MAGNETIC FIELD MEASUREMENT AND SPECTROSCOPY IN MULTIPOLE FIELDS [*]

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Summary

The magnetic field measurements in multipole magnetic fields, which are used to measure magnetic field parameters in the strong-focusing lenses for the Stanford two-mile linear accelerator, will be reviewed. Specifically, the theory and the measurement processes used to determine such important parameters as the magnetic center in multipoles, the length of the gradient fields, and the harmonic content in strong-focusing lenses will be described. The results of these accurate measurements will be related to the optical parameters of the multipole lenses.

Field Distribution In Multipoles

The Ideal Quadrupole

The quadrupole magnet was introduced in 1952 by Courant, Livingston and Snyder[1] along with the strong-focusing synchrotron, and by Christofilos[2] as a means of focusing charged particle beams. In order to study the field configuration, let us consider the interior of the quadrupole magnet as shown in Fig. 1a, which is bounded by four equipotential electrodes maintained respectively at the potential $\pm\mu$.

Solving for the scalar magnetic potential in a quadrupole by starting with the two-dimensional Laplace equation, assuming the existence of a product solution in r and θ and then applying the boundary conditions of four-fold symmetry, results in a magnetic potential

$$u_2(r,\theta) = \sum_{n=2,6,10}^{\infty} B_{2n} (\sin n\theta)\, r^n$$

and the magnetic field intensity is $\vec{H} = -\vec{\nabla} u$. A constant-gradient quadrupole is one in which the first term is the only non-vanishing term, i.e., $B_{2n} \neq 0$, but $B_{2n} = 0$ for $n = 6, 10, 14 \ldots$

It is convenient to express the scalar potential for a quadrupole in the XYz coordinate system. Using the linear transformation $X = r \sin\theta$, $Y = r\cos\theta$, one gets

$$u_p = 2B_2 XY + 6B_6 XY \left\{ X^4 - \frac{10}{3} X^2Y^2 + Y^4 \right\} + \ldots$$

Origins of Higher Poles in Quadrupole Magnets

Figure 1 shows some of the differences between an ideal quadrupole and a practical one. In the ideal quadrupole the pole surfaces are shaped according to the equation $X \cdot Y = \pm R^2/2$. One can see that in the practical quadrupole the pole surfaces have the required hyperbolic shape over a considerable extent, but must be truncated laterally at some point to allow sufficient space for the excitation windings. In order to discuss the effects of mechanical imperfections in the practical magnet, designate the pole tip spacing along X as A and the pole tip spacing along Y as B. Let the letters a, b, c, and d stand for the spacing between adjacent poles measured at the point of truncation.

Consider first a mechanically perfectly fabricated quadrupole as far as symmetry of the location of the four poles is concerned, that is, where $A = B = 2R$ and $a = b = c = d$. In this case, the fabrication of the poles themselves would be the only source of the higher poles. Let us further assume that the poles themselves are symmetrical about their own centerline axes along X and Y. The fact that the extent of the hyperbolic pole pieces is not infinite would result in a pole configuration that has the four-fold quadrupole symmetry; however, because the magnetic equipotential of the pole stops at the point of truncation, the field would appear too low at the truncation. Near the points of truncation, the field of the pole (N) suffers a weakening of the N field and can be represented as a virtual S field superimposed on the N field. The cause of this weakening can be attributed to two factors, a leakage of flux beyond the truncation point and a saturation of the pole at the truncation point. The multipole so produced is the duodecapole, as each pole acts as three poles.

If the pole is made by taking a circular approximation to the required hyperbolic shape, even higher poles will be present in the quadrupole. If the pole is made symmetrically, these higher poles will result in some of each of the possible poles having four-fold symmetry, that is, each will have an odd number of poles in each quadrant. Therefore, the higher poles that can possibly exist in the magnet when all elements of construction are perfect (i.e., $A = B = 2R$ and $a = b = c = d$) are 4-pole, 12-pole, 20-pole, 28-pole, or $4(2n - 1)$ poles where $n = 1, 2, 3 \ldots$

Now assume that the poles are perfect hyperbolas but that the mechanical construction is such that the opposite pole spacing A is not equal to B, but $a = b = c = d$. This is one way in which the octupole perturbation can be generated. The other usual way is when $A = B$, $a = c$, and $b = d$, but $a \neq b$. From a slight extension of this analysis one can see how these misalignments can account for the whole set of multipoles with two-fold symmetry. These are the multipoles contained in the set octupole, 16-pole, 24-pole, 32-pole, or $2(4n)$ poles where $n = 1, 2, 3 \ldots$

In quadrupoles constructed such that $a \neq c$ or $b \neq d$, various higher poles can occur; these are in general poles that are assymmetric, that is, they have neither two- nor four-fold symmetry. These poles are the dipole, sextupole, decapole, 14-pole, 18-pole, or $2(4n\pm1)$ poles where $n = 0, 1, 2, 3 \ldots$

Spectroscopy of Multipoles

Interpretation of Harmonic Spectrum

It is apparent that one of the most important

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methods of evaluating a multipole magnet is the determination of the harmonic content of its field. In any practical multipole magnet there are some higher harmonic fields present, and these, if sufficiently large, can affect the beam dynamics in the magnet. In some cases it is desirable to build magnets in which there is a large harmonic content in order to correct optical errors. One can use the information about the harmonics of a multipole magnet to design special pole faces[3,4] or fringing fields for the magnetic multipoles.

The magnetic scalar potentials for various multipoles can be written as

$$u = \sum_{n=2,6,10,14,18,\dots} B_{2n} (\sin n\theta)\, r^n$$

for quadrupole fields,

$$u = \sum_{n=3,9,15,21,\dots} B_{3n} (\sin n\theta)\, r^n$$

for sextupole fields, and

$$u = \sum_{n=4,12,20,\dots} B_{4n} (\cos n\theta)\, r^n$$

for octupole fields.

When the proper boundary conditions are satisfied, only one term remains in the summation. For example, in the case of the quadrupole only B_{22} is the non-vanishing coefficient when the equipotential pole tip surfaces are in the form of equilateral hyperbolas in the X-Y coordinate system. By measuring $B_{2,6}$, $B_{2,10}$ ... in a quadrupole, one actually gets a measure of how well the pole faces approach the theoretical shape. In the case of pole saturation, because of the distortion of the ideal equipotential surfaces, the higher harmonic content increases. In a quadrupole, for example, $B_{2,6}$, $B_{2,10}$ will be non-vanishing at high field values even when at low field only B_{22} is not zero. If the symmetry conditions in a quadrupole are not completely satisfied, other coefficients like B_{3n}, $n = 3, 9, 15, 21 \dots$, and B_{4n}, $n = 4, 12, 20 \dots$ will be present. Then measuring B_{33} and B_{44}, one might draw conclusions about the quadrupole symmetry. One can analyze other multipoles in a similar manner.

To summarize, one might say that by measuring B_{mn} in a multipole (mn), where $n = m, m+2n, m+4n, \dots$, one gets a measure of how well the actual multipole approaches the theoretical multipole with ideal boundary conditions, and by measuring $B_{m'n'}$ where $m' = m+1, m+2, \dots$, $n' = m+1, (m+1)+2n, (m+2)+2n$, one obtains a measurement of the symmetry of the multipole.

Harmonic Measurement System

The existing harmonic content with all the amplitudes (B_{nm}) can be considered as the spectrum of the multipole. Naturally the amplitudes of the higher harmonics decrease rapidly with harmonic numbers. For example, in a quadrupole magnet, the pure quadrupole field (B_{22}) is much larger than other multipole field components, and some provision must be made to cancel or at least reduce the quadrupole field coefficient sufficiently so that its presence does not mask the other multipole coefficients.

Basically, the harmonic measurement system is a coil rotating in the aperture of the magnet at a fixed frequency. The output from the coil is Fourier-analyzed with a narrow bandwidth wave analyzer and the amplitude of each Fourier coefficient is noted. In this system the Fourier coefficient corresponding to the frequency of rotation ω of the coil is the dipole field; the coefficient corresponding to frequency 2ω is the quadrupole field; the coefficient of 3ω is the sextupole field, and so forth for higher fields. The rotating coil can be calibrated in multipole calibrating magnets of known field strength, or its response can be calculated for a given coil geometry.

Coil Design and Calibration

The rotating coil used for harmonic analysis in a multipole field should be sensitive to the harmonic field components which are being measured. If a field component with large harmonic number is measured, it is desirable to suppress the coil sensitivity for the other harmonics, particularly when the corresponding fields are large in magnitude. For example, if one desires to measure B_{26} in a quadrupole field, it is necessary to minimize the coil response for B_{22}; otherwise the small signal corresponding to B_{26} would be lost in the large signal background. With special coil design one can decrease the coil sensitivity for any one harmonic.

W. H. Lamb calculated[5] the induced voltage in a rotating asymmetric coil where two return bundles are used. The return bundles are located at an angle α from the main bundle so that $\theta' = -\alpha$ for one and $\theta' = +\alpha$ for the other. Figure 2 shows the arrangement of the wire bundles on the rotating coil. The induced voltage in this coil is given as

$$E_n = \mu_o \ell \omega \lambda \sum \frac{n}{n+1} \sin n\theta' \left[\left(r_{1a}^{n+1} - a^{n+1} \right) - \left(r_{1b}^{n+1} - b^{n+1} \right) 2 \cos n\theta' \right]$$

With this formula the coil response E_n/E_1 can be calculated, and using this formula it is possible to make the voltage response of the coil for the n-th harmonic vanish. For example, the condition that the voltage response be zero for B_{22} is such that

$$\left[\left(r_{1a}^3 - a^3 \right) - \left(b^3 - r_{1b}^3 \right) 2 \cos 2\alpha \right] = 0$$

Because this particular coil has a response characteristic that is more sensitive for measuring B_{33} than it is for B_{22}, it is particularly useful for measuring the sextupole field in a quadrupole-sextupole magnet. Figure 3 is a representative spectrum of a quadrupole magnet made with a simple asymmetric loop with the return wire placed on the axis of the coil.

Magnetic Center Location

Experimental

In general, the magnetic center of a quadrupole magnet does not correspond to the mechanical center. For alignment of a quadrupole, the relationship of the magnetic center to the mechanical center must be known.

Rotating coils provide one method. Because the field at the center of a quadrupole is zero, the output from a symmetrical rotating coil is a minimum when the coil is at the center. Thus, by using a rotating coil and moving it around until its output is a minimum, one can locate the center. It is very difficult to reference the spatial location of the magnetic center as determined by this method to the mechanical structure of the magnet because of (1) uncertainty of the location of the coil axis and (2) runout of the coil shaft. Considering these factors, probably the best center determination possible by this method is ±0.005 inches.

In our case the method of magnetic center determination is the use of a colloidal suspension of ferrous oxide particles. This technique was proposed and used by R. M. Johnson[6] to locate the magnetic center in quadrupole fields. The physical mechanism of this method was explained recently[7] as scattering of polarized light on aligned colloidal particles in multipole fields. In this system a small vial of the suspension is placed in the magnetic quadrupole field such that the mechanical center falls within the area of the vial. White plane-polarized light is directed through the vial of solution from one end of the magnet. The experimental arrangement is shown in Fig. 4. The observer at the opposite end of the magnet then looks at the vial through a plane-polarizing analyzer which is crossed with the polarizer of incoming light such that complete cancellation of light should occur when the magnetic field is turned off. With magnetic field, complete cancellation does not occur except along two mutually perpendicular axes which cross at the magnetic center of the quadrupole. The accuracy of this type of center determination is of the order of ±0.001 inch. The vial with the polarizer and analyzer is mounted in a small carriage which can be moved along the Z axis of the magnet. With this device the "magnetic center line" can be measured.

The scattering centers in the colloidal solution are Fe_3O_4 crystallites. The preparation of such a colloidal solution is described by D. J. Craik and P. M. Griffiths.[8] The individual crystallites of the magnetite (Fe_3O_4) have been measured with an electron microscope by Craik[9] and it was found that the particles are of the order of 100 Å.
The alignment of these magnetite crystallites in the magnetic field might be explained by the theory of paramagnetic alignment.

Symmetry Relations in Multipole Fields

The theory of anisotropic light scattering is complicated and a rigorous solution of the problem exists only in a few special cases. In our case the symmetry properties of the magnetic multipoles allow a number of simplifications in the calculation of the intensity distribution of the scattering pattern. Such a symmetry relation in a quadrupole field is that any line passing through the center of symmetry with an angle θ, with respect to the X axis, will cross the magnetic field lines at an angle β, where $\beta = -\pi/2 + 2\theta$. In order to prove this relation, write the magnetic field in a quadrupole in the following form

$$\vec{H} = -\vec{i}\,\frac{\partial u}{\partial X} - \vec{j}\,\frac{\partial u}{\partial Y}$$

where $u = 2B_2XY$ is the scalar magnetic potential. Thus

$$\vec{H} = 2(\vec{i}Y + \vec{j}X)$$

The line which gives direction of the magnetic field at point Q intersects the X axis with an angle γ (see Fig. 5) which is given by

$$\tan\gamma = \frac{(\vec{H})_Y}{(\vec{H})_X} = \frac{X}{Y} = \frac{r\cos\theta}{r\sin\theta} = \cot\theta = \tan(\pi/2 - \theta)$$

or $\gamma = \pi/2 - \theta$. Hence, since $\gamma + \pi - \theta + \beta = \pi$, $\beta = \pi/2 + 2\theta$. But β is defined as the angle between two vectors; therefore, one must consider β and $\beta + \pi$ as the angles between the direction of the magnetic field line at point Q and the line passing through the center. This yields

$$\boxed{\begin{aligned}\beta &= \frac{\pi}{2} + 2\theta \\ \beta &= -\frac{\pi}{2} + 2\theta\end{aligned}}$$

which agrees with the observed placement of lines.

Measurement of the Effective Length in Multipoles

The action of a transverse magnetic field on a particle beam can be characterized by the integral

$$\int_{-\infty}^{\infty} B_r(r,z)\,dz$$

where the line integral is taken along the particle trajectory in the magnet system, and $B_r(r,z)$ is the magnitude of the transverse field component at a distance r from the center line (Oz) of the multipole field.

It is also very useful, especially for magneto-optical calculations, to define equivalent lengths for the multipole field components in a magnet system. Using the analog to the definition of the equivalent length in a dipole field,

$$L_1 = \frac{1}{B_0}\int_{-\infty}^{\infty} B(z)\,dz$$

one can define the effective length of the quadrupole field as

$$L_2 = \frac{1}{\frac{\partial B}{\partial r}(0,r)}\int_{-\infty}^{\infty} \frac{\partial B(z,r)}{\partial r}\,dz$$

The effective length of a quadrupole is one of its most important characteristics because it is used in the matrix element when calculating the beam dynamics in a magnetic lens system. In general, the effective length is a function of the radial position r from the magnetic axis of the quadrupole. There are several methods of finding the effective length. One involves using normal mapping procedures, plotting the field at a point r as a function of the axial position z for $-\infty < z < \infty$ and integrating the area under the curve from $-\infty$ to ∞. This area is then divided by the maximum field, and thus the effective length of the dipole field L_B as a function of radial position is obtained. From this, using the formula

$$L_2(r) = L_1(r) + r\,\frac{\partial L_1(r)}{\partial r}$$

the length of the quadrupole field L_2 is calculable.

A second method of effective length determination involves the use of four coils rotating on a single shaft.[10] Two of the coils are long compared to the field while two are located in the central field of the magnet. The outputs from the long and short coils add in a quadrupole field but exactly cancel in a dipole field. The total output sinusoidal wave from the long coils is divided down on a precision potentiometer and compared with the total output sinusoidal wave from the short coils. The phase of the outputs is exactly the same because the long and short coils are built in the same plane. The two signals are thus compared until the divider potentiometer is set for complete cancellation of signals. Cancellation is facilitated by inversion of one signal with respect to the other, so that when the signals are equal they appear as a null. Then, measuring the ratio of the induced voltages and knowing the coil dimensions, the effective length of the quadrupole field is calculable. In this way, accuracy of about 0.1% is assured.

Gradient Measurement in Multipoles

One of the best methods of specifying the quality of a given quadrupole is the constancy of the gradient $\partial B_r/\partial r$ over the aperture of the magnet. Because the direction of the field vector is a function of azimuthal angle in the aperture, the gradient is usually determined along the two axes, one principal and one secondary. The simplest method of examining the deviation of the gradient along an axis is by normalizing the gradient at a point to the gradient at the center of the magnet. Thus referring to the axis X, the gradient deviation would be expressed as a function of X as

$$\frac{\partial B_Y}{\partial X} \Big/ \left(\frac{\partial B_Y}{\partial X}\right)_{X=0} ,$$

and along the axis x the equivalent expression would be

$$\frac{\partial B_x}{\partial x} \Big/ \left(\frac{\partial B_x}{\partial x}\right)_{x=0}$$

Among the methods available to measure these quantities, one is to use a pair of closely matched linear hall probes mounted so that they are spaced δY for the measurement of the gradient versus displacement in X and Δx for the measurement of the gradient versus displacement in x (see Fig. 6).

Since the procedure for making the measurement is essentially similar along the two axes, this description will describe the measurement in x only. The difference between the hall probes output is determined for the case when the probes are at the center of the magnet, and this difference signal is then nulled with an external voltage. Next, the probes as a unit are displaced along the principal axis and the change in difference versus position from the center is recorded. This operation yields the quantity $\partial B/\partial x - \partial B/\partial x|_{x=0}$. Normalizing this to the gradient at the center of the aperture $\partial B/\partial x|_{x=0}$, one obtains

$$\frac{\delta\left(\frac{\partial B}{\partial x}\right)}{\left.\frac{\partial B}{\partial x}\right|_{x=0}}$$

the nonlinearity of the gradient over the aperture. In an ideal quadrupole magnet this would be zero for all values of x. In a practical quadrupole there is some nonlinearity caused by the factors mentioned earlier. Referring to that discussion, one can construct what nonlinearity of gradient will result from the truncation of the poles and from various misalignments and asymmetries in the construction.

Acknowledgements

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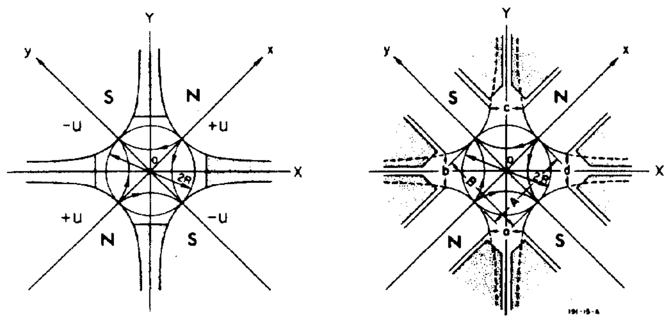

a) IDEAL (THEORETICAL) QUADRUPOLE

b) PRACTICAL QUADRUPOLE. THE SHAPE OF THE POLE SURFACES DOES NOT EXACTLY CORRESPOND TO A TRUE HYPERBOLA AND THE POLES HAVE BEEN TRUNCATED LATERALLY TO PROVIDE SPACE FOR COILS.

Fig. 1. (a) Ideal (theoretical) quadrupole. (b) Practical quadrupole. The shape of the pole surfaces does not exactly correspond to a true hyberbola and the poles have been truncated laterally to provide space for coils.

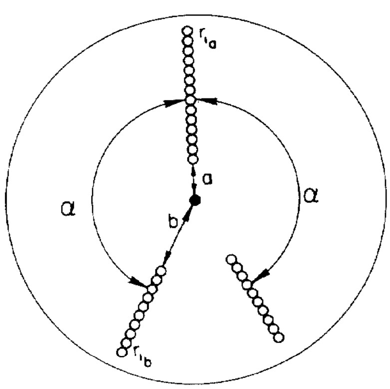

Fig. 2. Geometry of coil arrangement on rotating asymmetrical coil.

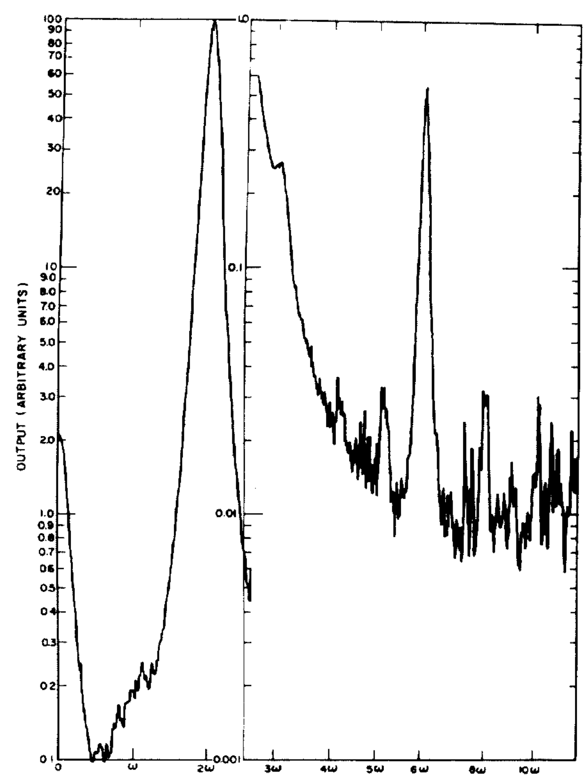

Fig. 3. Typical spectrum of quadrupole magnet, coil rotating at frequency ω.

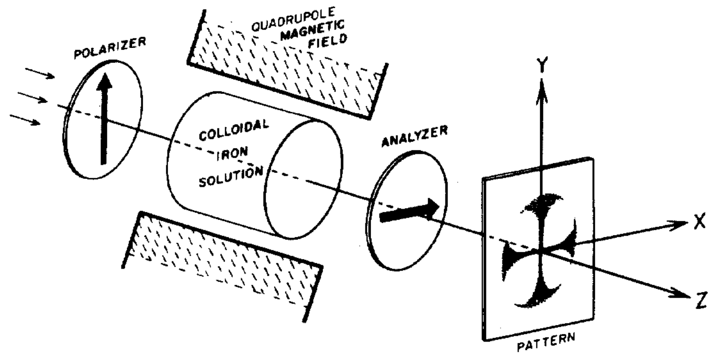

Fig. 4. Experimental setup for magnetic center location in quadrupole magnetic field.

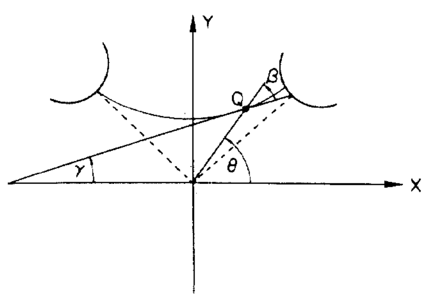

Fig. 5. Interrelation of angles γ, θ, and β in a magnetic field with quadrupole symmetry.

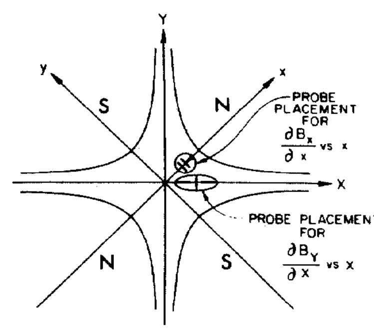

Fig. 6. Probe arrangement for making gradient measurements in quadrupoles.