BEAM LOADING IN LINEAR ACCELERATORS

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Abstract

The interaction of the bunched particle beam in a linear accelerator with the waveguide structure produces several effects which go under the general name of beam-loading. These effects include reduction, redistribution, and phase shift of the rf field in the waveguide, changes in the tuning of the waveguide structure, excitation of other waveguide modes, and various transient phenomena.

A discussion of these various effects and methods for their calculation will be presented.

Introduction

The process of accelerating particles in an accelerator implies that energy is removed from the accelerator. In the linear accelerator this transfer of energy from the accelerator to the beam can be so efficient that the behavior of the accelerator can not be even qualitatively described without consideration of beam loading. The energy of the particle beam can easily be changed by a factor of two or more due to beam loading, and changes of the radio frequency power density at some points in the accelerating waveguides of an order of magnitude are observed.

The tightly bunched characteristic of the accelerated particle beam in the linear accelerator together with the pulsed nature of these beams and the narrow band-pass-characteristic of the accelerating waveguides produce a number of effects which have become important in present day linear accelerators. These effects include transient phase and amplitude oscillations in the rf transmitted down the waveguide, limitation on the amount of beam which can be accelerated in very short beam pulses, excitation by the bunched beam of spatial harmonics in the waveguide, and beam excitation of the waveguide at frequencies in other pass-bands leading to deterioration of beam optics in general and deflection of the beam into the waveguide walls in some cases. These effects will be increasingly important in the next generation of linear accelerators, especially for accelerators designed for accelerated beams of tens of amperes in short nanosecond bursts.

This report is concerned primarily with disk-loaded accelerator structures, although the general techniques should be applicable to other accelerators. Most of the examples to be presented have been calculated for traveling wave accelerators whose structure does not change down the length of the waveguide, the constant-structure accelerator, although some information is presented on other accelerator types.

The report is divided into two parts. First we present a brief outline of beam loading theory in linear accelerators based upon the rf power diffusion equation in the waveguide structure. This is the most common approach to beam loading and gives reliable results where dispersive effects and effects due to the phase bunching of the particle beam can be ignored. Next we present a more general approach based upon filter theory which allows inclusion of dispersive effects and effects due to the bunched nature of the particle beam. Examples are presented which illustrate various effects.

I. Beam Loading Theory based upon Power Diffusion Equation

The most common approach to beam loading in linear accelerators is to start with the power diffusion equation

$$\frac{dP}{dz} = -2IP - iE, \qquad \text{(I-1)}$$

where P is the rf power density, I is the field attenuation factor, and i is the instantaneous beam current averaged over the rf structure of the beam. The term iE is introduced to represent beam loading. Using the definition of shunt impedance, r, of the waveguide,

$$r = \frac{E^2}{\frac{dP}{dz}} = \frac{E^2}{2IP} . \qquad \text{(I-2)}$$

Equation I-1 can be converted to an equation for the electric field in the guide,

$$\frac{dE}{dz} = -IE - Iir . \qquad \text{(I-3)}$$

From I-3 one obtains an expression for the electric field, E, as a function of z.

$$E(z) = E_o e^{-Iz} - ir\left(1-e^{-Iz}\right). \qquad \text{(I-4)}$$

Equation I-4 is a steady state equation. E_o is the peak electric field at the beginning of the waveguide. It is multiplied by an attenuation factor e^{-Iz} and reduced by a beam loading term $-ir\left(1-e^{-Iz}\right)$. Equation I-4 can be thought of as two fields: the usual term from the power source looks like $E_o e^{-Iz}$, the beam loading term is $-ir\left(1-e^{-Iz}\right)$, and the sum of these fields is the field which particles in the waveguide see. This separation into two fields is a natural result of the superpostion theorem.

Equation I-4 was developed assuming that the particle beam bursts entered the waveguide at the crest of the rf field. This is not always the case. If the beam bursts enter at angle Φ with respect to the peak rf field from the power source, the field the beam particles see will be

$$E(z) = E_o \cos \Phi \, e^{-Iz} - ir \left(1-e^{-Iz}\right). \qquad (I\text{-}5)$$

The energy gain V of the beam particles is obtained by integration of Equation I-5 over the length L of the waveguide section.

$$V = E_o L \cos \Phi \left(\frac{1-e^{-IL}}{IL}\right) - irL\left(1 - \frac{1-e^{-IL}}{IL}\right). \qquad (I\text{-}6)$$

Equation I-6 can be expressed in the convenient form

$$V = V_o \cos \Phi \left[1 - \frac{i}{2i_m}\right], \qquad (I\text{-}7)$$

where

$$V_o = E_o L \left(\frac{1-e^{-IL}}{IL}\right). \qquad (I\text{-}8)$$

is the energy gain for beam particles riding on the rf wave in the absence of beam loading, and

$$i_m = \frac{P_o}{V_o} \cos \Phi \left(\frac{e^{-IL}-1}{1-\frac{IL}{1-e^{-IL}}}\right) \qquad (I\text{-}9)$$

is the beam current corresponding to maximum efficiency for transfer of power to the particle beam. At current i_m the fraction of rf power from the power source, μ_m, transferred to the beam is given by

$$\mu_m = 1/2 \cos^2 \Phi \cdot \frac{e^{-IL}-1}{1-\frac{IL}{1-e^{-IL}}} \qquad (I\text{-}10)$$

Equations I-6 to I-10 represent the basic design information required for most constant structure traveling wave linear accelerators. Figure 1 illustrates the dependence of μ_m, i_m and V_o upon attenuation length, IL, of the waveguide.

Figures 2 and 3 illustrate the large redistribution of E(z) and of P(z) which can occur in the waveguide due to beam loading. Three cases are shown. First the case of no beam current, second the normal accelerating condition where the beam is phased to ride on the crest of the rf wave from the power source, and third the case where the beam is back-phased 180^o so that the beam loading wave adds to the field from the rf power source. For intermediate cases, where the particles enter the waveguide at some phase Φ between 0 and 180^o, the phase of the total rf field relative to the beam will vary continuously depending upon the beam current and the distance down the waveguide.

The solutions presented previously represent the steady state behavior of linear accelerators. It is possible[1,2] to extend this same general approach to include transient phenomena by including the fact that P(z), i, and E(z) are also time dependent quantities with power density P(z,t) and field E(z,t) traveling in the waveguide with group velocity v_g and waveguide filling time $t_f = \frac{L}{v_g}$. An example of the types of effect observed is given in Fig. 4 where the relative energy gain of beam current i_m is plotted as a function of time for different turn on times δ of a step function particle beam after rf power turn on.

The above description of linear accelerator behavior has been developed for the constant structure accelerator in which the waveguide parameters do not change over the length of the waveguide. Many linear accelerators are not of this form but have parameters which vary down the length of the waveguide section. These are often referred to as constant gradient accelerators since the group velocity in the waveguide can be reduced at such a rate as to overcome the normal attenuation of the rf wave as it travels down the waveguide and thus produce constant rf field in the absence of beam loading.

The theory of the constant gradient accelerator can be developed in a similar manner to that presented above[3] by including specifically the variation of waveguide parameters in the diffusion equation. The largest influence of the constant gradient structure on the effects thus far considered is an enhanced influence of beam loading upon the rf field toward the end of the accelerating guide[4]. For large beam currents, these fields can be so large as to exceed the breakdown limits of the structure and thus lead to serious sparking. The advantage of the use of constant gradient or variable structure waveguides for high current linear accelerators is primarily their ability to reduce the tendency toward excitation of higher frequency modes in the waveguide with consequent loss of beam (beam blow-up).

II. Filter Theory Approach to Linear Accelerators

The previous analysis rests upon assumptions which are strictly valid only for a continuous wave accelerator. These assumptions are that the rf field in the accelerator is at one precise frequency, and that sharply rising waveforms are transmitted in the accelerator without distortion. Due to the pulsed nature of these accelerators, the first of these assumptions is necessarily incorrect because of the frequency spreads inherent in switching transients. The second assumption can not be strictly true since the accelerator is essentially a band pass filter. It is still a good assumption, however, provided the frequency spread in the rf pulse is essentially all contained within the pass band of the accelerator and provided the phase shift varies linearly with the frequency over the frequency range of interest.

It is further assumed in the previous analysis, although reasonably well hidden, that the particle beam contains only one frequency component. Due to the very tight phase bunching in linear accelerators, this assumption is substantially incorrect. The beam is in fact very rich in harmonic structure.

In this section we present a very general approach[5] to the linear accelerator based upon filter theory which allows an examination of these dispersive effects. As in the previous section, the specific results presented will be for the constant-structure traveling-wave linear accelerator, however mention will be made of the constant gradient accelerator and the standing wave accelerator.

Two general classes of solution to the linear accelerator will be presented, one in which the specific dispersion effects of the narrow band-pass accelerator structure are included, and a second in which the dispersive effects of the waveguide are ignored, however the bunched nature of the particle beam is retained. For this second type solution the linear accelerator theory resulting corresponds essentially to the solutions found from the power diffusion equation as regards the fundamental accelerating mode. For higher spatial modes and for excitation of other waveguide pass bands the beam loading predictions are different.

The general nature of the dispersion diagram for a disk-loaded waveguide is indicated in Fig.5. The imaginary part, β, of the complex phase shift, Γ, in the waveguide is plotted against frequency for unit length of waveguide. (In the convention indicated in Fig. 5 and in the treatment which follows the unit length of waveguide is taken as twice the distance between loading disks.) At a given frequency ω within a pass-band, the propagation constant $\Gamma(\omega)$ can have several values;

$$\Gamma_{m,n}(\omega) = \Gamma_{m,o}(\omega) + 4\pi n i, \text{ for } \beta > o, \qquad \text{(II-1)}$$

and

$$\Gamma_{m,n}(\omega) = \Gamma_{m,-1}(\omega) + 4\pi n i, \text{ for } \beta < o, \qquad \text{(II-2)}$$

where $\Gamma_{m_1,o}(\omega)$ is the complex phase shift for $o \leq \beta \leq 2\pi$ in the m^{th} frequency mode and $\Gamma_{m_1,-1}(\omega)$ is the complex phase shift for $-2\pi \leq \beta \leq o$.

It is a general result of Floquet's theorem[6] for periodic structures that the value of a wave at point q in the structure $V_q(\omega)$ is related to the wave $V_o(\omega)$ at the beginning of the structure by the relation

$$V_q(\omega) = V_o(\omega) \cdot \sum_{n=-\infty}^{\infty} a_{m,n}(\omega) e^{-q\Gamma_{m,n}(\omega)}. \qquad \text{(II-3)}$$

or

$$V_q(\omega) = V_o(\omega)\, G_q(\omega), \qquad \text{(II-4)}$$

where

$$G_q(\omega) = \sum_{n=-\infty}^{\infty} a_{m,n}(\omega) e^{-q\Gamma_{m,n}(\omega)} \qquad \text{(II-5)}$$

The coefficients $a_{m,n}(\omega)$ are determined by the necessity of matching the boundary conditions on the walls of the waveguide. For the pass band of greatest interest, m=o, which is the accelerating mode, $a_{o,o}(\omega)$ is usually approximately one and the other modes are generally neglected. In most of the analysis which follows we will assume $a_{m,o}(\omega) = 1$, $a_{m,n}(\omega) = o, n \neq o$. This is the assumption usually made in calculating linear accelerator behavior. Thus $G_q(\omega)$ reduces to

$$G_q(\omega) = e^{-q\Gamma(\omega)}, \qquad \text{(II-6)}$$

where we have suppressed the subscripts on $\Gamma(\omega)$. $G_q(\omega)$ is the response at point q of the waveguide to a delta function impulse at the entrance to the waveguide. Knowing $\Gamma(\omega)$ one can apply inverse Laplace transforms to determine $G_q(t)$, the delta function response of the waveguide as a function of time. Applying the superpostion theorem allows one to determine the waveguide response for an arbitrary input rf waveform.

We have examined linear accelerator theory for two expressions for $\Gamma(\omega)$. In the first we include dispersive effects and attempt to make good analytic fits to experimental phase shift and attenuation data. For the usual constant structure accelerator one may write as a good approximation

$$\cosh \Gamma(\omega) = 1 + \left[\frac{\alpha_o}{\sqrt{2}} + \frac{\sqrt{2}}{\omega_c} (i\omega - i\omega_o) \right]^2, \qquad \text{(II-7)}$$

where ω_o is the mid-band frequency difference between the π mode and $\pi/2$ mode, and α_o is the attenuation coefficient at the mid-band frequency ω_o. A similar expression has been used by Robson[7] to fit waveguide dispersion curves neglecting attenuation.

From Equation II-7 the delta function response of the waveguide can be calculated.

$$G_q(t) = 2q \frac{J_{2q}(\omega_c t)}{t} e^{i(\omega_o t - \pi q)} e^{-\alpha_o \omega_c t/2}. \qquad \text{(II-8)}$$

A second approximation to $\Gamma(\omega)$ which ignores dispersive effects is to expand $\Gamma(\omega)$ around some operating frequency ω_a.

$$\Gamma(\omega) = \Gamma_a + \Gamma'_a(i\omega - i\omega_a) + \cdots, \qquad \text{(II-9)}$$

where

$$\Gamma'_a = \left.\frac{\partial\Gamma(\omega)}{\partial(i\omega)}\right|_{\omega_a}.$$

From Equation II-9 the delta function response of the waveguide is

$$G_q^o(t) = \delta(t - q\Gamma'_a) e^{i\omega_a t} e^{-\frac{\Gamma_a t}{\Gamma'_a}} \qquad \text{(II-10)}$$

In this case dispersive effects are neglected. The interpretation in terms of a delta function impulse moving down the waveguide is apparent.

Pulse transmission in the waveguide.

By taking the inverse transform of Eq. II-4 we obtain the response of the waveguide to arbitrary input signal.

$$V_q(t) = \int_o^t V_o(t-\tau)\, G_q(\tau)\, d\tau. \qquad \text{(II-11)}$$

In general the solutions found including dispersive effects require computer evaluation to be meaningful. For illustration we give solutions found using $G_q^o(t)$, Eq. II-10. Let:

$$V_o(t) = e^{i\omega_a t},\ t \geq o, \qquad \text{(II-12)}$$

a step function at t=o modulated by rf at frequency ω_a. The signal at point q in the waveguide is

$$V_q(t) = e^{i(\omega_a t-\beta_a q)}\, e^{-\alpha_a q},\ t \geq q\Gamma' \qquad \text{(II-13)}$$

$$= o \qquad ,\ t \leq q\Gamma',$$

where we have written the complex phase shift Γ_a as

$$\Gamma_a = \alpha_a + i\beta_a.$$

Figure 6 shows rf pulse transmission calculated for a specific waveguide using both Eq. II-8 and II-10 for $G_q(t)$. These calculations were for a specific S-band accelerator waveguide of 86 disks. The various waveforms shown are for different rise times of incident rf pulse $V_o(t)$. $Z_q^o(t)$ represents the solution II-13. In addition to the rf amplitude modulation exhibited in Fig. 6, there is considerable phase modulation during the rising portion of the curve.

Energy Gain at Vanishing Beam Current

In the absence of appreciable beam loading, a particle passing through the waveguide sees the rf field $V_q(t)$ in the waveguide at the time the particle reaches point q. The integration of this field over the waveguide is the particle energy gain. We define the time for a particle entering the waveguide at time t=o to reach point q to be

$$\text{transit time of particle to point } q = q\left(\frac{\pi+\Psi_o}{\omega_a}\right). \text{(II-15)}$$

If $\pi+\Psi_o-\beta_a=o$, the particle motion is synchronous with the wave.

For particles entering the waveguide at time t, their energy gain, $\Delta E(t)$, is

$$\Delta E(t) = \int_o^{q_m} V_q\left(t+q\ \frac{\pi+\Psi_o}{\omega_a}\right) dq, \qquad \text{(II-16)}$$

where q_m is the length of the waveguide.

Figure 7 shows an example of $\Delta E(t)$ for the same $2\pi/3$ waveguide discussed previously in Fig. 6 for the case where the particle and wave velocity are the same.

Linear Accelerators with Beam Loading

Consider a unit delta function current pulse entering the waveguide at time t=0. This current pulse will arrive at some point u at time $u\left(\frac{\pi+\Psi_o}{\omega}\right)$. As the current pulse passes point u it induces an impulse in the waveguide and consequent forward and backward waves are transmitted to other points of the waveguide. The general form of these transmitted waves is that of Eqs. II-8 or II-10, the response of the waveguide to a delta function impulse.

We wish to know the rf wave at a point q in the waveguide due to the interaction of the current pulse with the waveguide at all points u. If q>u the wave which reaches point q is a forward-going wave. If q<u the wave reaching q is a backward-going wave.

Thus the forward-going induced rf waves at point q are

$$V_q^f(t) = -K_f \int_o^q G_{(q-u)}\left[t-u\left(\frac{\pi+\Psi_o}{\omega}\right)\right]du,\ t-u\left(\frac{\pi+\Psi_o}{\omega}\right)\geq 0, \text{(II-17)}$$

with a similar expression for the backward going wave. The coefficient K_f is an undetermined coupling of the beam pulse to the waveguide. We assume this coefficient to be real and positive; however we have no valid justification for this. The negative sign is to account for the fact that the beam induced wave is such as to decrease the electron energy.

Equation II-17 represents the induced wave due to a single delta function current pulse entering the waveguide at time t=0. The induced wave due to arbitrary current pulse can be found by folding the actual current pulse with $V_q^f(t)$. We will consider one special case. We would like to determine the beam-loading wave for a series of delta function current pulses occurring at repetition rate ω. For this case the folding integral of $V_q^f(t)$ with the true current distribution reduces to a summation of time delayed waves of the form given in Eq. II-17. If the first current pulse enters the waveguide at time t=0, then the k^{th} current pulse will enter the waveguide at $t=\left(\frac{2\pi}{\omega}\right)k$. Thus the forward going wave, $W_q^f(t)$, for this repetitive pulse situation is

$$W_q^f(t) = \sum_{k=o} V_q^f\left(t-\frac{2\pi}{\omega}k\right), \qquad \text{(II-18)}$$

where $t-u\left(\frac{\pi+\Psi_o}{\omega}\right)-\frac{2\pi}{\omega}k \geq o$ and $V_q^f(t)$ is given by Eq. II-17.

We would like to know the change in energy of a beam particle due to this beam-induced wave. For a particle in the k_m^{th} pulse, the particle passes point q of the waveguide at time $t=\left(\frac{2\pi}{\omega}\right)k_m+q\left(\frac{\pi+\Psi_o}{\omega}\right)$. The beam induced rf at this time at point q is

$$W_q^f\left[t=\frac{2\pi}{\omega}\,k_m + q\left(\frac{\pi+\Psi_o}{\omega}\right)\right]. \qquad \text{(II-19)}$$

The energy change, $\Delta E_{k_m}^f$, of the k_m^{th} particle is obtained by integrating Eq. II-19 over the length of the waveguide.

$$\Delta E^f_{k_m} = \int_0^{q_m} W^f_q\Big[t = \frac{2\pi}{\omega} k_m + q\left(\frac{\pi+\Psi_o}{\omega}\right)\Big]dq,$$

$$\text{where } \frac{2\pi}{\omega}(k_m - k) + \left(\frac{\pi+\Psi_o}{\omega}\right)(q-u) \geq 0. \qquad \text{(II-20)}$$

$\Delta E^f_{k_m}$ represents the energy loss due to rf fields in the waveguide induced by the beam. To this should be added the energy gain of the beam due to rf from the power source, Eq. II-16, to obtain the total beam energy change in the waveguide.

Evaluations of Eqs. II-17 and II-20 are too cumbersome to place in this report but can be found in reference.[5] Except for effects which occur for extremely short beam bursts, beam loading effects are essentially the same as those calculated from the power diffusion equation. For extremely short high intensity beam bursts however, large effects are predicted, resulting in substantially greater beam loading than would be predicted by the power diffusion equation. Figures 8 and 9 illustrate this effect. What is plotted in these figures is the contribution to the beam loading energy loss, $\Delta E^f_{k_m}$, due to beam loading by the k_mth beam pulse and previous beam pulses. The total energy loss of the k_mth pulse is therefore the sum of k_m points from this curve. Figure 10 illustrates such a sum for three cases; $\Delta h=1$ corresponding to injection every rf cycle, and $\Delta h=3$ and $\Delta h=5$ corresponding to subharmonic injection every third and fifth rf cycle respectively.

The $\frac{2\pi}{3}$ mode, L-band waveguide used to calculate Figs. 8 to 10 has a steady state current limit i_m, Eq. I-9, of 0.684 amperes with 10 megawatts of input rf power. For this waveguide and input power, Figs. 9 and 10 indicate that a beam current of 24 amperes in a beam pulse one rf cycle wide would have a beam loading energy loss of 50%, when dispersive effects are included. If dispersive effects are neglected, the corresponding prediction is a beam current of about 10^3 amperes. While this result is quite sensitive to the detailed nature of the beam pulse, it does indicate the extreme importance of dispersive effects for high current, nanosecond pulsed accelerators.

Beam Excitation of other Spatial Modes in the Waveguide

We have thus far neglected the influence of all spatial harmonics in the waveguide except that represented by $\Gamma^n_{o,o}(\omega)$, Eq. II-5. The response function $G^n_q(t)$ for these spatial harmonics is simply related to $G_q(t)$ by (neglecting the coefficient $a_{o,n}(\omega)$),

$$G^n_q(t) = G_q(t)\, e^{-i4\pi nq}, \quad n \text{ integer}. \qquad \text{(II-21)}$$

It is usually argued that these spatial harmonics are incoherent with the beam pulses and have very little average effect. For rf in the waveguide from an external power source, this can be shown to be true;[5,6] however for beam induced rf waves it is not true in all cases. An evaluation equivalent to Eq. II-20 for beam energy loss due to exitation of the nth spatial harmonic, either forward going waves or backward going waves, shows that there is transient beam loading due to excitation of these spatial harmonics. The time for this transient effect depends upon the group velocity in the waveguide. For typical cases of $\pi/2$ or $2\pi/3$ waveguides, this transient effect will last about 100 rf cycles. In the special case however of accelerators operating with zero group velocity, the excitation of these spatial harmonics is completely coherent with excitation of the fundamental harmonic for both forward and backward going waves. In this special case beam loading will be about twice as large as that predicted if one considers only the fundamental forward going wave.

Application to Other Structures

The filter theory approach discussed previously has thus far assumed a constant-structure traveling-wave linear accelerator. In this section we would like to indicate the application of these techniques to other structures, namely the standing-wave linear accelerator, and the constant-gradient traveling-wave accelerator.

Standing Wave Structures

One converts a traveling-wave structure into a standing-wave structure by placing suitably reflecting terminations on the waveguide. Neglecting the problem of coupling to the power source, the response function $G_q(t)$ analogous to Eq. II-10 for this structure is easily determined by simply adding the various time delayed and attenuated waves. Thus for a standing-wave structure of length q_m ($2q_m$ loading disks in the waveguide), the transfer function which represents the forward-going waves is

$$G^o_q(t) = e^{i\omega_a t}\, e^{-\frac{\Gamma_a t}{\Gamma'_a}} \sum_n \delta(t - q\Gamma'_a - 2nq_m\Gamma'_a), \qquad \text{(II-22)}$$

where the limit on n is given by $t - q\Gamma'_a - 2nq_m\Gamma'_a \geq 0$. If we apply an rf signal to the input of this guide of the form given by Eq. II-12 the resultant field in the guide is given by

$$V_q(t) = e^{i\omega_a t}\, e^{-q\Gamma_a} \sum_{n=o} (e^{-2q_m\Gamma_a})^n, \qquad \text{(II-23)}$$

where $t - q\Gamma'_a - 2nq_m\Gamma'_a \geq 0$.

In the steady state limit, letting $\Gamma_a = \alpha_a + \beta_a$,

$$\lim_{t\to\infty} V_q(t) = e^{i(\omega_a t - \beta_a q)}\, e^{-\alpha_a q}\left(\frac{1}{1 - e^{-2iq_m\beta_a}\, e^{-2q_m\alpha_a}}\right). \qquad \text{(II-24)}$$

Equation II-24 exhibits the expected normal-mode behavior of the standing-wave structure. There are $2q_m+1$ resonances corresponding to frequencies having phase shift β_a between 0 and 2π.

Beam-loading effects may be calculated exactly as for the traveling-wave accelerator using Eqs. II-17 to II-20.

Constant-Gradient Accelerator

For the constant-gradient or variable structure accelerator in which the parameters of the waveguide vary along the guide, the expression for $G_q(\omega)$ given in Eq. II-6 should be replaced by

$$G_q(\omega) = F(\omega,q)\, e^{-\int_o^q \Gamma(\omega,q)dq}\;, \qquad \text{(II-25)}$$

the factor $F(\omega,q)$ being required for normalization. We will consider two examples of this type. It has been shown[3] that the attenuation of the waveguide may be corrected to produce a uniform field in the guide by varying the waveguide parameters such that

$$\alpha_a(q) = \frac{\alpha_o}{1-\alpha_o q},$$

$$\beta_a(q) = \beta_o, \qquad \text{(II-26)}$$

$$\text{and } \Gamma'_a(q) = \frac{\Gamma'_o}{1-\alpha_o q}\;,$$

where α_o, β_o and Γ_o represent the parameters at q=0. From Eq. II-25 one can show that for this case the transfer function corresponding to Eq. II-10 is

$$G_q(t) = \frac{1}{1-\alpha_o q}\delta\left(t-\frac{\Gamma'_o}{\alpha_o}\ln\left(\frac{1}{1-\alpha_o q}\right)\right) e^{i\left(\omega_a t - \frac{\beta_a q t}{\frac{\Gamma'_o}{\alpha_o}\ln\left(\frac{1}{1-\alpha_o q}\right)}\right)} e^{-\frac{\alpha_o t}{\Gamma'_o}} \qquad \text{(II-27)}$$

Beam loading effects may be calculated as before using Eqs. II-17 to II-19; however now Eq. II-17 becomes more complicated since care must be taken to use appropriate starting values of the parameters corresponding to the source point u.

A second illustrative example, of interest to study of the beam-blow up phenomena, is a waveguide whose phase shift β_a varies along the guide. We consider the special case of

$$\beta_a(q) = \beta_o(1+\gamma q), \qquad \text{(II-28)}$$

with α_a and Γ'_a assumed independent of q. The transfer function in this case corresponding to Eq. II-10 is

$$G_q(t)=\delta(t-q\Gamma'_a)\left[e^{-i\left(\omega_a t-\frac{\beta_o t}{\Gamma'_a}\right)-\frac{\alpha_a t}{\Gamma'_a}}\right]e^{-\frac{i\beta_a t}{\Gamma'_a}\frac{\gamma q}{2}}\,. \qquad \text{(II-29)}$$

Beam Excitation of other Waveguide Pass-bands

The bunched nature of the particle-beam in the linear accelerator implies that the beam has frequency components which can excite other waveguide passbands. The most important effect of this sort is the phenomena known as beam blow-up due to excitation of waveguide modes having large transverse accelerating fields. Wilson[8] has presented an explanation of this effect in analogy with the theory of the backward wave oscillator. Similar calculations have been made by Gluckstern[9] for standing-wave linacs.

The calculations in reference 8 have been concerned primarily with dynamical effects. These effects will not be discussed here, but rather coherence phenomena will be discussed. A calculation[5] very similar to the development of Eq. II-20 can be made for beam excitation of higher pass-bands. These calculations indicate that the energy given beam pulses due to excitation of some higher pass-band by a series of beam bursts occurring at times $(\frac{2\pi}{\omega a})h$, h integer, is of the form

$$\sum_h [\text{slowly varying coefficient}]\, e^{i2\pi\frac{\omega_{a1}}{\omega_a}h}\;, \qquad \text{(II-30)}$$

where ωa_1 is the frequency at which the higher pass-band has phase velocity equal to the particle velocity. For many traveling wave electron linacs $\omega a_1/\omega a \approx 3/2$ and successive terms in the summation of Eq. II-30 tend to cancel. This is true for beam bursts occurring at any odd subharmonic of ωa. If however beam injection occurs at even subharmonics of ωa, the cancellation of successive terms in Eq. II-30 does not occur and the tendency for beam-blow up is considerably enhanced. This is of concern, for example, in high energy proton linear accelerators where the transition to disk-loaded structures is planned with an even integer ratio between frequencies of the two portions of the accelerator. Calculations[9] for typical standing-wave structures using a normal mode treatment have indicated the effect is not too severe. The validity however of a normal mode calculation is not clear (see Eqs. II-23 and II-24) since summations over normal mode frequencies are only an approximation to integrals over all frequencies.

Suppression of beam blow-up by varying the waveguide parameters has been observed on many accelerators. Equation II-29 can be used to estimate this effect. We define a coherence length, ℓ, as the distance excited waves in the guide must move to fall out of phase by $\pi/2$. Due to the term $e^{\frac{i\beta_o t\gamma q}{2\Gamma'_a}}$ in Eq. II-29, this coherence length is

$$\ell = \sqrt{\frac{\pi}{\beta_o\gamma}}\;. \qquad \text{(II-31)}$$

Beam blow-up can not occur if the guide length required to start the backward wave oscillator with a given current is larger than ℓ. In practical cases, this provides a powerful suppressant to the beam blow-up phenonena.

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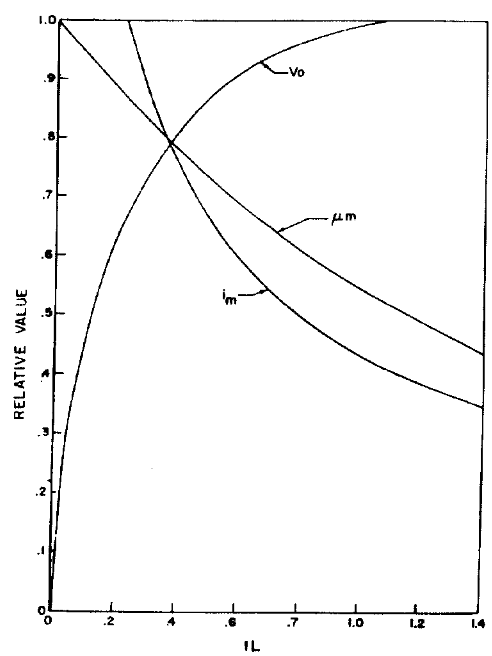

Fig. 1. Plot of μ_m, i_m, and V_o versus guide attenuation length for a constant structure traveling wave accelerator assuming input power P_o and guide wavelength L are held constant.

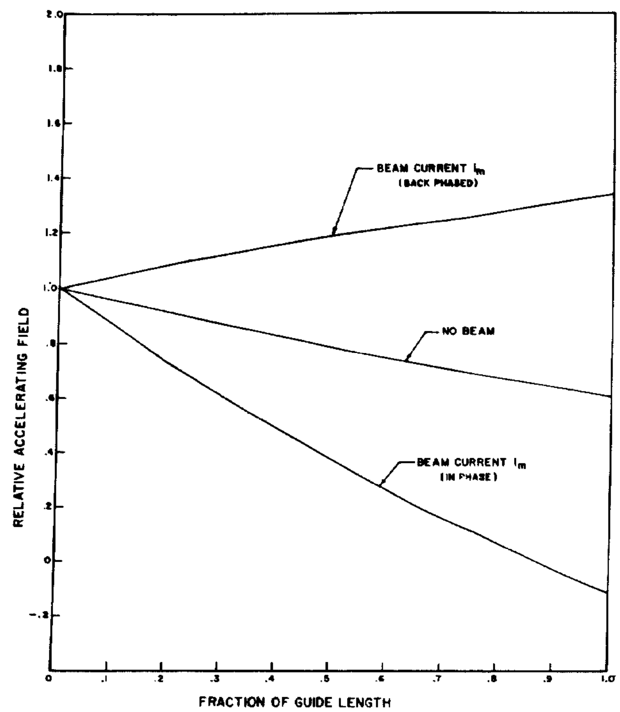

Fig. 2. Relative accelerating field for constant gradient accelerator (IL=0.5) for no beam current, beam current i_m with ϕ=0, and beam current i_m with ϕ=180°.

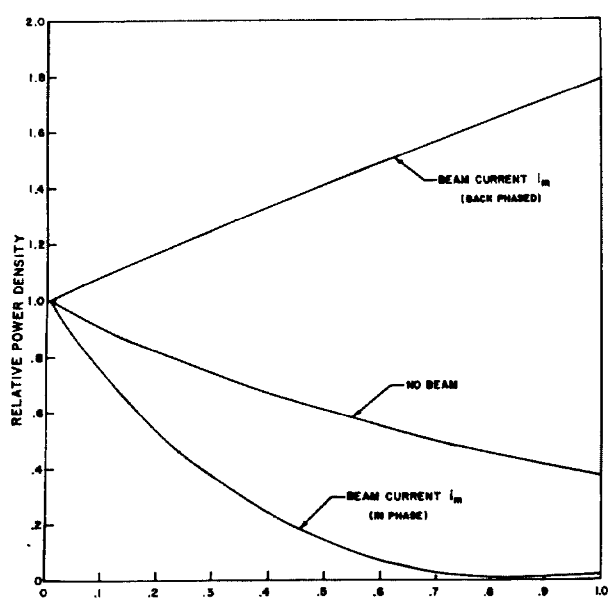

Fig. 3. Same as Figure 2 but for rf power density in waveguide.

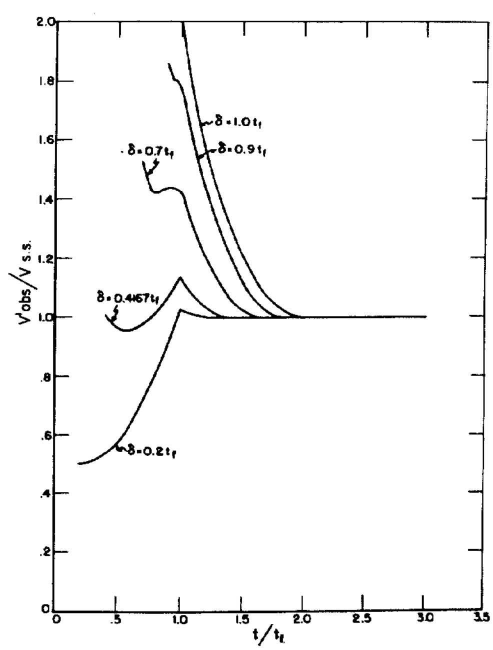

Fig. 4. Relative energy gain of beam current i_m for different beam turn on times during the rf filling time of the waveguide.

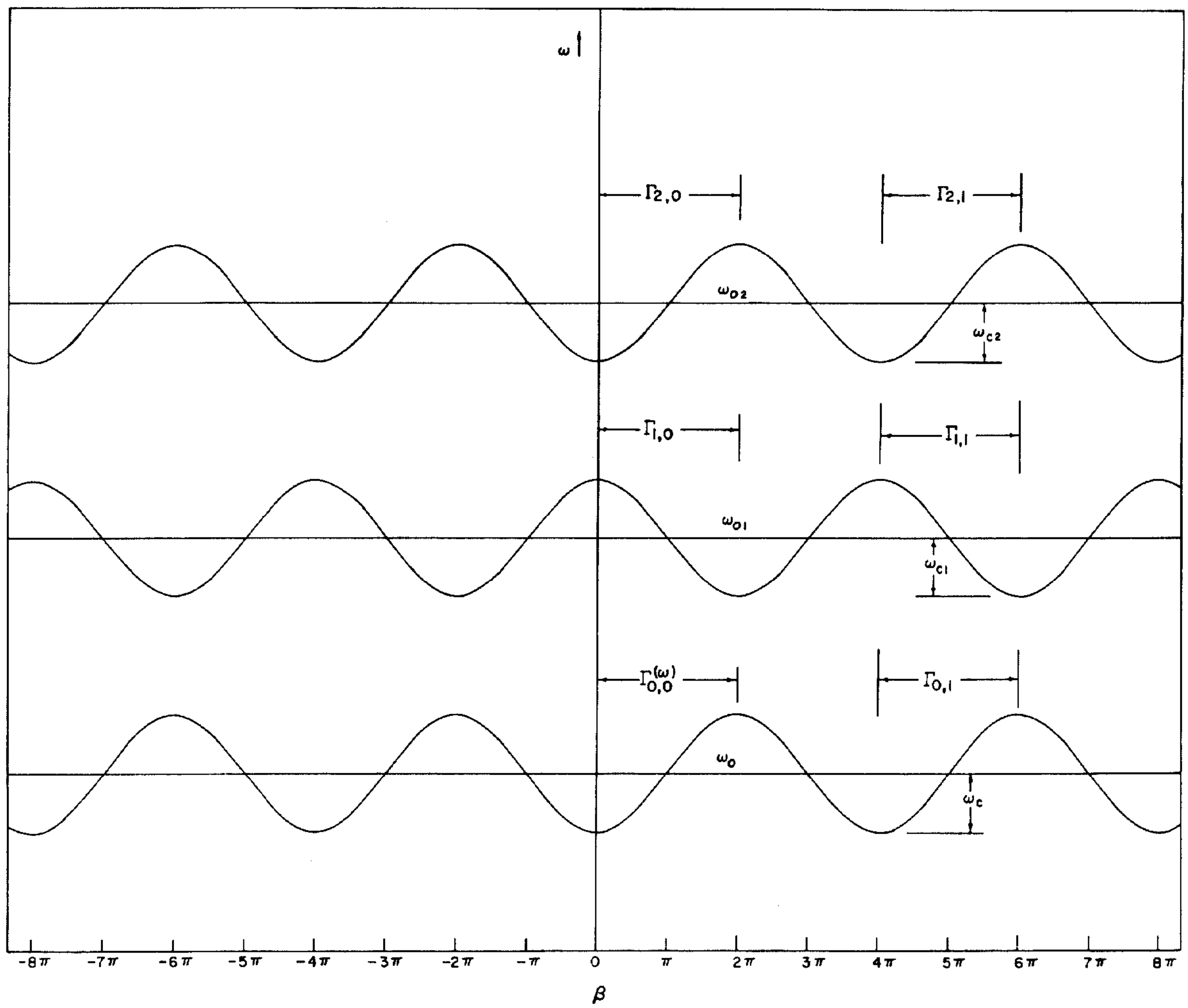

Fig. 5. The imaginary part, β, of the complex phase shift, Γ, for a typical disk-loaded waveguide structure plotted against frequency, ω. Three of the many pass-bands are indicated. Particle acceleration is usually in the region marked by $\Gamma_{0,0}(\omega)$. Microwave particle separators operate in the region marked $\Gamma_{1,0}(\omega)$.

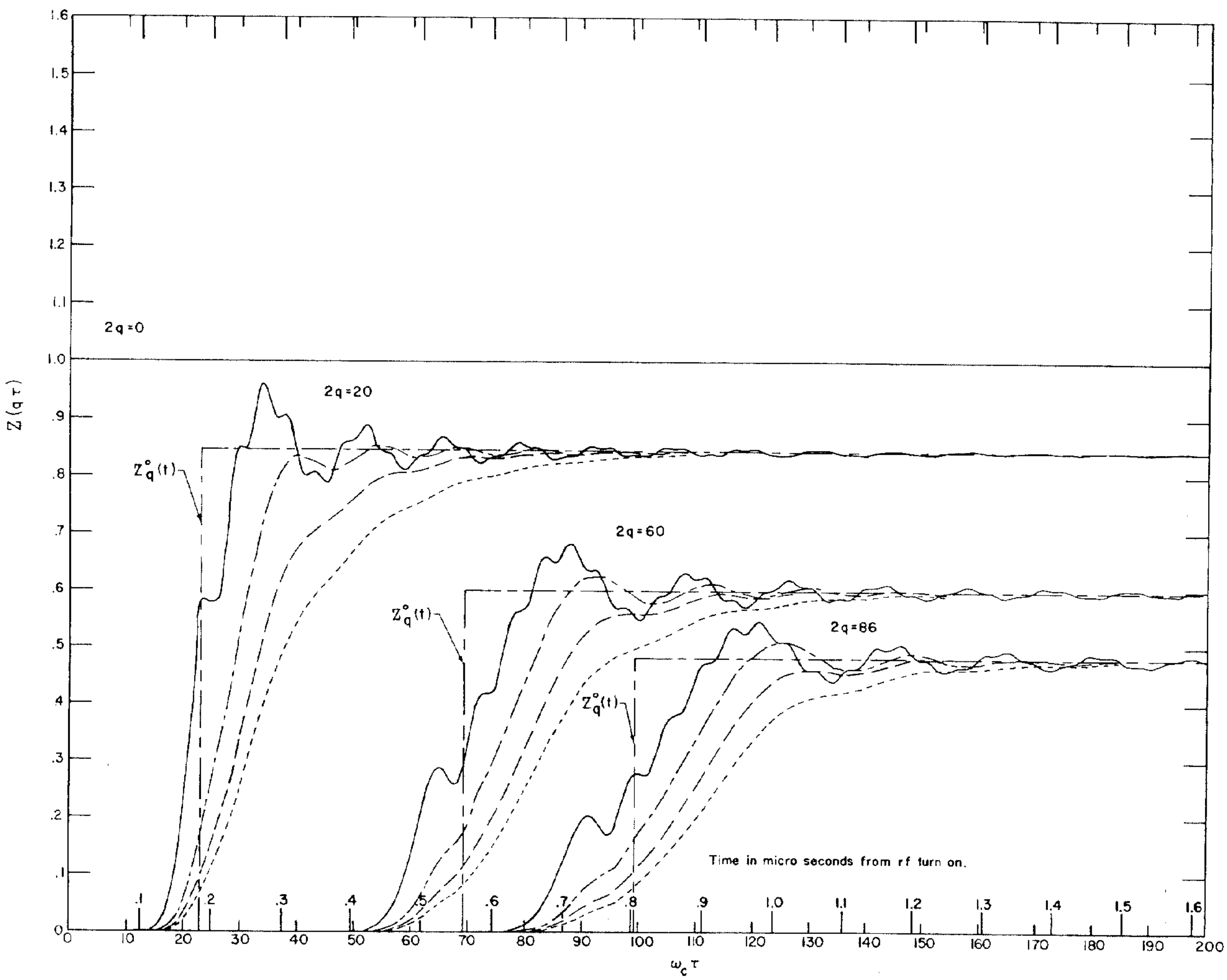

Fig. 6. Pulse transmission in $\frac{2\pi}{3}$ mode S-band accelerator of 86 disks for different values of input risetime plotted against time in units of $\omega_c t$. The amplitude $z(q,t)$ is shown for the signal at different points in the waveguide calculated using Eq. II-8. z_q^o (t) is calculated using Eq. II-10.

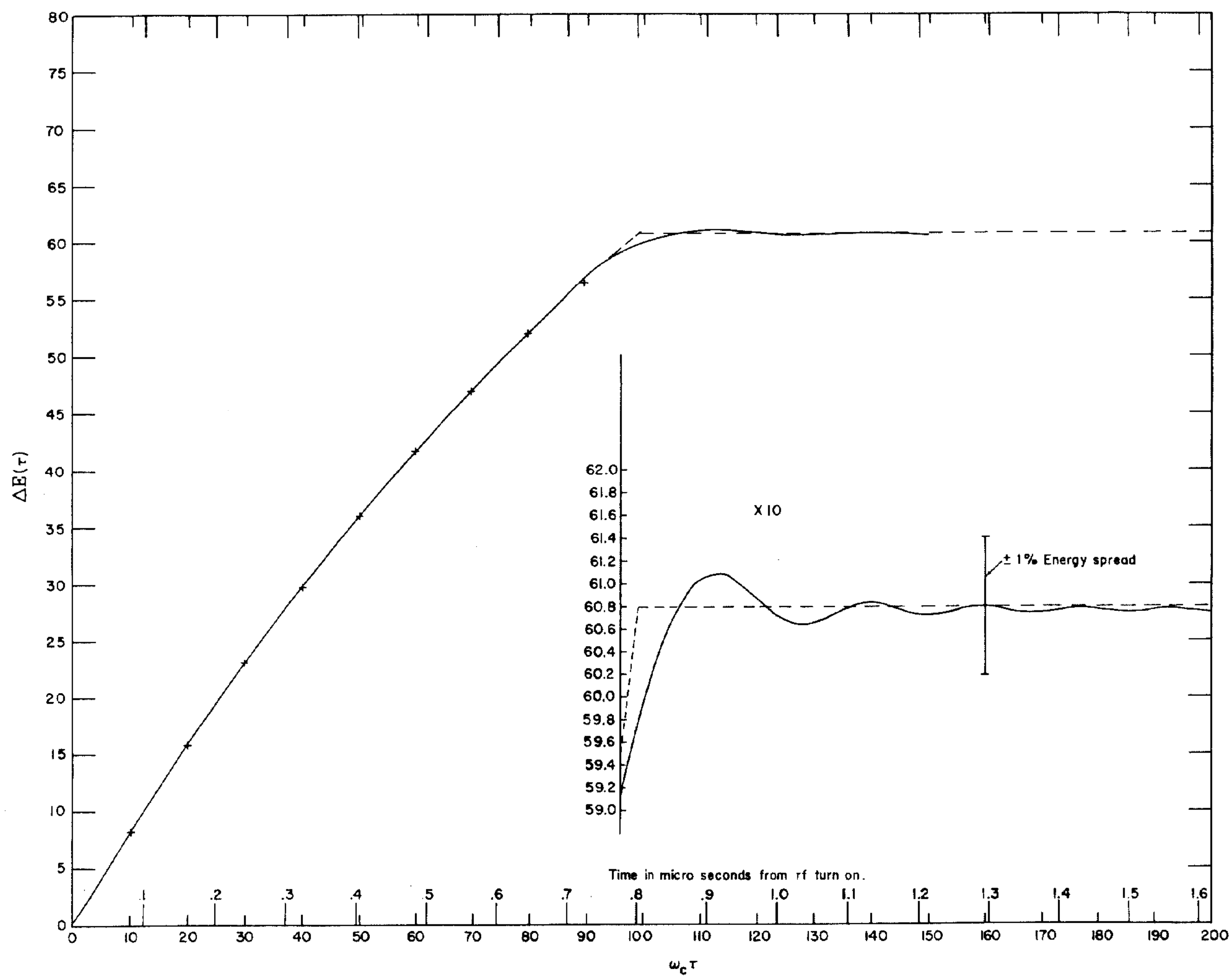

Fig. 7. Relative energy gain, $\Delta E(t)$, at vanishing current plotted versus time in units of $\omega_c t$ for the same case as Fig. 6. The dotted curve is calculated using Eq. II-10, the solid curve using Eq. II-8.

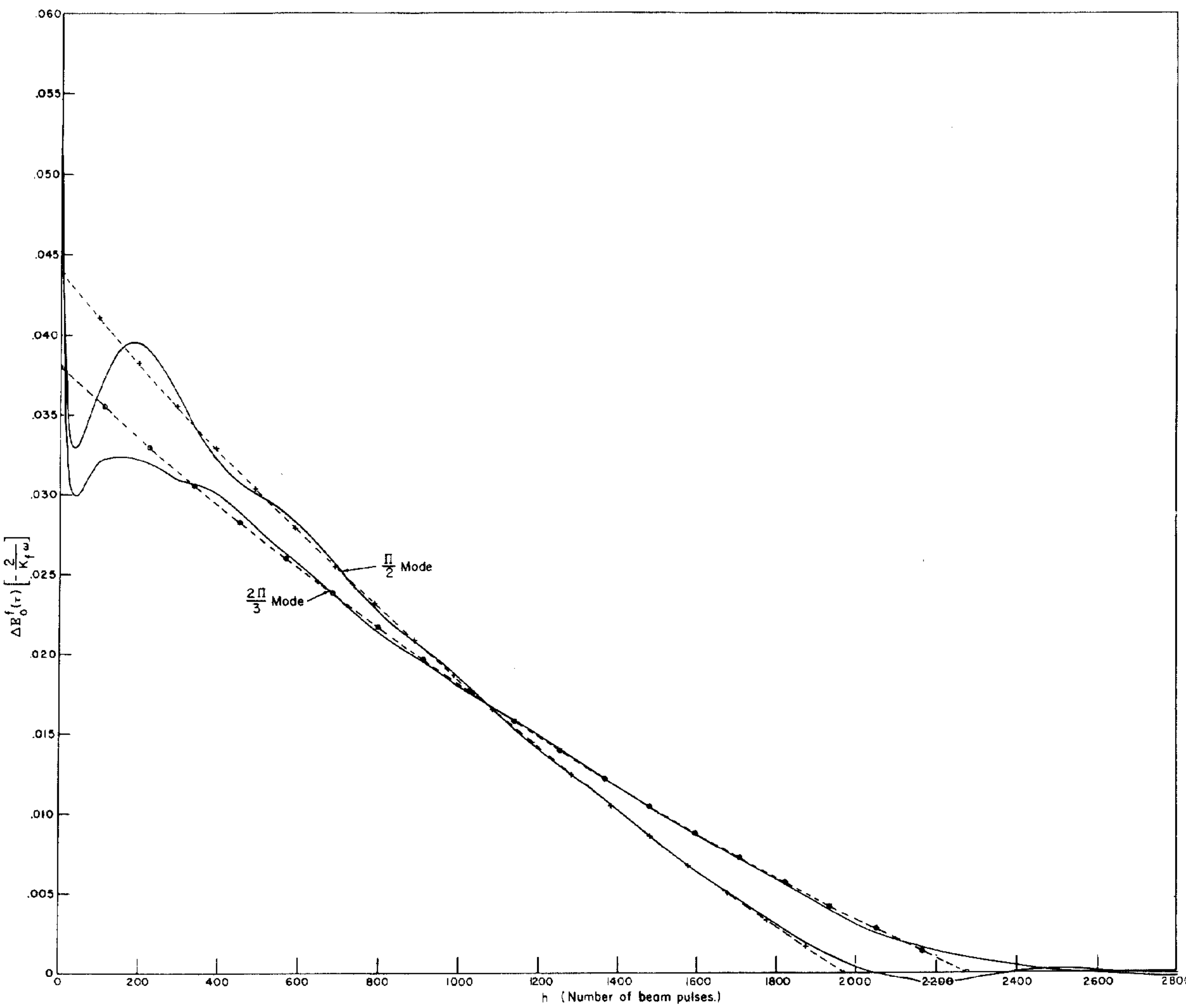

Fig. 8. Relative beam loading energy loss for a series of δ function current pulses passing through two L-band waveguides. The solid curves are calculated using Eq. II-8, the dotted curves using Eq. II-10. The meaning of the curves is that the value of the curve for h=0 corresponds to the energy loss of a beam burst to itself. For h=1, the value of the curve corresponds to energy loss due to the previous beam burst, etc. The total energy loss in the k_m^{th} pulse if $\sum_{h=0}^{km} \Delta E_o^f(t)$. The solid curve is calculated using Eq. II-8, the dotted curve using Eq. II-10.

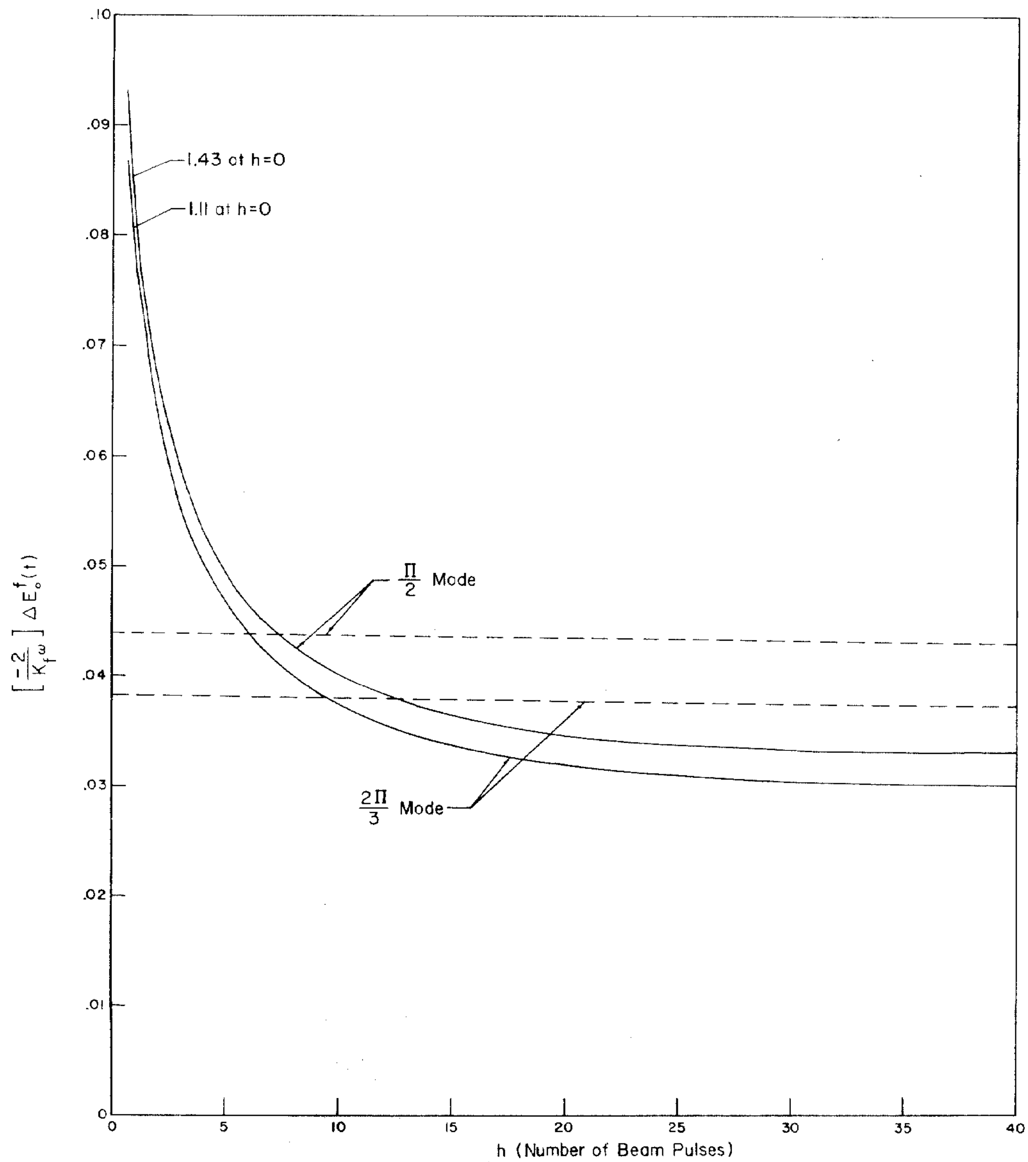

Fig. 9. Same as Figure 8, but with expanded scale. Note the increased beam loading indicated for very short beam pulses when dispersive effects are included.

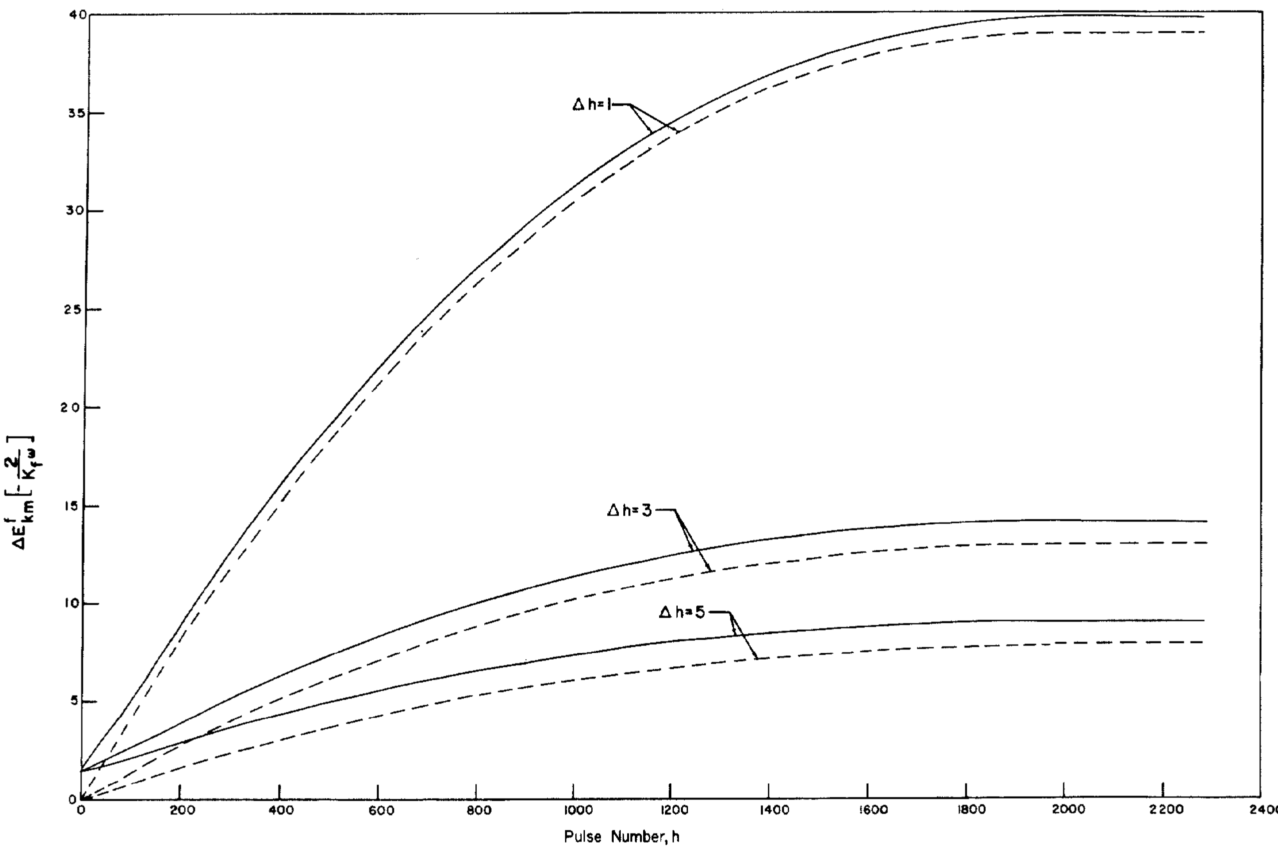

Fig. 10. ΔE^{f}_{km} for the $\frac{2\pi}{3}$ mode waveguide of Figs. 8 and 9. This curve is generated by taking the running sum of points from Figs. 8 and 9. Curves marked Δh=1,3,5 correspond to beam bursts ocurring every rf cycle, every third cycle, and every fifth cycle respectively. The solid and dotted curves have the same meaning as in Fig. 8.