UNIFORM-FIELD WEDGE MAGNETS: RESULTS

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Introduction

The formulas of the preceding paper[1] will be applied to the symmetric case and to actual magnets here. For a beam of particles initially parallel to the optic axis, the transverse focal distance is

$$q_2 = \frac{f_2(f_1 - \alpha')}{f_2 + f_1 - \alpha'} - \Delta_2 \quad , \qquad (1)$$

and the radial focal distance is given by

$$q_r^{-1} = \tan(\alpha - \beta_1) - \tan\beta_2 \quad . \qquad (2)$$

These distances are measured in units of the central radius of curvature, ρ, and from the exit effective field edge.

Double-Focusing Magnet with $\beta_1 = \beta_2$

Let us consider the case of a symmetric magnet—one in which entrance and exit quantities are identical. Rewriting the above equations, we obtain

$$q_2 = \frac{f(f - \alpha')}{2f - \alpha'} - \Delta \qquad (3)$$

in the transverse direction and

$$q_r^{-1} = \tan(\alpha - \beta) - \tan\beta \qquad (4)$$

in the radial direction.

Exact computations were made using these equations to find β and the normalized image distance, $q_2 = q_r \equiv q$, as a function of α and kg. The results are shown in Figs. 1 and 2.

For small angles, Eqs. (3) and (4) yield

$$\beta \simeq \tfrac{1}{4}\alpha + \tfrac{1}{2}\Delta \simeq \tfrac{1}{4}\alpha + \tfrac{1}{2}kg \qquad (5)$$

through the second order. This limiting case is shown by the dashed line in Fig. 1.

Double-Focusing 45° Magnet with $\beta_2 \approx 0$

Two 45° deflection magnets and a quadrupole magnet in the center were combined to form a 90° achromatic system.[2] The magnets were designed to have the trajectory between bending magnets normal to the adjacent edges; thus, $\beta_2 = 0$ for the first magnet. The magnetic field was measured using a 0.1% rotating-coil gaussmeter. These results were utilized in a computer ray-tracing program, using steps of 0.125 in.

The magnet has a wedge angle of 21°8'. In order to obtain a double focus at 30.00 in. from the actual exit pole edge, the computer results required that ρ = 11.00 in. and that the entire magnet be rotated so that β_1 = 24° 50' and β_2 = -0° 58'.

The magnet gap is 1.75 in. (g = 0.159) and the effective field extends 1.374 in. beyond the pole edge (h = 0.785). The computer program then yields q = 2.602.

For these values, Eq. (2) produces q_r = 2.603 for the radial focus, in excellent agreement with the computer result. In order to make q_2 of Eq. (1) fit the computer value, one must set $k_1 = k_2$ = 0.489. Using the method of Enge[3], we find q_2 = 2.54 for this case.

For this magnet, the effective field extends only 4.6% of the distance to the image and an error in h produces a correspondingly smaller relative error in q. A change in the assumed value of k produces a larger effect, $\delta q/q = 0.7\ \delta k/k$.

The transverse image distance was found to be 30 ± 1 in. from the actual edge using a highly collimated electron beam from the NRL Linac. The complete magnet system of two such bending magnets and a quadrupole has also been tested with the Linac beam. With a total deflection angle of 90.0 ± 0.1 degrees, the 1/8 in. beam spot did not shift by more than 1/16 in. over the pass band of the system, 8.6% in momentum.

Double-Focusing 35° Magnet

A magnet of 35° deflection and about 5 ft. focal length was desired. The magnet built has a gap of $g\rho$ = 1.75 in. and a wedge angle of 13.60°. Field measurements show the effective field to extend 1.194 in. beyond the pole edge (h = 0.682).

In order to obtain this deflection with a beam approximately through the center of the magnet, a radius of curvature of ρ = 18.3 in. was adopted. For a double focus, the computer program required that β_1 = 4.06° and hence β_2 = 17.34°. The focus is at 62.10 in., measured normally from the pole edge. From the effective field edge, this is $(62.10-1.194)/\rho\cos\beta_2 = q = 3.487$ radii along the trajectory. For these conditions, Eq. (2) shows $q_r = 3.482$ for the radial focus. Agreement with Eq. (1) is obtained using $k = 0.493$.

The symmetric case, $\beta_1 = \beta_2 = 10.70^\circ$, was also calculated, but with the trajectory constrained to go through the exact "center" of the magnet. The computer results are $\rho = 18.169$ in., $q_r = 3.811$ for radial focusing and $q_2 = 3.362$ for transverse focusing. With this radius the formulas yield $q_r = 3.809$ and the computed value of q_2 for $k = 0.475$.

Note that k must change by 0.018 to fit these computer ray-tracing results.

Choice of k

The value of k found to agree most closely with the computer predictions for our magnets is $k = 0.486$. The ratios of pole width to gap are 4.6 for the 45° magnet and 5.0 for the 35° magnet. The coil arrangement is between curves a and c of Fig. 3 of Enge[3]. Thus his "long-tail" field should fit these cases quite well, a conclusion verified by the field measurements. For the long-tail field, Enge obtains values of 0.487 and 0.475 by two methods. It may be concluded from these considerations that for magnets similar to those considered here a reasonable value is

$$k = 0.485 \pm .01.$$

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References

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3. Harald A. Enge, Rev. Sci. Instr. 35, 278 (1964).

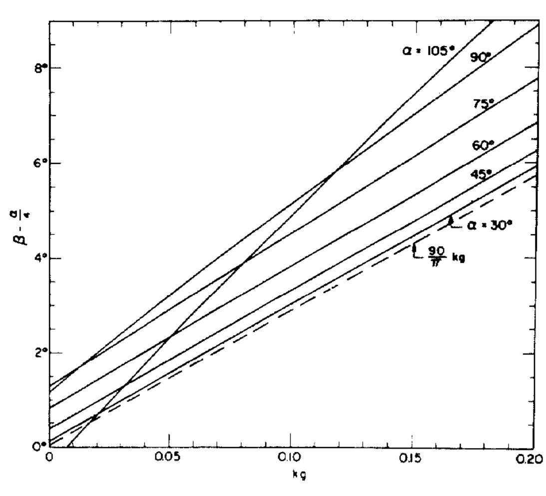

Fig. 1. Edge rotation, β, for a symmetric double-focusing magnet. The dashed line corresponds to the limiting case, Eq. (5) of the text.

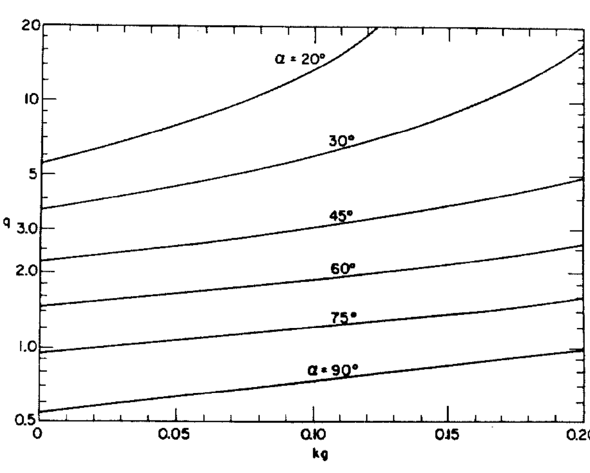

Fig. 2. Normalized image distance, q, for a symmetric double-focusing magnet.