A STORAGE RING FOR 10 BEV MUONS *

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Summary

It is proposed to construct a strong focusing ring of magnets whose function is to trap muons of 5 - 10 Bev, and to study the interaction of the trapped particles in targets placed in the ring. It is shown that such a ring is especially suitable for use with the Stanford linear accelerator because of the hundred-fold increase in effective duty cycle. Other advantages of the scheme are that the muons are nearly mono-energetic, free of contamination, well collimated, and confined to a small beam well suited for performing scattering experiments. Typical parameters of the trapping ring are presented.

Introduction

The Stanford two-mile linear accelerator will provide an intense source of muons having energies ranging from a few Bev up to nearly the maximum primary electron energy. For example, Ballam[1] has computed that 25 Bev electrons incident on a 1/2-radiation-length target will produce 10 Bev muons with an intensity of 10^{-3} electron^{-1} ster.$^{-1}$ Bev^{-1}. (See Fig. 1). The muons can be quite efficiently separated from electrons and strongly interacting particles by filtering through a low-Z absorber of length equivalent to 15 - 20 nuclear mean free paths. The energy loss by ionization (3 - 4 Bev) and the multiple scattering (5 - 10 mrad) are tolerable as long as the final muon energy is to be greater than 5 Bev or so. Such a beam would be very useful in pursuing studies of muon elastic and inelastic scattering from hydrogen and other nuclei. Somewhat paradoxically, it appears that it is quite difficult to design suitable experiments which make use of the full intensity of such a beam. This is because of the highly unfavorable duty cycle of the linear accelerator (a few hundredths of a percent) and the difficulty of collimating high energy muons. In addition, some experiments require that the muon energy be restricted within a band of a few percent; this is difficult and usually accomplished only with a large loss in intensity. Other experiments (particularly elastic scattering, for which the cross sections may be as small as 10^{-37} cm^2) require that the detectors subtend very large solid angles -- a feature which is usually prohibitively expensive and complicated unless the beam dimension is very small; however, the natural width of the beam is usually somewhat large, and is augmented by the spread produced by multiple scattering in the filter. Again it appears that small beam size is possible only at the cost of beam intensity.

The device which is described in this paper is designed to trap muons of 5 - 10 Bev in stable orbits, and thus to enable one to study interactions of the muons with targets placed in their path. The device appears to eliminate all of the difficulties mentioned above, and yet to provide satisfactorily large effective beam intensity. It was originally proposed by the first named author in 1962, and was described in an unpublished report written for the Stanford Linear Accelerator Center[2]; further studies are outlined in an internal SLAC note[3] of September, 1964. In this report, we shall summarize the material presented previously, and add an account of recent results obtained in a computer study of the magnet system.

General Features

Muons are to be produced in a thick target (2 radiation lengths of copper) by the primary electron beam. After passing through a crude momentum analyzer (whose function is principally to eliminate most of the remaining electrons), the beam is to be guided to the vicinity of a ring of magnets similar to that used in a strong focusing synchrotron. The muons are then to be injected into the ring by means of a pulsed magnet system; those muons having the proper energy will then be trapped in the ring and circulate until they either decay or are ejected from their stable orbits by electromagnetic interaction with material placed in their path. Only a crude vacuum is necessary to insure that residual gas has negligible effect on the stored beam. It is, in fact, possible to place hydrogen or other targets of quite appreciable thickness (fractions of a g/cm^2) in the ring, and to study interactions of the beam with the targets over the entire circulation period.

Although the storage ring is intended to be used over a range of energies of 5 - 10 Bev, the remarks in the succeeding paragraphs are, for the sake of clarity, based on the assumption of 10 Bev energy. The laboratory mean life of 10 Bev muons is about 220 µsec, while the lineac repetition period is less than 3 msec.; thus it is clear that the effective duty cycle should be of the order of 5%, which represents a large improvement over the best duty cycle obtainable in the usual lineac beam. Other advantages, however, are at least as important as this one:

(1) The muon beam remaining in the ring after an initial period of 40 μsec or so is essentially free of any contamination by electrons, π's and K's. Electrons rapidly lose energy by synchrotron radiation and are lost from the ring in a fraction of a turn at 10 Bev, and a few turns at 5 Bev; the laboratory mean lives of π's and K's are less than 1% of that of the μ's, so that they too disappear very quickly.

(2) The muon beam has very small angular divergence (typically a few mrad), small size (a few cm in transverse dimension) and small momentum spread (a few percent at most).

(3) The period of revolution is of the order of 1 μsec; thus muons on the average traverse the target a few hundred times before they are lost by decay. The effective target thickness may then be as much as 50 - 100 g/cm^2; such a thickness of liquid hydrogen is impossible in a linear beam array.

Design of the Ring

The conclusion of the early studies^{1,2} was that a ring of magnets suitable for this application should have approximately the following properties:

a) Synchrotron frequencies = 6 to 8, with corresponding momentum compaction of 0.03 to 0.015.

b) Acceptances in phase space of at least 5 cm-mrad vertically and 20 cm-mrad radially.

c) Momentum acceptance of at least ± 2%.

d) Diameter somewhat less than 100 m. (This is dictated by the available site at the accelerator); thus the average guide field should be at least 7 kg.

e) Straight sections having a free drift space of at least 5 meters; one would be used for injection, and two diametrically opposed sections would contain experimental targets.

With these approximate restrictions in mind, we have investigated the properties of strong focusing magnet systems with some care, using the SYNCH 7094 computer program of Garren and Eusebio^4, as modified for use at Brookhaven by Courant^5. The first set of computations was aimed at designing a suitable ring of alternating-gradient magnets. The equilibrium guide field was taken as approximately 13 kg., and the field gradient $n \cong 100$. Four straight sections were to be constructed on the Collins principle^3, i.e., the straight sections match both the radial and vertical betatron functions β_r and β_z, and provide one quarter wavelength betatron phase advance per section. The unit cell and the Collins straight section are shown in Fig. 2. The resulting radial betatron function, evaluated over one superperiod (one quarter of the ring), is shown in Fig. 3. We see that the maximum value of β_r is about 15 m., corresponding to a betatron form factor of approximately 2. (The latter result is in agreement with the predictions of the usual betatron orbit theory). The radial phase space acceptance is then dependent only on the radial half-aperture, a, as shown below:

$$A_r = a^2/\beta_r(\max).$$

One evidently obtains the required acceptance by choosing $a \gtrsim 6$ cm. The aperture limitation is in fact in one of the Collins straight section quadrupoles. Thus, assuming that a = 6 cm and that the Collins magnets have suitably large aperture, one can obtain a radial acceptance of about 27 cm-mrad. The vertical betatron function has a form very similar to the horizontal function; the required vertical phase space acceptance (5 cm-mrad) may be achieved by choosing the vertical half-aperture $b \gtrsim 2.5$ cm. The off-momentum radial excursion is computed in first order; it is denoted by the function x_{eq}, and is shown in Fig. 4. The momentum acceptance is defined by, $\Delta p/p = a/x_{eq}(\max)$. One sees that the maximum value of x_{eq} (about 30 m.) corresponds to a momentum acceptance of ± 2%, which is approximately the value specified previously.

The vertical and radial betatron frequencies are 6.4 and 6.3 respectively; although not much attention has been directed at the exact values of these frequencies, it is clear that one can adjust them to avoid all of the usual strong resonance values. The complete layout of the A-G ring is shown in Fig. 5.

Most recently, we have begun to investigate the possibility of constructing the ring using separated function guide and strong focusing magnets. In order to obtain a diameter of 90 m (just under the maximum allowable) the guide magnets must have a field of 20 kg; the strong focusing magnets are quadrupoles. The principal disadvantage of this scheme over the more usual A-G system is that of somewhat greater cost and size. The separated function ring, however, offers two very appealing features: (1) it is in principle trivial to change the equilibrium energy over a wide range; this is certainly not so for the A-G ring, where a change in energy might require complete reshimming and considerable re-aligning. (2) The alignment problem is greatly eased; it is evident that the positioning of zero-gradient bending magnets is not at all critical, while the symmetry of strong-focusing quadrupoles makes critical positioning a rather easy task.

It was found that a separated function ring could indeed be designed to have nearly the same parameters as the A-G ring (except for the increased diameter). The resultant parameters are as follows:

Guide field (zero-gradient magnets) = 20 kg
Field gradient (unit cell quadrupoles) = 1 kg/cm
$\nu_r = 6.35$
$\nu_z = 6.59$

$A_r = 28.6$ cm-mrad (for $a = 7.5$ cm)
$A_z = 9.8$ cm-mrad (for $b = 3.75$ cm)
$\Delta p/p = \pm 2.4\%$
(corresponding to $x_{eq}(\max) = 3.1$ m)

These parameters should not be taken as final, since the studies are continuing. We hope in particular to reduce the value of $x_{eq}(\max)$ by refinement of the straight section design.

Application as Muon Trap

Fig. 6 shows the proposed layout of the ring at the SLAC site. Before proceeding to the discussion of the ring as a muon trap, one must note some relevant features of muon production by electrons. The full energy electron beam of the Stanford accelerator (nominal energy $\lesssim 25$ Bev in the first stage of operation) will have very small phase space emittance ($\approx 10^{-5}$ cm-mrad in each direction). Thus one should be able to focus the beam to a spot only a few mm in diameter. The limitation on beam size, in fact, arises from the extremely high energy deposition in the target, which can easily vaporize any target material if the beam density is too large. It appears that, for this reason, the beam area must be at least 0.3 cm^2 for a beam of 30 μa. incident on a copper target, i.e., a circular beam must have a radius of about 3 mm. The incident electron beam produces an electromagnetic cascade in the target; photons of the cascade then produce muons in pairs, with energy and angular distributions as described by the Bethe-Heitler formula. 50% of the muons are produced within a cone having half-angle, $\theta_{1/2} \approx \mu/E$, where μ is the muon mass, E the muon energy; thus at 10 Bev, the half-angle is only 10 mrad., and 95% of the muons are emitted within a cone of half-angle 25 mrad. The two-radiation-length target introduces some multiple scattering (3 mrad.) but produces negligible broadening of the effective source size. The phase space emittance of the muon beam for 95% muon acceptance is then approximately 7.5 cm-mrad in either direction. It is quite possible to produce different emittances in the two orthogonal directions by forming the electron beam into an ellipse having the proper eccentricity. Thus we see that the muon beam emittance can be made to match almost perfectly to the acceptance of the ring. The pulsed injection system, although not yet designed, is expected to distort the beam emittance appreciably; it is hoped, however, to attain an efficiency of at least 0.5 for injection of the entire muon beam (at one energy) into stable orbits in the ring. On this assumption, one can compute the expected intensity of stored beam:

$$N = 2 \times 10^{8}\ \text{sec}^{-1}\ \text{Bev}^{-1} \text{ for } 30\ \mu\text{a. of } 25 \text{ Bev electrons.}$$

For a total momentum acceptance band of 4%, we then have 8×10^{7} muons trapped per second. We now compute the amount of hydrogen (in g/cm^2) which the average muon traverses before either decaying or losing so much energy by ionization loss as to be lost from the ring; this is found to be about 50 g/cm^2. Finally, therefore, we find that the total muon traversals of hydrogen per second are approximately equal to $4 \times 10^{9}\ \text{g-cm}^{-2}\text{-sec}^{-1}$. With such a flux, one finds that an interaction rate of one per hour corresponds to a reaction cross section of about $1.2 \times 10^{-37}\ \text{cm}^2$. The actual counting rate for a hydrogen scattering experiment is given by the reaction rate multiplied by the fraction of solid angle included by the detectors. There is good reason to believe that one could construct detectors subtending half of the total solid angle (this is practical because of the small stored beam size and small beam divergence). Thus one can hope to study reaction cross sections considerably less than $10^{-36}\ \text{cm}^2$. This would represent an improvement in sensitivity of about 1000 over that attainable with presently available muon beams, with the added advantage of excellent energy definition of the beam.

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References

1. J. Ballam, unpublished SLAC report M-200 (1960, Stanford U.)
2. J. Tinlot, unpublished SLAC report SLAC-5 (1962, Stanford U.)
3. J. Tinlot, internal note (in print, 1964, SLAC, Stanford U.)
4. A. Garren and J. Eusebio, unpublished report (1964, Brookhaven Lab.)
5. E. D. Courant, private communication.

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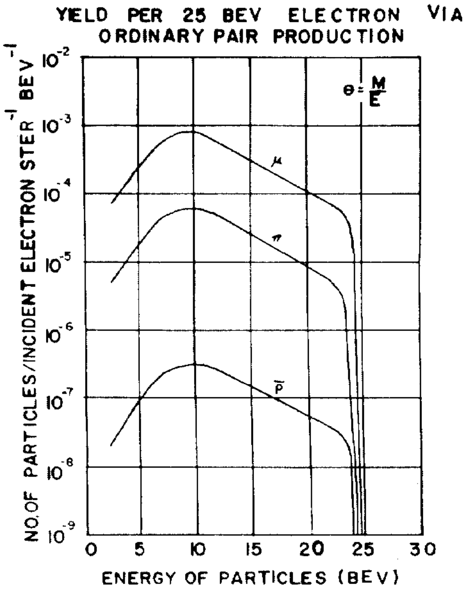

Fig. 1. Probability of production of μ's, π's and K's by 25 Bev electrons.

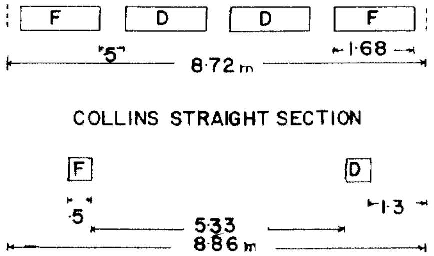

Fig. 2. Storage ring unit cell and Collins straight section (A-G ring).

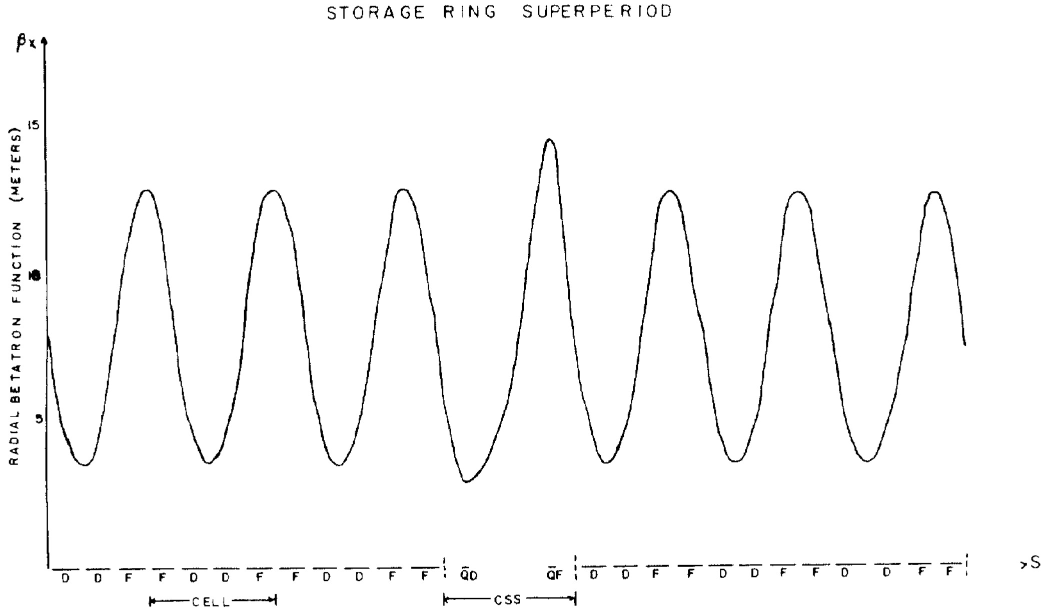

Fig. 3. Radial betatron function for a super-period (A-G ring).

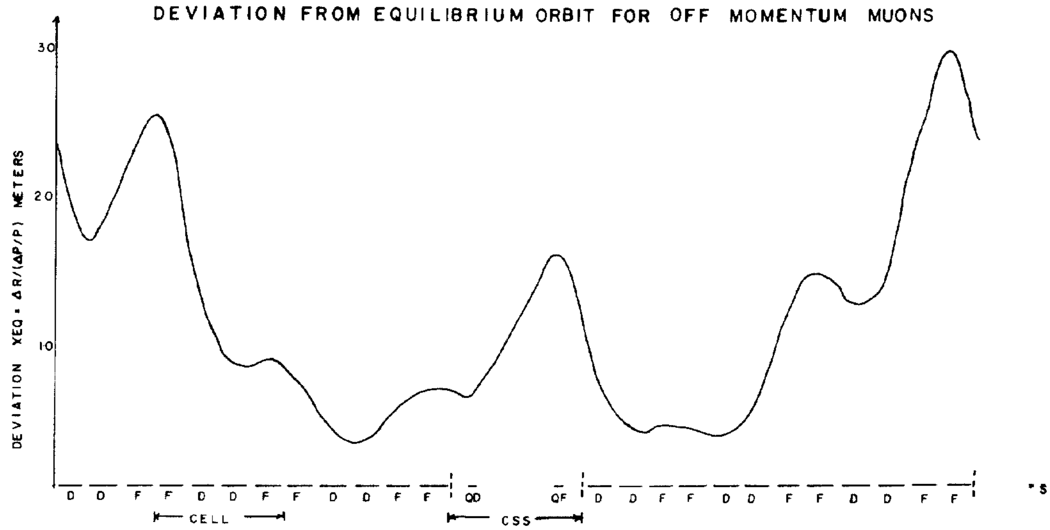

Fig. 4. Deviation from equilibrium orbit for off-momentum particles (super-period of A-G ring).

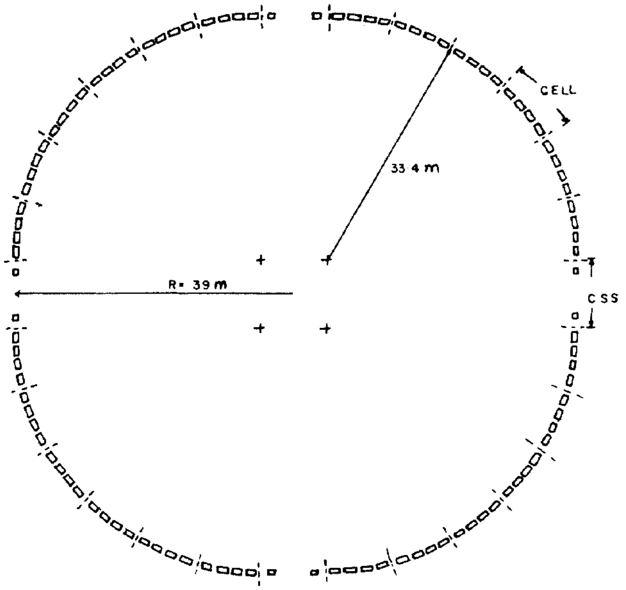

Fig. 5. Muon storage ring.

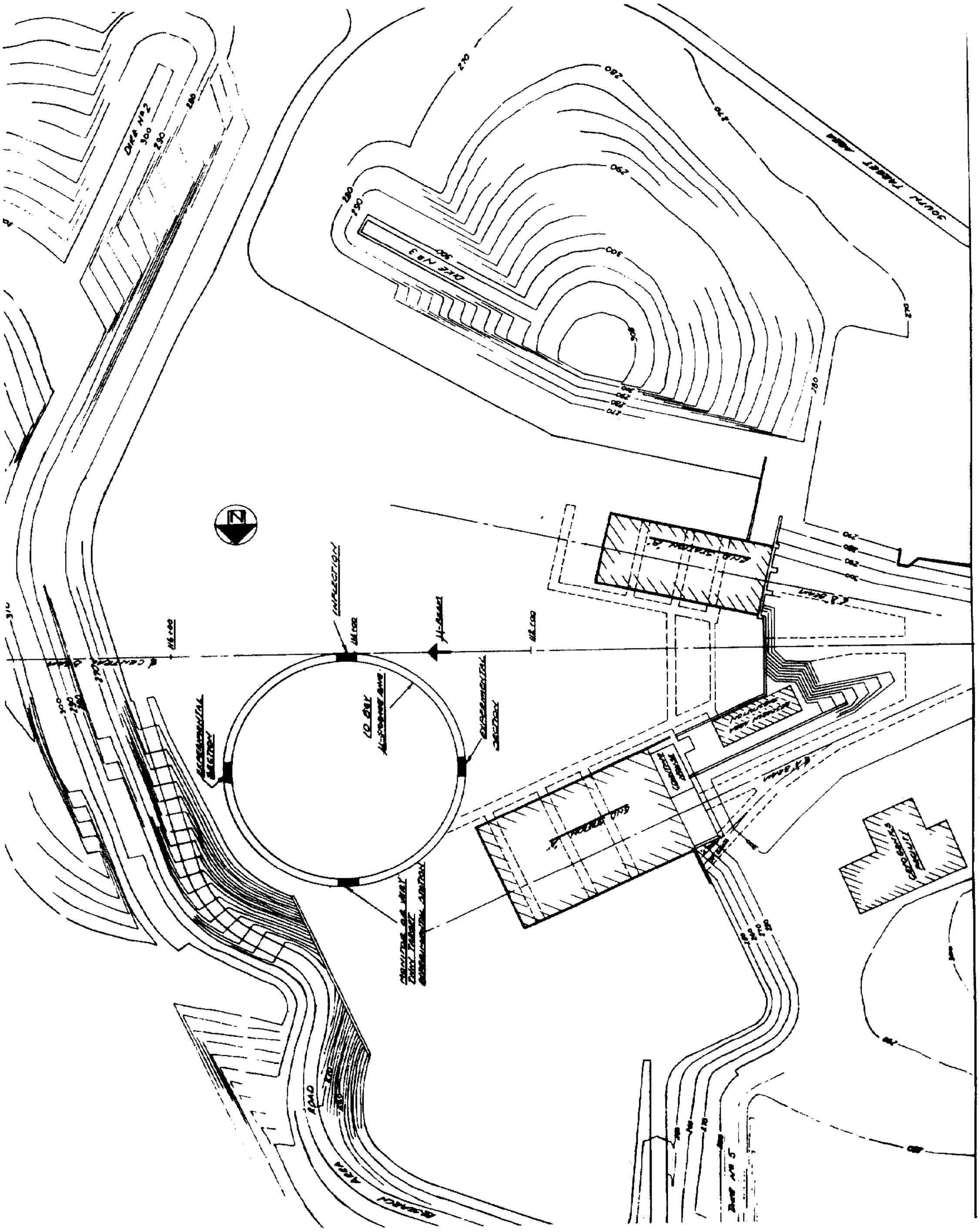

Fig. 6. Proposed placement of the muon ring at the SLAC site.