COMPUTER PROGRAM FOR A HIGH-CURRENT ELECTRON ACCELERATOR

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SUMMARY

A comprehensive computer program for a high-current accelerator operating in the 100 ampere/10 MeV range has been developed. The accelerator is a narrow-bandwidth, coupled-cavity device with or without feedback between input and output cavities. Equations for the transient behavior of both the pumped or buildup phase and the accelerator phase have been programmed to allow for a wide range of design parameters such as number of cavities, cavity-cavity coupling, beam-cavity coupling, cavity loss or Q, stagger tuned cavities, input-output cavity coupling, and injected current as a function of time. The sinusoidal electric field in each cavity is expanded with the resonant modes, excited by the beam and boundary-matching magnetic current on the center holes. The irrotational field is computed as the unretarded Coulomb field of the beam as a collection of discs and their images. Feedback and feedforward waves in a waveguide which couples input and output cavities are represented by cutoff and propagating waveguide modes decomposed into Fourier components. Some computational properties of the program are discussed.

DESCRIPTION OF THE PROGRAM

The program has been developed for a high speed large memory digital computer to analyze the detailed behavior of a resonant ring accelerator in particular. This structure is pumped to a high energy during a time on the order of a microsecond and then depleted of energy during a time on the order of 15 nanoseconds or 45 rf periods of the microwave (S-band) frequency. In a high-current accelerator the space charge effects upon the solenoidal field and upon the irrotational or Coulomb repulsion between current elements (discs) are not negligible. In this program they are accounted for during each interval of time when the amplitudes of the solenoidal resonant cavity modes are integrated forward in time and the equations of motion for the various discs of charge are solved according to the total instantaneous axial electric field on each disc.

Specifically the program is written to describe the following phenomena. The entering prebunched current is decomposed into thin discs of varying charge and perhaps velocity, the total number limited by computer storage. As a disc approaches the first cavity it moves according to the Coulomb field of neighboring discs, their images, and the field created by magnetic current placed on the first centerhole to match boundary conditions there. A disc just leaving the last cavity is influenced by similar field components. A disc inside a given cavity moves according to the total axial electric field consisting of (a) a solenoidal portion represented by resonant cavity modes, the amplitudes of which are determined by the total cavity current and boundary-matching magnetic current on the centerholes and surfaces joining a feed-through rectangular waveguide, if one is present, and (b) the irrotational or unretarded Coulomb field of the discs in the given cavity and their images, the latter chosen to satisfy the boundary condition of normal electric field on the transverse walls of the given cavity <u>before</u> the holes are cut. In any cavity the magnetic current creates no Coulomb electric field.

The amplitude of each resonant cavity mode (so-called short-circuit mode) is evaluated by a straightforward procedure[1] of solving Maxwell's equations within a closed volume containing electric currents inside and magnetic surface currents on part of the surface. Loss in the cavity walls and/or loss due to external cavity loading is accounted for by a phenomenological term in the equation of motion for each mode amplitude. The magnetic current on each centerhole is angularly flowing and a function of radial position on the hole, since the resonant modes and discs of charge are not angularly varying. This current is expanded in radial moments, each of which is determined by many radial moments of the total solenoidal and irrotational electric field on the centerhole so as to guarantee continuity of normal electric field across the hole. In addition the amplitude of each resonant cavity mode in the first or last cavity may be influenced by tangential electric field on a surface which joins the cavity to a feedthrough waveguide connecting the input and output cavities. This tangential electric field is expanded with the cutoff and propagating waveguide modes of (a) the wave arriving from the other cavity, (b) the "driving" wave which is present during the buildup phase, and (c) the wave which is launched to provide for continuity of tangential magnetic field across the surface which joins the first or last cavity to the waveguide. To obtain physical realism and minimize the accumulation of computational error it is necessary to expand the time dependent amplitudes of the waveguide modes into Fourier components over a frequency range of, say, twice the cutoff frequency of the dominantrectangular waveguide mode.

The irrotational component of the space charge field at any point within a cavity due to a disc or one of its images within that cavity, or on a centerhole as due to a disc or one of its images within the cavities on either side, is computed from analytic approximations to the exact field in the vicinity of a disc as given by a Legendre-polynominal expansion[2]. This representation is preferable to a slowly convergent expansion in irrotational modes because of the limitation on computer storage space.

APPROXIMATIONS

1. The accelerator chain of cavities is angularly symmetric; if a feedthrough waveguide couples input and output cavities in a nonsymmetric fashion the excitation of angularly-varying cavity modes is neglected.
2. The cavities all have the same radii, so that the resonant frequency of the first or TM_{010} mode is the same for all.
3. The surface which joins the input and output cavities to a rectangular feedthrough waveguide, if present is assumed to extend over the entire guide cross section; a smaller iris would require the solution of a set of simultaneous equations for the amplitudes of the guide cutoff and propagating modes launched at any instant of time. Curvature of these surfaces is neglected.

EQUATIONS

The m^{th} solenoidal resonant cavity mode is defined with vector (denoted by a bar) patterns $\bar{E}^n_m$ and $\bar{H}^n_m$ within cavity n at point $\bar{r}$ as

$$\nabla\times\bar{E}^n_m(\bar{r}) = k^n_m\bar{H}^n_m(\bar{r}),\quad \nabla\times\bar{H}^n_m = k^n_m\bar{E}^n_m \tag{1a}$$

$$\int_V[\bar{E}^n_m(\bar{r})]^2 dv = \int_V[\bar{H}^n_m(\bar{r})]^2 dv = \tau^n,\ \text{volume of cavity n} \tag{1b}$$

$$\bar{E}^n_m(\bar{r})\times\bar{n} = 0 \text{ on cavity n surface} \tag{1c}$$

If only the m=1 or TM_{010} mode is employed to approximate the fields the electric field $\bar{E}^n(\bar{r},t)$ in units of energy is

$$\frac{ec}{\omega_1 m_o c^2}\bar{E}^n(\bar{r},t) = \frac{ec}{\omega_1 m_o c^2}V^n(t)\bar{E}^n_1(\bar{r}) = W^n(t)\bar{E}^n_1(\bar{r}) \tag{2}$$

where e is the absolute charge of an electron of rest mass m_o, c is light velocity, $\omega_1=k_1c$, the same for all the cavities, and $k_1R=2.405$. W^n is

$$W^n(t) = \frac{1}{2\pi}\frac{\left(\begin{array}{c}\text{energy electron would gain from}\\ \text{field } V^n \text{ in a free space wavelength}\end{array}\right)}{\text{electron rest energy}} \tag{3}$$

Furthermore, if the coupling holes between cavities are small the equation for W^n is, with $\psi=\omega_1 t$,

$$W^n(\psi) = \epsilon^{-w^n\psi/2Q^n}[W^n(0)\cos(w^n\psi) + \dot{W}^n(0)\sin(w^n\psi)/w^n] - \frac{1}{2}(w^n)^{-1}\cdot \int_0^\psi[m^{n,n-1}W^{n-1}(\psi')+m^{n,n+1}W^{n+1}(\psi')]\epsilon^{-(\psi-\psi')w^n/2Q^n}\sin w^n(\psi-\psi')d\psi'$$
$$+\frac{\tau^1}{\tau^n}\int_0^\psi\sum_{d,\text{discs}} Q_d\frac{v_d(\psi')}{c}\bar{E}^n_d\cos w^n(\psi-\psi')\epsilon^{-(\psi-\psi')w^n/2Q^n}d\psi' \tag{4}$$

4. The waveguide of a resonant ring accelerator is driven during the pumping phase, through a coupler assumed to be unidirective. The guide transmission factor in either direction between input and output cavities is assumed to be frequency independent.
5. Radial motion of the discs of charge is neglected.
6. The circular walls of the cavities are assumed not to influence the Coulomb (irrotational) electric field due to the discs and their images chosen to keep the electric field normal to the walls between cavities <u>before</u> the center holes are cut.

Here

Q^n = quality factor of the m=1 mode in cavity n

$\cdot$ = $d/d\psi$

$m^{n,n\pm1}$ (<0) is a dimensionless coupling coefficient[3] between cavities n and n±1.

$w^n = 1+\frac{1}{2}|m^{n,n-1}|+\frac{1}{2}|m^{n,n+1}|$, the first resonant frequency of cavity n with its centerholes present, normalized to ω_1.

If the two holes are the same size $m^{n,n-1} = m^{n,n+1}$ and w^n is the normalized $\pi/2$ phase shift frequency in an infinite chain of such cavities.

$$Q_d = \frac{ec}{\omega_1 m_o c^2} \frac{|q_d|}{\varepsilon_o k_1 \tau^1} = (\omega_{pd}/\omega_1)^2,$$

where q_d is the negative charge of disc d and ω_{pd} is the resonant frequency of an infinite plasma of charge density q_d/τ^1.

$\overline{\overline{E}}_d^n$ = axial electric field pattern averaged over disc d.

If n=1, signifying the first or input cavity, the right side of (4) would omit the $m^{n,n-1}$ term and would contain the term

$$-\frac{1}{w^1} C_{1g} \int_o^{\psi} [\frac{1}{A_g} \int_{S^1} \frac{ec}{\omega_1 m_o c^2} \overline{E}(\overline{r}, \psi') x \overline{H}_1^1 \cdot \overline{n} dS^1] \epsilon^{-(\psi-\psi')w^1/2Q^1} \sin w^1(\psi-\psi') d\psi' \quad (5)$$

where $C_{1g} = A_g/(k_1 \tau^1)$, a dimensionless coupling coefficient in Fig. 1, A_g being the waveguide cross sectional area.

A term analogous to (5) would appear in the equation for $W^N(\psi)$ of the last cavity, excited by electric field on S^N.

If the waveguide excitation is represented by the dominant TE_{01} ("01" in this notation) mode then $\overline{E}(\overline{r}, t)$ on S^1 is composed of (a) the driving field coupled into the feedthrough waveguide through a directional coupler, (b) the incoming or feedback wave which has been launched from the last cavity, and (c) the outgoing or feedforward wave which is launched from the first cavity in order to satisfy the boundary condition of continuous tangential $\overline{H}$ field on S^1. The Fourier decomposition of $\overline{E}$ on S^1 is, with $\psi = \omega_1 t$ and $i = \sqrt{-1}$,

$$\frac{ec}{\omega_1 m_o c^2} \overline{E}(\overline{r}, \psi > 0) \text{ on } S^1 = \left\{ \text{Re} \frac{1}{\pi} \int_o^{\infty} dw \epsilon^{iw\psi} \int_o^{\psi} d\psi' \epsilon^{-iw\psi'} \begin{bmatrix} 1/\sqrt{1-(w_{01}/w)^2}, \text{propagating} \\ 1/\sqrt{(w_{01}/w)^2 - 1}, \text{cutoff} \end{bmatrix} \right.$$

$$\cdot \left(\frac{1}{A_g} \int_{S^1} \overline{e}_{01}(\overline{r}) x \frac{ec}{\omega_1 m_o c^2} \sqrt{\frac{\mu_o}{\epsilon_o}} \overline{H}_F(\overline{r}, \psi') \cdot \overline{n} dS^1 \right) - \text{Re} \frac{1}{\pi} \int_o^{\infty} dw \epsilon^{iw\psi} \int_o^{\psi} d\psi' \epsilon^{-iw\psi'}.$$

$$\begin{bmatrix} \exp(-i\sqrt{(w/w_{01})^2 - 1}\, \psi_g)/\sqrt{1-(w_{01}/w)^2}, \text{propagating} \\ i\exp(-\sqrt{1-(w/w_{01})^2}\, \psi_g)/\sqrt{(w_{01}/w)^2 - 1}, \text{cutoff} \end{bmatrix} \left\{ \frac{1}{A_g} \int_{S^1} \overline{e}_{01}(\overline{r}) x \frac{ec}{\omega_1 m_o c^2} \cdot \right.$$

$$\left. \sqrt{\frac{\mu_o}{\epsilon_o}} \overline{H}_B(\overline{r}, \psi') \cdot \overline{n} dS^N \right) + W_d \cos w_d \psi \Big\} [\overline{e}_{01}(\overline{r})]_{S^1} \quad (6)$$

Here $\overline{H}_{F,B}$ are the magnetic fields of the feed forward waves launched from S^1 and the feedback waves launched from S^N, respectively. W_d is the amplitude of the driving electric field on S^1 in units of energy. ψ_g is $k_{01} L_g$, the electrical length of the feedthrough waveguide at cutoff frequency ω_{01}.

$$\overline{H}_F(\overline{r}, \psi') \text{ on } S^1 = \overline{H}_{\text{cavity } 1} - \overline{H}_{\text{feedback}} - \overline{H}_{\text{drive}} \quad (7a)$$

$$\overline{H}_B(\overline{r}, \psi') \text{ on } S^N = \overline{H}_{\text{cavity } N} - \overline{H}_{\text{feed forward}} \quad (7b)$$

where $\overline{H}_{\text{feedback}}$ is given by the second term of (6) but without the relevant factor in brackets, with the overall sign changed to +, and with $[\overline{e}_{01}]_{S^1}$ changed to $[\overline{h}_{01}]_{S^1}$.

More generally $W_m^n(\psi)$, the amplitude of the m^{th} mode in cavity n, would be determined by the total magnetic current $\overline{K}_M(\overline{r}, \psi)$ on the centerholes by the term

$$\frac{1}{2w_m^n \tau^n} \int_o^{\psi} \int_{\text{holes}} \frac{ec}{\omega_1 m_o c^2} \overline{K}_M(\overline{r}, \psi') \cdot \overline{H}_m^n(\overline{r}) dS.$$

$$\epsilon^{-w_m^n(\psi-\psi')/2Q_m^n} \sin w_m^n(\psi-\psi') d\psi' \quad (8)$$

instead of by the first integral term in (4). If the total normal incident field on a hole is $\overline{E}_z^1$, expanded in radial moments or powers of ζ/b (b = hole radius) as

$$\frac{ec}{\omega_1 m_o c^2}\,\frac{1}{2}E_z^i(\zeta,\psi') = \sum_{n=0}^{M} A_{2n}(\psi')(\zeta/b)^{2n} \quad (9)$$

and $\bar{K}_M(\zeta,\psi')$ is expanded as

$$\frac{ec}{\omega_1 m_o c^2}\,\bar{K}_M(\zeta,\psi') = \frac{\bar{\zeta}\times\bar{a}_z}{\sqrt{(b^2-\zeta^2)}}\sum_{n=0}^{M} B_{2n}(\psi')\left(\frac{\zeta}{b}\right)^{2n} \quad (10)$$

$\bar{a}_z$ being the unit vector in the z-direction, then the Bs are determined by the As so as to maintain continuity of normal electric field[4] as

$$B_{2m} = \left[(-)^m\frac{2A_{2m}\Gamma(m+2)}{(m+1)\Gamma(m+\frac{3}{2})} - \frac{1}{m!}\sum_{n=m+1}^{M}(B_{2n}n!)\cdot S(m,n)\right]/S(m,m) \quad (11)$$

where Γ is the ordinary gamma function, and

$$S(m,n) = \sum_{\alpha=m}^{n}\frac{(-)^\alpha}{(n-\alpha)!}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\,\frac{1}{(\alpha-m)!}\,.$$

Eqs. (9) - (11) enable a computation of the cavity excitation term (8) from knowledge of the total axial electric field on each centerhole.

The analytic expressions for the irrotational field in the vicinity of a disc due to its charge are not quoted here.

The equations of motion for each disc to be solved during each short time interval $\Delta\psi$ are written

$$\Delta v_d(\psi)/c = -[1-(v_d(\psi)/c)^2]^{3/2}\,\frac{1}{\pi}\,\frac{ec}{\omega_1 m_o c^2}\cdot \int_0^1 E_{z,\,total}(\psi)2\pi\frac{\zeta}{a}d\frac{\zeta}{a}\Delta\psi \quad (12)$$

"a" being the disc radius. Last of all,

$$\Delta k_1 z_d(\psi) = \frac{v_d(\psi)}{c}\Delta\psi \quad (13)$$

expresses the change in position of disc d.

RESULTS

To date the initial pumping phase of a resonant ring structure has been studied in the absence of a beam via Eqs. (4) - (7). In order to reduce computation time, integral Eq. (4) has been applied over a succession of short time intervals comprising the interval of interest rather than over one long interval. Early in the program the feedback and feedforward waves were not decomposed into Fourier components but instead were approximated by propagating TE_{01} waves of a single velocity which were launched and reacted back upon the cavities according to instantaneous steady-state equations. This representation caused intolerable error in computation of the cavity voltages over a few rf cycles. With the feedthrough waves decomposed into Fourier components traveling at different velocities through the waveguide the feedthrough effect is less of a step function on both the input and output cavities. Even so, the computations to date indicate that the errors in computing cavity voltages by Eq. (4) from the succession of convolutions over short time intervals tend to be inacceptably large. Subsequent work may demonstrate that the voltages are computed more accurately from the difference equation approximations to their differential equations rather than from integral equations.

REFERENCES

1. R.M. Bevensee, Electromagnetic Slow Wave Systems (J. Wiley and Sons, 1964) Chap. 5, Section 3C.
2. P.M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw Hill Book Co., 1953) p. 1267.
3. R.M. Bevensee, op. cit., Chap. 10, Section 2.
4. R.M. Bevensee, op. cit., Chap. 3, Section 4.

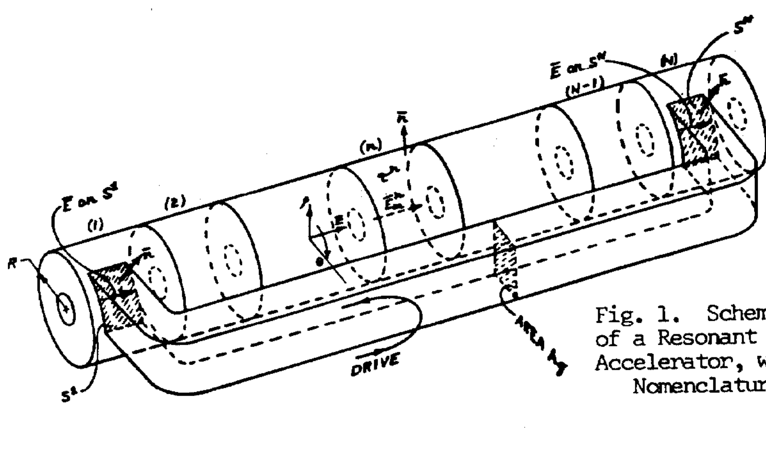

Fig. 1. Schematic of a Resonant Ring Accelerator, with Nomenclature.