each turn of the machine and, to be most effective, should be spaced about a quarter-wavelength of oscillation. It would appear that this scheme could increase the tolerance by the square root of the number of turns. A more detailed investigation is in progress.

References

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Table III: Relative Sensitivity of Random Errors in a 200-800 MeV SOC.

Types of Errors		Relative Sensitivity: Correlated	Uncorrelated
1. Non-collinearity	Δ	1.0	—
2. Displacement	δ	1.4×10^{-2}	0.84
3. Skew	ε	10^{-3}	—
4. Orientation in z-x plane	$x\Delta\phi$	1.4×10^{-2}	0.84
5. Length	$\Delta\ell/\ell$	1.4×10^{-2}	—
6. Gradient	$z/2\Delta G/G$	1.4×10^{-2}	0.84

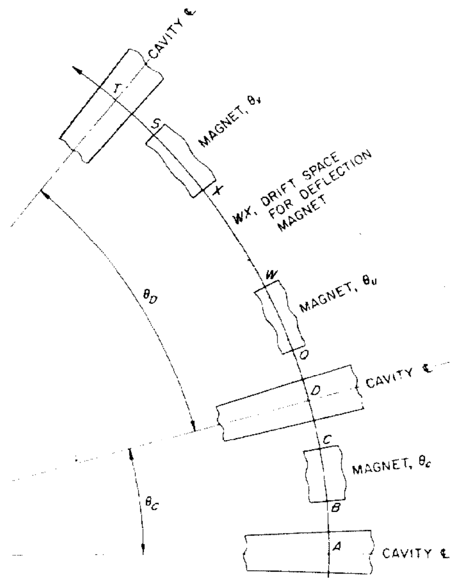

Fig. 1. SOC, normal and deflection sectors.

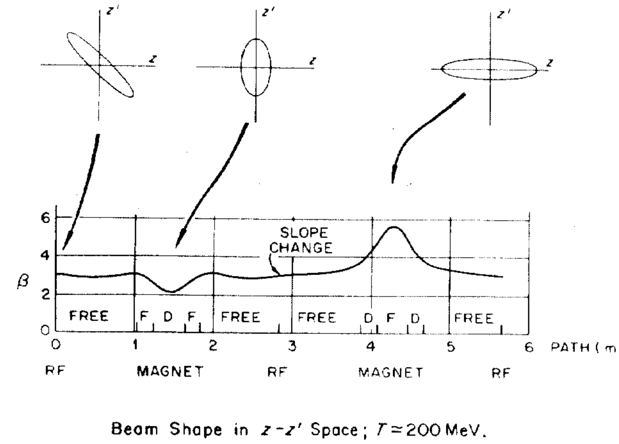

Fig. 2. β and beam shape in typical sector.

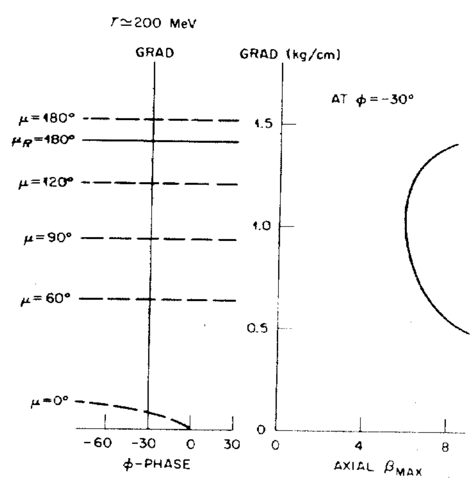

Fig. 3. Stability limits and β_{max} in typical sector, for alternating triplets.