PROBLEMS IN THE DESIGN OF CRYOGENIC TARGET FACILITIES FOR ELECTRON SCATTERING EXPERIMENTS

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Summary

This paper describes a proposed target facility for use in a program of electron scattering experiments at the Naval Research Laboratory electron linear accelerator on inelastic electron scattering experiments on low atomic number elements as H^1, H^2, H^3, He^3 and He^4, all of which in the liquid or solid state provide compact, high density, well localized, targets of pure elements. Problems discussed are: (a) electron energy dissipation in cryogenic targets by continuous electron beams of 5-60 MeV energy, (b) target design dimensions such as target diameter to beam diameter ratios, permitting maximum incident beam current, (c) treatment of pulsed electron beams using "instantaneous line source theory" in estimation of maximum permissible current densities, (d) vapor production and mechanical shock in targets based in part on bubble chamber theory, and (e) thermal radiation into the target shields from the scattering chamber wall.

The Cryogenic Target Ensemble

The interchangeable triple target system considered for the scattering assembly is shown in Fig. 1. The two upper targets are separately fed by H_2 and D_2 lines to wells in a copper block which also carries the third, carbon or optional solid, target. Change of targets is effected by action of a central push tube connected, as shown, together with a flexible sylphon to the target block which slides within a helium-cooled shield. Helium gas is condensed on the sylphon wall, flows to the bottom of the space between the rod and sylphon, where its subsequent evaporation removes heat from the target block. The helium gas pressure in this space determines thermal conduction flux between the central tube and sylphon wall, and provides a means of controlling the target block temperature. These features were suggested by Andonian Associates Inc. Waltham, Mass. who hold proprietary rights. Surrounding the central system are radiation shields, connected to the liquid nitrogen section of the Dewar, which part is built directly on the flange cover of the scattering chamber. The target system may be rotated by rotation of the whole Dewar on slide seals (not shown) or by a similar rotation of the inner He bath system. The shields have annular slots permitting entrance of an electron beam and exit of electrons scattered in desired angular ranges.

Electron Energy Dissipation in Cryo-Targets

The electrons are considered as "extreme relativistic" so that use may be made of the Bohr-Bethe equation (Heitler, "Quantum Theory of Radiation", 3rd ed. p.370).

$$-\left(\frac{dE}{dx}\right)_{col} = NZ\, \phi_o\, \mu\, 3/4 \ln \frac{E^3}{2\, I^2\, Z^2\, \mu} \qquad (1)$$

which gives the collisional energy loss, per unit length, of an electron of energy E, in a medium containing N atoms of atomic number Z per cm^3. The other constants ϕ_o, μ and I are respectively, the Thomson cross section ϕ_o (6.653×10^{-25} cm^2), μ the electron rest mass energy, 0.511 MeV, and I = 11.5 ev. Combining constants and reducing to quantities of interest the above becomes

$$-\left(\frac{dE}{dx}\right)_{col} = 5.87 \times 10^{-25}\, NZ \left\{9.869 \times \log_{10} \frac{E^3}{Z^2}\right\} \qquad (2)$$

MeV/cm/electron

From Table I "normal" H_2 and D_2 are liquid below 20.39°K and 23.57°K respectively, at 1 atmosphere pressure. The molar volumes of these liquids are given by:

$$V_{H_2} = 24.747 - 0.08005\, T + 0.012716\, T^2 \ cm^3/mol \qquad (3)$$

$$V_{D_2} = 22.965 - 0.2460\, T + 0.0137\, T^2 \ cm^3/mol. \qquad (4)$$

The values of these at their respective boiling points are V_{H_2} (20.39°) = 28.402 cm^3/mol; V_{D_2} (23.57°) = 24.779 cm^3/mol.

By Avogradro's number, 6.02472×10^{23} molecules/mol, the number of atoms per cm^3 of H_2 and D_2 at their atmospheric boiling points are

$$N_{H_2}\ (20.39^o) = 4.24525 \times 10^{22}\ H^1\ atoms/cm^3 \qquad (5)$$

$$N_{D_2}\ (23.57^o) = 4.86276 \times 10^{22}\ H^2\ atoms/cm^3. \qquad (6)$$

Equation (2) may now be written to express the heat input (or energy loss/cm) produced by a 1-ampere beam which delivers 6.2422×10^{18} electrons per second. Using the conversion factor, 1 MeV = 3.8296×10^{-14} calories,

$$\Delta H = 5.871 \times 10^{-19} \left\{9.869 + \log_{10} E^3/Z^2\right\} \quad NZ \text{ watt-sec/amp/cm}$$

$$= 1.403_5 \times 10^{-19} \left\{9.869 \times \log_{10} E^3/Z^2\right\} \quad NZ \text{ cal/sec/amp/cm.} \qquad (7)$$

Using Z = 1 and the values N_{H_2} and N_{D_2} computed above, the values of ΔH were computed for values of E up to 60 MeV. These are shown in Fig. (2) together with corresponding $-\left(\frac{dE}{dx}\right)$ values for a single electron. The ΔH values refer to the number of electrons in one ampere without regard to cross sectional distribution, and material 1 cm thick.

Heat Input Due to a Continuous Electron Beam

Using the above equations for energy delivered per electron or per ampere into unit thickness of liquid target, the temperature field produced by the passage of a continuous beam of electrons may be computed from the Fourier heat conduction equation(1).

$$c\rho \frac{d\theta}{dt} = K \Delta^2\theta + Q \qquad (8)$$

where a falling temperature gradient is understood. θ and t are the temperature and time respectively, and c, ρ, K and Q are specific heat, density, thermal conductivity, and heat produced per unit volume per unit time, assuming a uniform rate. Under these conditions a steady state will set in making $\frac{d\theta}{dt} = 0$ after some time. Rewriting the La Placian in cylindrical cordinates (r, θ, z) and assuming symmetry around the beam the above equation is

$$\Delta^2\theta = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + \frac{d^2\theta}{dz^2} = -Q/K. \qquad (9)$$

The problem is made as simple as possible by assuming a beam of circular cross section passes through a cylindrical target mass in the direction of the cylinder axis and symmetrically disposed about it. If it is further assumed that radiation from the front and back surface of the target may be neglected at the low temperatures, the equation becomes

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} + \frac{Q}{K} = 0 \qquad (10)$$

the solution of which is

$$\theta = C_1 + C_2 \ln r - \frac{Q}{K}\frac{r^2}{4}. \qquad (11)$$

The solution is not valid for values of $r \to o$ but the ln r term may be rejected to obtain a solution valid for points near the cylinder axis.

For values of $r > r_i$, the electron beam radius, no heat is produced, (Q = 0) and the differential equation is simply

$$\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) = 0 \qquad \left(r_i < r < r_o\right) \qquad (12)$$

where r_o is the radius of the liquid cylinder. The solution is

$$\theta = g_1 \ln r + g_2 \qquad (13)$$

At the beam edge $\left(r = r_i\right)$ the two solutions must yield the same temperature and temperature gradient

$$C_1 - \frac{Q}{K}\frac{r_i^2}{4} = g_1 \ln r_i + g_2 \qquad (14)$$

$$-\frac{Q}{2K} r_i = g_1/r_i. \qquad (15)$$

At r = o eq. (11) gives $\theta_c = C_1$. This value and the value of $g_1 = -\frac{Q}{2K} r_i^2$ given by (15) gives

$$g_2 = \theta_c - \frac{Q}{2K}\left(1/2 - \ln r_i\right) r_i^2. \qquad (16)$$

With constants evaluated, the solutions for the two regions within and without $r = r_i$ are:

$$\theta = \theta_c - \frac{Q}{K}\frac{r^2}{4}; \left(r < r_i\right) \qquad (17)$$

$$\theta = \frac{Q}{2K} r_i^2 \ln \frac{r_i}{r} + \left(\theta_c - \frac{Qr_i^2}{4K}\right); \left(r_i < r < r_o\right). \qquad (18)$$

In this application, the temperature at r_o is fixed by the temperature θ_o (the reservoir temperature), and by the second equation above

$$\theta_c = \theta + \frac{Q\, r_i^2}{4K}\left(1 + 2 \ln \frac{r_o}{r_i}\right). \qquad (19)$$

Target Design Dimensions

The above equation allows calculation of the value of $\frac{Q}{K}$ necessary to maintain a temperature difference $\left(\theta_c - \theta\right)$ between target center and target holder.

$$\frac{Q}{K} = \frac{4\left(\theta_c - \theta_o\right)}{r_i^2\left(1 + 2 \ln \frac{r_o}{r_i}\right)} \qquad (20)$$

which could also be specified by specifying values of θ_c and θ_i by means of eq. (17).

$$\frac{Q}{K} = \frac{4}{r_i^2}\left(\theta_c - \theta_i\right) \qquad (21)$$

The value of Q is related to the value of ΔH given by equation (7) by

$$Q = \frac{\Delta H}{\pi r_i^2} = \frac{4 K \left(\theta_c - \theta_o\right)}{r_i^2 \left(1 + 2 \ln \frac{r_o}{r_i}\right)} \quad (22)$$

giving

$$(\Delta H)_m = \frac{4 \pi K \left(\theta_c - \theta_c\right)}{1 + 2 \ln \frac{r_o}{r_i}} \quad (23)$$

where $(\Delta H)_m$ now indicates the maximum thermal input, per ampere, per cm of target thickness, permissible to maintain a specified temperature difference between target center and target edge at the holder reservoir. This equation is plotted in Fig. (3) using values of r_o/r_i as absissae and values of $\left(\theta_c - \theta_o\right)$ from 5 to 25°K as curve parameters. Alternatively, using $(\Delta H)_m$ or the corresponding values of beam energy E as curve parameters, values of $\frac{r_o}{r_i}$ may be given as functions of $\left(\theta_o - \theta\right)$. These show a very rapid increase for r_o/r_i with $\left(\theta_c - \theta_o\right)$ for all energies.

Instead of using $(\Delta H)_m$ as ordinates, the maximum beam current which will give the same thermal input at a particular electron beam energy may be plotted against r_o/r_i with $\theta_c - \theta_o$ as curve parameter. (The values of ΔH from eq. (23) are thermal values not as yet related to current while values of ΔH given by eq. (7) are heat inputs per ampere of beam).

In this way one obtains curves of maximum permissible beam current at some stated beam energy plotted against $\left(\theta_c - \theta_o\right)$, where one may use r_o/r_i as a curve parameter and obtain thereby linear plots, the logarithmic factors now being constants. Fig. (4) gives such a curve set for 60 MeV electron beams in H_2. Such curves show that increasing the value of r_o/r_i decreases the maximum electron beam current that may be used, and it would be best to use a target cylinder radius equal to the electron beam radius. If, to avoid accidental striking of the beam on the target holder, a choice of $r_o/r_i = 1.5$ is made, the maximum permissible beam current is halved.

It may be noted that r_i^2 may be eliminated between equations (20) and (21) which specify $\frac{Q}{K}$ giving

$$2 \ln \frac{r_o}{r_i} = \frac{\theta_i - \theta_o}{\theta_c - \theta_i}. \quad (24)$$

If the three temperatures θ_c, θ_o are given at the outset then r_o/r_i is fixed by these. If r_o/r_i is given, any one of the three may be given in terms of r_o/r_i and the other two temperatures. We have

$$\theta_o = \theta_i - 2 \left(\theta_c - \theta_i\right) \ln r_o/r_i \quad (25)$$

$$\theta_i = \left(\theta_o + 2 \theta_c \ln r_o/r_i\right)/\left(1 + 2 \ln r_o/r_i\right) \quad (26)$$

$$\theta_c = \left(\left(\theta_i - \theta_o\right) + 2 \theta_i \ln r_o/r_i\right)/\left(2 \ln r_o/r_i\right) \quad (27)$$

Using the second and taking a value of $r_o/r_i = 1.25$

$$\theta_i = \frac{\theta_o + 0.4463 \theta_c}{1.4463}.$$

With $\theta_c = 20$°K and $\theta_o = 10$°K $\theta_i = 13.08$°K which is below the solidification temperature (13.81°K) of liquid H_2. Thus the edge of the beam will be incident on solid H_2 in this case, and indicates that θ_i should be considered in making numerical calculations.

Pulsed Beams. Instantaneous Line Source Theory

The continuous current condition assumed above, in a way, approximates an average current in pulsed operation, but the transient effects due to current pulses must be examined for possible influence on design parameters adopted. An electron, being small compared to atomic dimensions, may be regarded as an ideal, almost instantaneous, line source of heat delivering amounts of energy per unit path length computed above. Because short times are involved, adiabatic approximation may be used for preliminary estimation of the electron heating effect in a material channel of arbitrary size. The passage of electrons through liquid targets is closely related to phenomena observed in bubble chamber operation, which in turn have been used here to obtain information on vapor production in these targets. Adiabatic estimates for channels of the radial dimensions postulated by Seitz[2] for critical bubble formation radius, and also for a 1 cm² x 1 cm channel exposed to a 1 μa-sec electron stream yield heating effects too small to raise material to the vaporization point. If however, a channel of radius corresponding to relaxation effects in line source theory is taken, temperature increases sufficient for vaporization are obtained. Thus line source theory will define a channel radius for non-adiabatic conditions within which vaporization temperatures exist.

The solution of the Fourier equation for heat conduction may be written in this case (c.f. Carslaw and Jaeger[1])

$$\theta = \frac{Q}{4\pi k t} e^{-R^2/4kt} \quad (29)$$

giving the temperature θ to be found at a distance R from an "instantaneous" line source at a time t after the source has acted in an extended medium of diffusivity k. The latter is defined by $k = \overline{K}/\rho c$ where $\overline{K}$ is the thermal conductivity, ρ the density and c the specific heat of the medium. Q is called the source strength and is related to the actual heat deposition per unit length ΔH used above, by $\Delta H = Q\rho c$ and is thus the temperature increase in unit volume of thermal capacity ρc due to liberation of ΔH units of heat. Differentiation of (29) with respect to + and setting to zero, shows that θ is a maximum at a distance R_m from the line source, at a time

$$t_{max} = R_m^2/4k \qquad (30)$$

which by (29) gives the corresponding maximum temperature

$$\theta_{max} = Q/e\pi R_m^2 \qquad (31)$$

The course of temperature as a function of time t and location in the medium R is easily viewed in Fig. (5) which shows that at a distance R from a line source the temperature will never exceed some value θ_m at any time, and thus this value of R serves to define a channel radius within which temperatures greater than θ_m are bound to appear in some ranges of the time variable. Using data from Table I and letting θ_m be the temperature interval $(33.19 - 20.34)^{o}K$ between the critical point and boiling point of H_2, one finds $Q = 0.906 \times 10^{-13}$ (^{o}K) for a 60 MeV electron and a "maximum" radius of $R_m = 2.879$ ($\times 10^{-8}$) cm at which θ_m is just reached in 0.697×10^{-13} sec. For values of $R<R_m$, temperatures exceeding this particular value of θ_m (chosen to represent an excursion to a vaporization point at the surface of a cylindrical channel of radius R_m) will occur at times $t < t_m$. Vapor conditions prevail in the channel but not outside of it. From the definition of $Q = \Delta H/\rho c$ and equation (31) above it is now possible to relate the values of R_m and θ_m to an average current density in the channel which will bring about the above temperature conditions together with an operative time τ

$$\bar{J} = \frac{e' \, e \, \theta_m \, \rho c}{\tau \left(\frac{dE}{dx}\right)_{col}} \quad \text{(e' is the electronic charge)} \qquad (32)$$

This value of current density may be defined as that current density which will bring about "incipient vaporization" in the channel of size noted. Since the temperature "relaxation time" is always less than $t_m = R_m^2/4k$ for all points within the channel it can also serve as a limiting current density or current density above which explosive vapor generation may certainly be expected. Using numerical data as above this is of the order of 41 amperes/cm^2 in a 1.5 μs pulse.

Vapor Production and Mechanical Shock

Bubble chamber data (2), (3) indicate that a relativistic electron produces about 15 bubbles of about 0.3 mm in size, per cm of path or 2.12×10^{-4} cm^3 of vapor under certain conditions. Theory indicates that these grow from "nucleus" bubbles of 7.18×10^{-19} cm^3 volume each. One coulomb of electrons, on this basis, may produce as little as 67.1 cm^3 of "nucleus" vapor or as much as 1.325×10^{13} cm^3 of final visible vapor. Fortunately this process has severe limitations which space does not permit discussion of here, but the above facts indicate that the vapor produced by a current density J acting for a pulse time τ may produce in 1 cm thick liquid targets, a volume $V = 2.12 \times 10^{-6}$ $J\tau/e$ cm^3 of final vapor or 1.078×10^{-18} $J\tau/e$ "nucleus" vapor from which some limiting values of J may be given in terms of permissible values of V.

Under adiabatic conditions a 1.5 μs 0.4 amp pulse of electrons will vaporize 0.00716 cm^3 of liquid H_2 at 20.4^{o}K per cm^3 of target yielding a volume expansion of 1.7% per pulse. This expansion of cold liquid of .071 g/cm^3 will produce a transient pressure of 10.4 lbs/in^2. Allowing for heating from 20.4 to 33.2^{o}K indicates 38 pulses instead of 1 will be required for this effect. The above vapor rules indicate for a pulse of size mentioned 2.6×10^{-6} cm^3 of "nucleus" vapor and 8×10^{8} cm^3 of possible final vapor if the efficiency of single electron vapor production in a bubble chamber were attained. Actually the lower limit is too low and the upper cannot be realized because pressure and temperature conditions are altered by an electron stream so that vapor bubble growth is prevented. More theoretical and experimental study on this subject is required.

Thermal Radiation

Formulas were derived for calculating thermal radiation input from the scattering chamber wall into the cryogenic target and into the outer radiation shield for a number of shielding configurations including that shown in Fig. 1.

The thermal input to the target is well under 100 milliwatts and that into the outer shield about 20 watts. A detailed treatment is given in a report in press.

References

(1) H. S. Carslaw and J. C. Jaeger "Conduction of Heat in Solids" Oxford, Clarendon Press 2nd ed. 1959, pp. 181 and 255-258.

(2) F. Seitz, "On the Theory of the Bubble Chamber" Physics of Fluids 1, No. 1, 2-13 (1958).

(3) A. G. Tenner, "Nucleation in Bubble Chambers," Nucl. Inst. and Methods 22, 1-42 (1963).

(4) H. W. Wooley, R. B. Scott, and F. G. Brickwedde "Thermal Properties of Hydrogen in its Isotopic and Ortho-Para Modifications" J. Rsch. Nat. Bur. Stds. 41, 379-475 (Nov. 1948) (NBS Rsch Paper RP 1932).

Table I. Properties of "normal" H_2 and "normal" D_2 at their boiling points at one atmosphere pressure see ref. (4).

Property	"Normal" H_2	Normal D_2
Boiling point ^oK	20.39^o	23.57^o
Molar volume, cm^3/mol	28.402	24.78
Molecules/cm^3	$2.121_3 \times 10^{22}$	$2.431_4 \times 10^{22}$
Density g/cm^3 (ρ)	0.0709_7	0.1625_7
Latent heat of vaporization cal/g (λ)	107.5	75.04
Thermal diffusivity $K = \bar{K}/\rho c$	3.044	
Total specific heat (c)		
cal/g/deg K	2.258	1.47
cal/mol/deg K	4.55	5.92
Triple point (20.4^o Eq. mixture)	13.81^oK 52.8 mm Hg.	18.69^oK 128.5 mm Hg.
Critical point	33.19^oK 12.8 atm.	

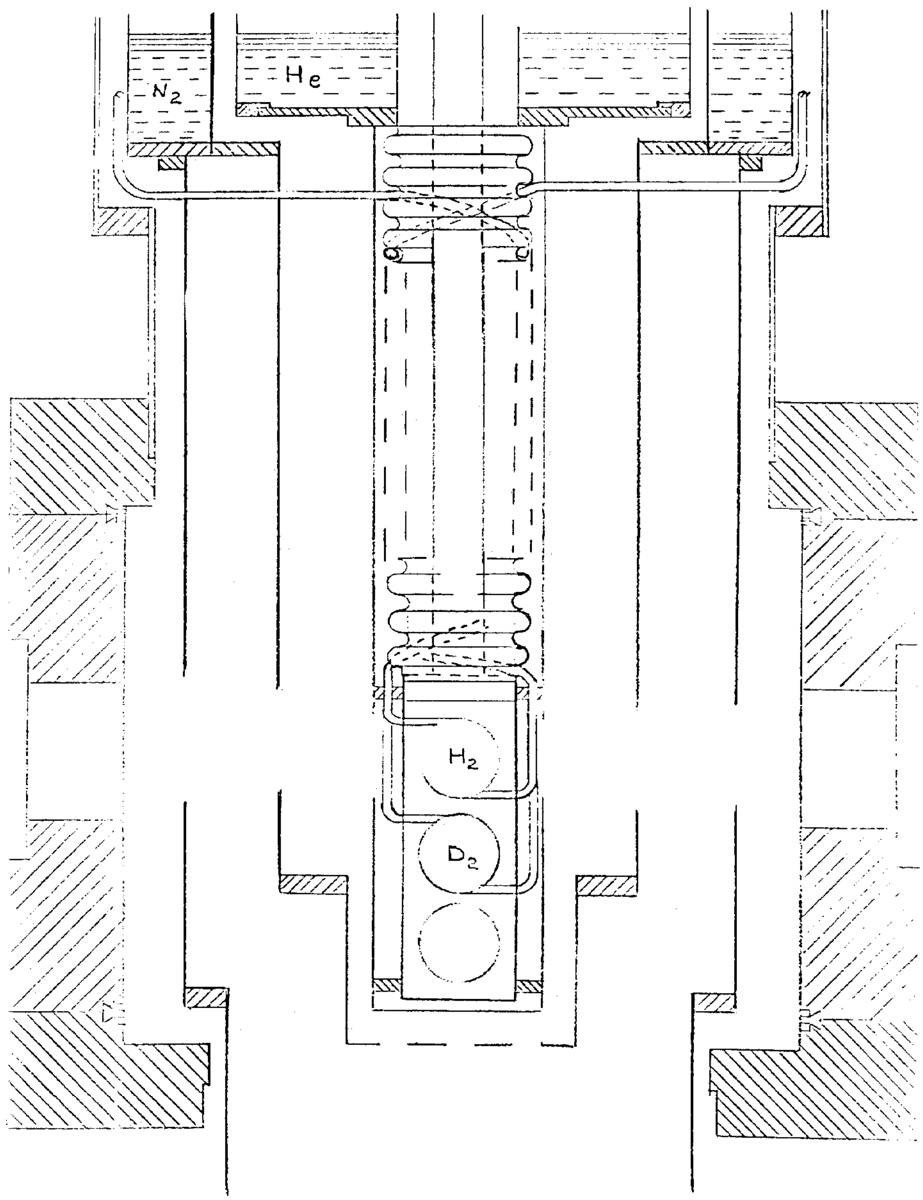

Fig. 1. Interchangeable triple target system. (Certain features are proprietary to Andonian Associates, Inc., Waltham, Mass.).

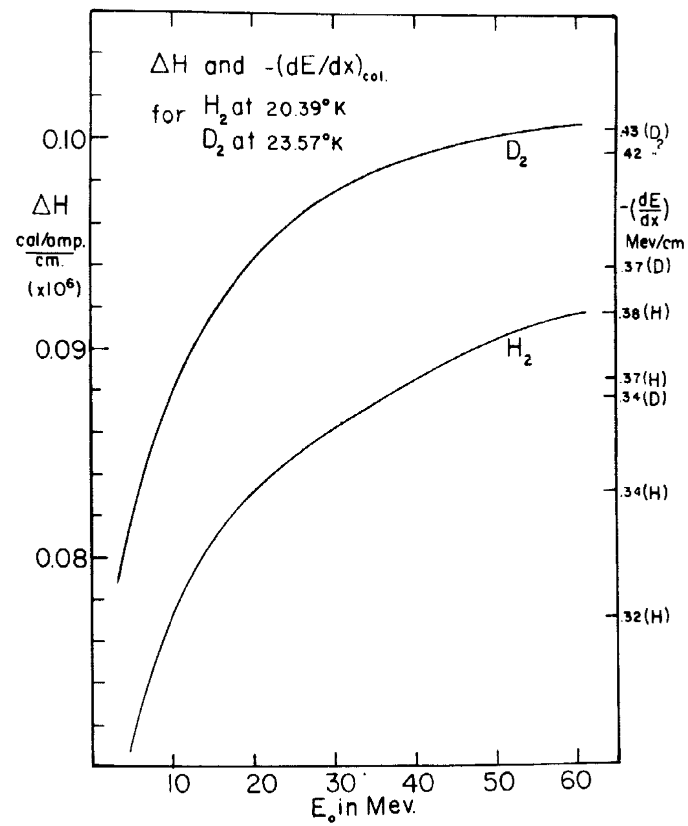

Fig. 2. Single electron collisional energy loss per cm of path and electron thermal input to targets for electron energies 5-60 MeV.

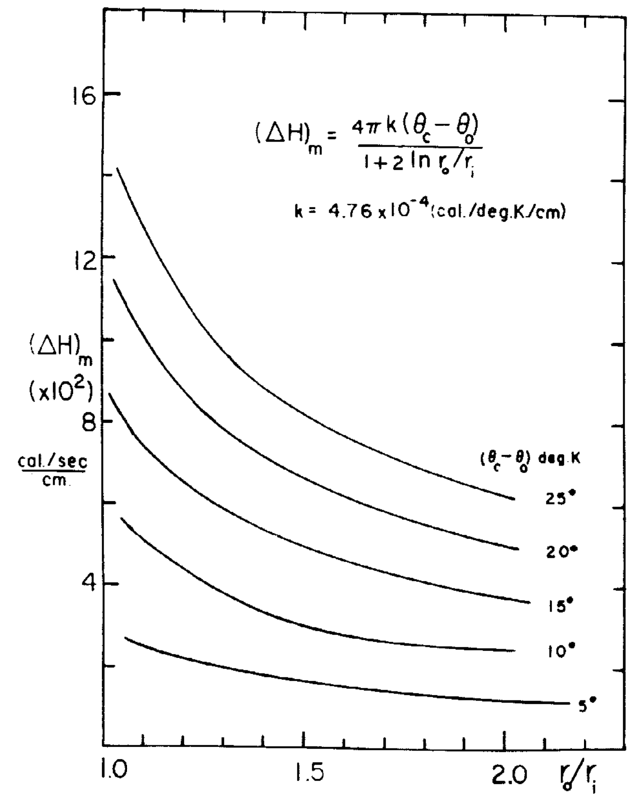

Fig. 3. Maximum permissible thermal inputs, per ampere, per cm of target thickness for maintaining a specified temperature difference between target center and target holder ($\theta_c - \theta_o$).

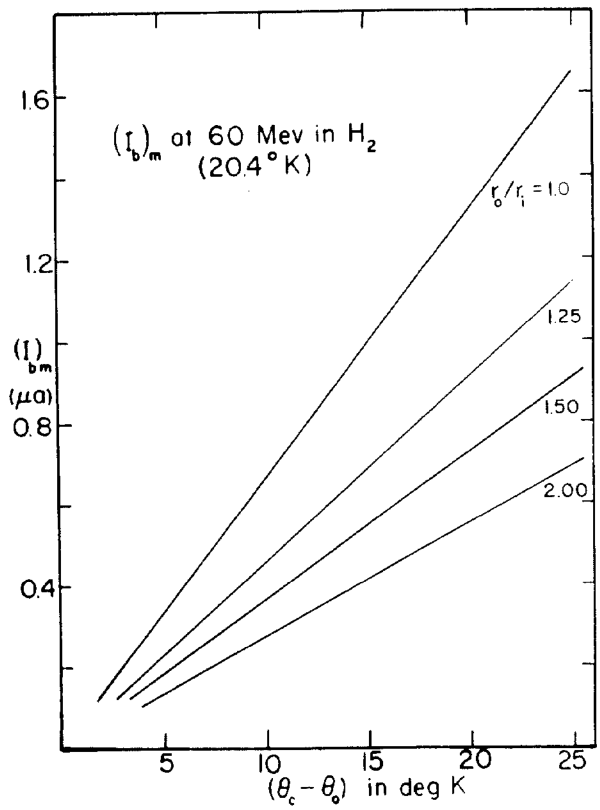

Fig. 4. Maximum permissible continuous beam currents for 60 MeV electrons in H_2 at 20.4°K as a function of $(\theta_c - \theta_o)$ for several ratios of target to beam radius (r_o/r_c).

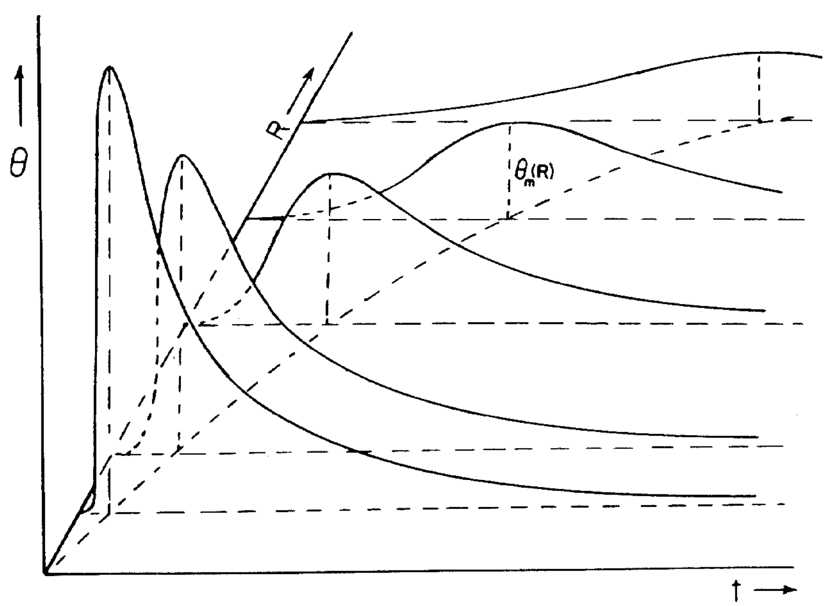

Fig. 5. Temperature as a function of time t and radial distance R at points in a medium after action of an "instantaneous line source" of heat.