# COMPUTER CALCULATIONS OF THE MAGNETIC FIELD OF ALTERNATING GRADIENT SYNCHROTRON MAGNETS*

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## I. Introduction

The computer program SIBYL for computing the magnetic fields of magnets has been applied to computing the magnetic field of the Brookhaven AGS magnets and a Brookhaven septum magnet. This report presents the SIBYL computer results and compares them with experimentally measured results.

The SIBYL computer program was applied to computing the magnetic field along the azimuthal center of the AGS magnets. This calculation gave the magnetic gradient along the azimuthal center of the magnet. In order to investigate the orbits of an accelerator, it is necessary to have information about the azimuthal variation of the magnetic field, or what is usually called the effective length of the magnet. The SIBYL computer program has been applied to the calculation of the effective length of AGS magnets.

The work of E.D. Courant[1] on the Brookhaven AGS indicates that for the usual AGS acceleration a knowledge of the field along the azimuthal center of the magnet and of the effective length of the magnet is sufficient to allow one to compute the orbit properties of the accelerator with considerable accuracy.

The effective length of the magnet has some variation in the radial direction since the magnet gap is varying in order to produce the desired field gradient. This variation in the effective length is equivalent to changing[2] the magnetic gradient by about 1½%. The SIBYL program introduces an error of about 1% in computing the gradient along the azimuthal center. The magnet effective length was computed only at the radial center of the magnet since neglecting the radial variation of the effective length introduces only a small error. It seems likely, however, that the SIBYL program can compute the radial variation of the effective length if the vertical magnetic gap does not change rapidly in a radial distance of one gap length.

It may be concluded that the results found using the SIBYL program for the magnetic field along the azimuthal magnet center, and for the magnet effective length, allow one to compute the orbit properties of an accelerator with an accuracy of a few percent. The largest error would be in the absolute magnitude of the $\nu$-values which would be in error by a few percent.

### Problem Solved by SIBYL

The SIBYL program is a two-dimensional program which omits effects due to the finite longitudinal extent of the magnet. The magnet coils are rectangular in cross section, and 40 such coils may be entered into the program. An example of the type of magnet that can be handled by SIBYL is shown in Fig. 1.

SIBYL calculations include the effect of the iron saturation. A B-H curve of the magnet iron is entered as a table.

### Possible Uses of SIBYL

The possible applications of the SIBYL program include the following:

1. The shape of the magnet pole may be found to give a desired magnetic field.

2. The effects of the magnet iron saturation at high magnetic field may be computed.

3. One may compute the leakage field in a septum magnet where the leakage field is due to the finite permeability of the magnet iron.

### Accuracy of SIBYL Results

The accuracy of the SIBYL computer results has been investigated here by comparing the computer results with experimentally measured magnetic fields. This is not quite correct as the physical magnets include effects which are not present in the computer program, such as remanent field effects, and effects due to the finite longitudinal extent of the magnet. In addition, the experimental results may be in error due to measurement errors.

The magnetic field in AGS type magnets can be computed by SIBYL with an error of about 1%. The gradient in AGS type magnets can be computed with an error of about 4%. The change in the magnetic field gradient across the horizontal aperture is found with an error of about 10%.

The leakage field in a septum magnet is computed by SIBYL with an error which is about 1% of the maximum field of the magnet.

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## II. Results for AGS Magnets

### Fringing Fields and Gradients

In Fig. 2 we compare the results of a SIBYL calculation of the median plane fringing field in an AGS open magnet with measurements by J. Palmer and R.H. Phillips.[3] The measured permeability of AGS type steel was used for this calculation, and the field at the aperture center line, $B_o$, was specified in the input data (13.1 kG in the present example). The program calculates the ampere turns required to produce the specified field strength, which is approximately 50,000 ampere-turns/coil in the case of 13 kG. Similar calculations have been made at the injection field, $B_o = 120$ G, and at an intermediate field, $B_o = 5000$ G. Calculations with $B_o$ greater than approximately 14 kG were not run, owing to the lack of permeability data above 20 kG, which is the maximum median plane field $B_{max}$ corresponding to this value of $B_o$. The results, when expressed as distributions of $B/B_o$ versus radius, differ by at most a few percent and are in satisfactory agreement with the corresponding measured distributions, which have an uncertainty of the order of one percent. (The very low experimental value for $B/B_o$ at R = 10" is thought to be in doubt.)

In Fig. 3 we compare calculated and measured field gradients over the central region of the gap in an open magnet, for injection, intermediate and high values of $B_o$. K is $(dB/dR)(1/B_o)$. A discrepancy of about 5% at injection field and 1% at intermediate and saturation fields is present, with the measured values of K being systematically high compared with SIBYL. It is reasonable to expect the best agreement at intermediate excitation, where the permeability is near its maximum value. Discrepancies of the order of a few percent, moreover, are not surprising, owing to such effects as eddy currents, remanent field, finite azimuthal extent, and finite packing of steel laminations in the real magnet.

As a further illustration of SIBYL, Fig. 4 shows a typical set of contours of constant permeability, or "isoperms", of the AGS iron during high excitation. Low values of $\mu$ indicate local regions of saturation. In addition to the permeability description the program output contains a map of the vector potential in both the air and iron, printed out at every specified number of iterations. Since the lines of vector potential correspond to the "lines of force", these types of data are most informative in the analysis of the onset of local saturation for a given iron magnet geometry.

### Results for the Magnet Effective Length

The present computer program is not designed to handle the type of magnet geometry relevant to the azimuthal field variation in an AGS magnet which, in fact, is a three-dimensional problem. Instead, the azimuthal fringing field at the equilibrium orbit is approximated by calculating the radial fringing field in a fictitious AGS magnet in which the hyperbolic pole contour is replaced by a flat pole tip, and whose gap height equals the gap height at the equilibrium radius in the actual AGS magnet.

We have compared the fringing field so calculated with the measured[3] fringing fields off the end of an open AGS magnet for various field strengths. The SIBYL results for intermediate excitation are compared with the corresponding measured distribution in Fig. 5. $B_o$, the field at magnetic center, is 5 kG. The uncertainty in the measured field values is of the order of one percent.

From these distributions we calculate the equivalent or effective length of the azimuthal fringing field by numerical integration, according to

$$\ell_B = \frac{\int_{\theta_o}^{\infty} B d\theta}{B_o} - \frac{L}{2}$$

Here L/2 is the geometrical half-length of the magnet (L/2 = 37.5" for a B-magnet), and $\theta_o$ is at the center of the magnet, where the field strength is $B_o$. The resulting effective lengths are given in Table I, in which we list $\ell_B$ based on measured and calculated field distributions at three levels of magnet excitation (injection field, intermediate field and saturation field).

TABLE I

Effective Lengths Based on Measured and Calculated Fringing Fields

| Magnetic Field (Gauss) | Effective Length (Inches) Computer | Effective Length (Inches) Measured |
|---|---|---|
| 120 | 2.2 | 2.1 |
| 5000 | 2.2 | 2.3 |
| 13000 | 2.1 | 2.0 |

The effective lengths derived from the SIBYL results vary more slowly with excitation than those based on the measured distributions. At intermediate excitation the calculated value is about five percent low compared with the experimental value, and at low and high excitation it is five percent high.

### Results for a Magnet with a Magnetic Septum

As a further example of computer applications Fig. 6 shows the fringing field as calculated by the present program outside a 15 kG magnet with a magnetic septum. In this magnet one coil is in the opening of the C-frame so that no external fringing field should exist for infinite steel permeability. The introduction of finite permeability gives rise to a fringing field of the order of 2% of $B_o$ and of reversed sign in the vicinity of the outside exciting coil.

Similar calculations have been made for the septum magnet in the AGS external beam. In this case as well a fringing field exists, whose magnitude and sign is quite sensitive to the detailed coil geometry in the magnet, and which is in qualitative agreement with the measured fringing field.

## Acknowledgements

We are much indebted to Dr. David Berley for data on the bending magnet with a magnetic septum, to Mr. C. Lasky for data on the AGS extraction septum magnet, and to Dr. R.A. Beth for data on the B-H curve for the AGS magnets.

## References

1. E.D. Courant, BNL Accelerator Dept. Internal Report EDC-36, 1960.
2. G.K. Green and E.D. Courant, Encyclopedia of Physics, Vol. 44, p. 330, Springer, Berlin, 1959.
3. J. Palmer and R.H. Phillips, Brookhaven Memo, January 15, 1959;
   R.H. Phillips, Brookhaven Memo, April 16, 1959.

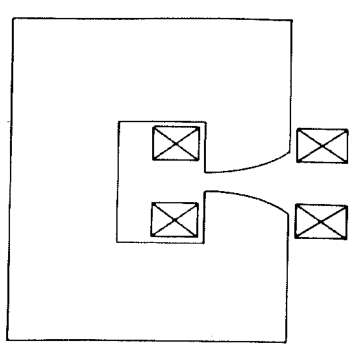

Fig. 1. Shape of a two-dimensional magnet solved by the SIBYL computer program.

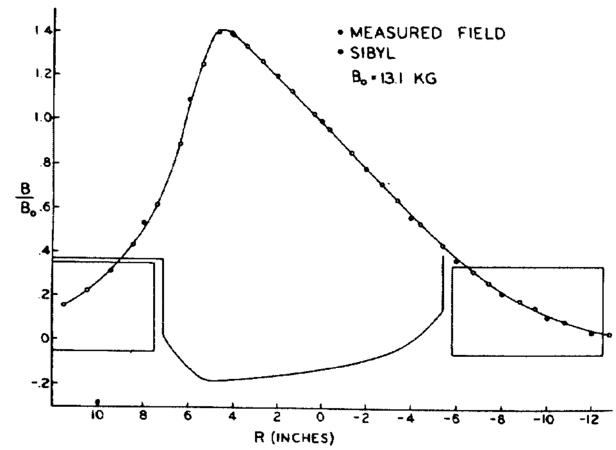


Fig. 2. Calculated and measured fringing field distribution in an open AGS magnet.

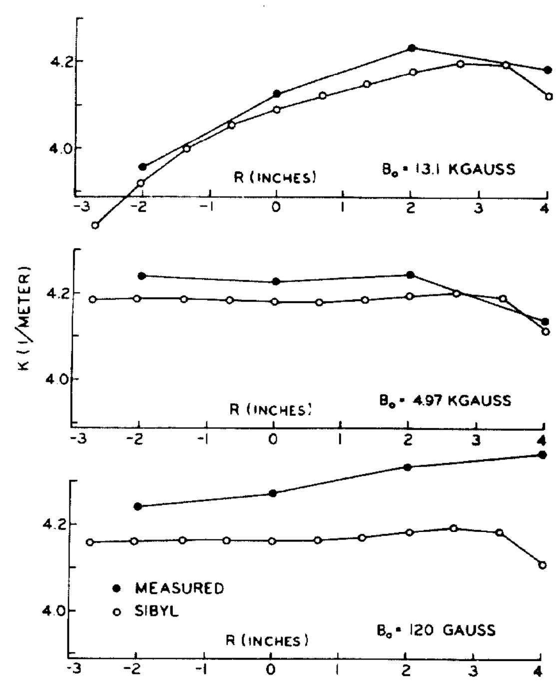


Fig. 3. Calculated and measured field gradients in an open AGS magnet. The high field end corresponds to negative R.

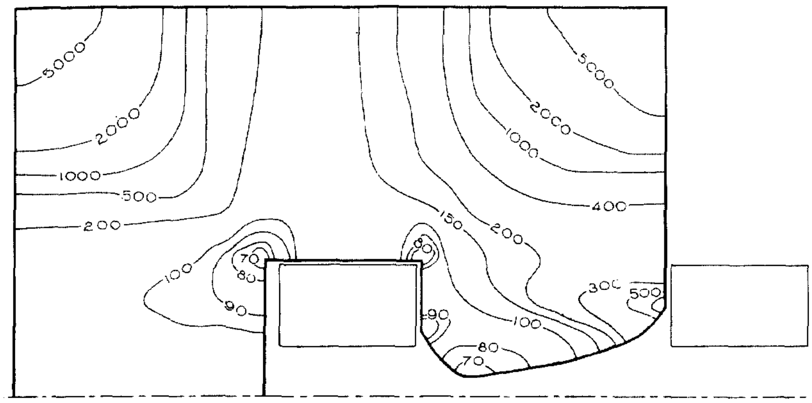


Fig. 4. Permeability map of AGS iron during high excitation.

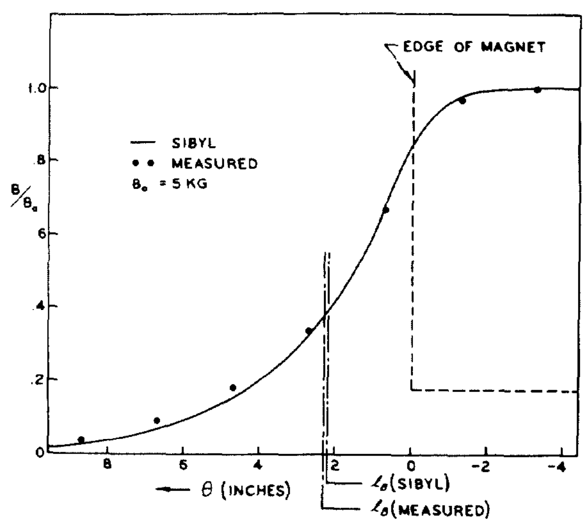


Fig. 5. Calculated and measured fringing field off the end of an AGS magnet.

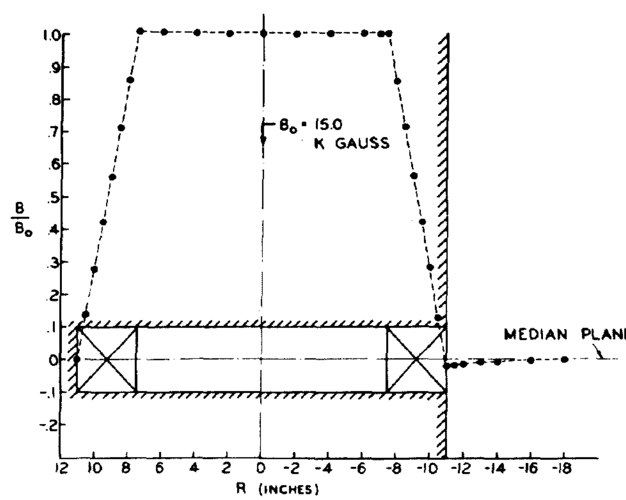


Fig. 6. Calculated fringing field distribution in a 15 kG magnet with a magnetic septum.