UNIFORM-FIELD WEDGE MAGNETS: DESIGN

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Introduction

Achromatic deflection systems have become increasingly popular in recent years, particularly to enhance the versatility of medium-energy electron linear accelerators. With the aid of suitably located slits, the pass-band of a well designed system can be varied from 0.1% or better to 10%. The key element in these systems is usually a uniform-field wedge magnet which relies on rotated edges to obtain transverse focusing and, at the same time, to decrease the usual radial focusing. The design of such magnets is complicated by the effect of the fringe fields. These fields are usually taken into account by assuming that the field extends a certain distance from the actual edge and drops sharply to zero. This procedure has been found to work very well for calculation of the radial motion. However, until recently no good approximation has been available for predicting the motion transverse to the median plane. The fringe effect is more serious if one desires a small high-field magnet to reduce the cost, or a large gap, or both. In extreme cases the computed transverse focal length may be in error by 20-30% if one assumes that the transverse deflection occurs at the real edges of the magnet.

We assume here that the entrance and exit fringe fields may be replaced by suitably located thin lenses for the purpose of calculating the transverse focusing. Recent work by Enge[1] has given a strong impetus to the use of this method. Our approach differs from that of Enge in that the thin lens assumption is employed in a straightforward geometrical manner.

Geometry

Figures 1 and 2 show the geometry and define most of the quantities used in the equations. We consider only magnets with mirror symmetry about the median plane and with pole edges perpendicular to that plane. All linear dimensions are measured in units of the central bending radius ρ. Subscripts 1 and 2 indicate quantities for the entrance and exit fields, respectively. The magnet gap is g units; α is the total deflection angle; β is the rotation of the pole edge from the normal to the trajectory at infinity. Following common practice, we define β so that it is positive for positive transverse focusing. Object and image distances are p and q, respectively.

The effective field boundary is $hg/\cos\beta$ beyond the pole, as shown in Fig. 2. The parameter h is about 0.6 to 0.8 for most cases and can be determined for a given case by integration of the fringe field or estimated from curves given by Enge[2].

The distance between the thin lens used for transverse focusing and the effective field boundary is k gaps when measured normal to the pole edge. An empirical value[3] based on several magnets is $k = 0.485 \pm .01$. The particle is assumed to be deflected by Δ radians between infinity and the thin lens. The correction angle, Δ radians, is

$$\Delta = kg/\cos\beta \quad .$$

The effective angle between the trajectory at the lens location and the normal to the magnet edge is therefore $\beta-\Delta$. The focal length of each lens, measured from the lens position, is given by[4]

$$f = \cot(\beta-\Delta) \quad .$$

The net deflection angle between lenses is

$$\alpha' = \alpha - \Delta_1 - \Delta_2 \quad .$$

Transverse Formulas

The usual thin lens formula applied to the entrance edge (see Fig. 2) yields

$$\frac{1}{p_1+\Delta_1} + \frac{1}{\alpha-\Delta_1+p_2} = \frac{1}{f_1} \quad .$$

The relation for the exit edge is

$$-\frac{1}{p_2+\Delta_2} + \frac{1}{q_2+\Delta_2} = \frac{1}{f_2} \quad .$$

These equations may be solved for p_2 and then for q_2.

For the case of a beam of particles entering parallel to the optic axis, we set $p_1 = \infty$ and obtain

$$q_2 = \frac{f_2(f_1-\alpha')}{f_2+f_1-\alpha'} - \Delta_2 \quad .$$

Radial Formulas

The radial image distance is[5]

$$q_r = \frac{\sin\alpha\cos\beta_1\cos\beta_2 + p\cos(\alpha-\beta_1)\cos\beta_2}{p\sin(\alpha-\beta_1-\beta_2) - \cos(\alpha-\beta_2)\cos\beta_1} \quad .$$

If the entering beam is parallel to the optic axis, this simplifies to

$$q_r^{-1} = \tan(\alpha - \beta_1) - \tan \beta_2 \quad .$$

Enge[1] showed that the direction of each ray and the image distance are the same for a real (extended) fringe field and the equivalent sharp-cutoff field. However, there is a slight radial shift of the input and output trajectory.

Acknowledgment

The concept of replacing a magnet edge by a suitably located thin lens was first brought to the attention of the authors by Dr. K. L. Brown, to whom acknowledgment is gratefully given.

References

1. Harald A. Enge, Rev. Sci. Instr. 35, 278 (1964).
2. Figure 3 of reference 1.
3. W.L. Bendel and T.F. Godlove, following paper.
4. R.M. Sternheimer, Methods of Experimental Physics (Academic Press, New York, 1963), Vol. 5, Part B. p. 733.
5. Reference 4, Eq. 4.2.186.

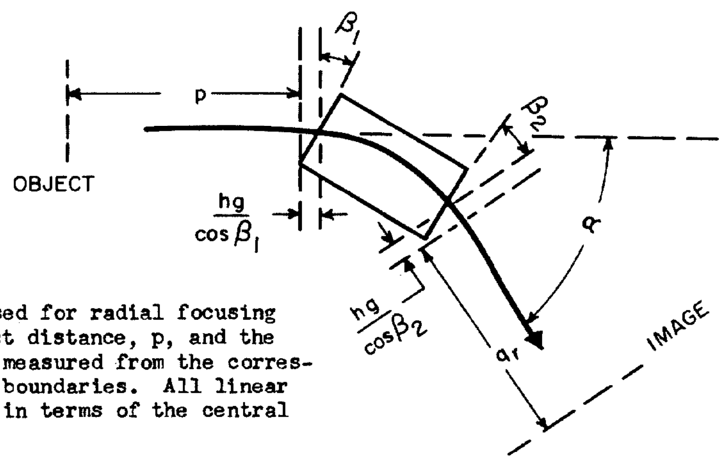

Fig. 1. The geometry used for radial focusing calculations. The object distance, p, and the image distance, q_r, are measured from the corresponding effective field boundaries. All linear dimensions are measured in terms of the central bending radius ρ.

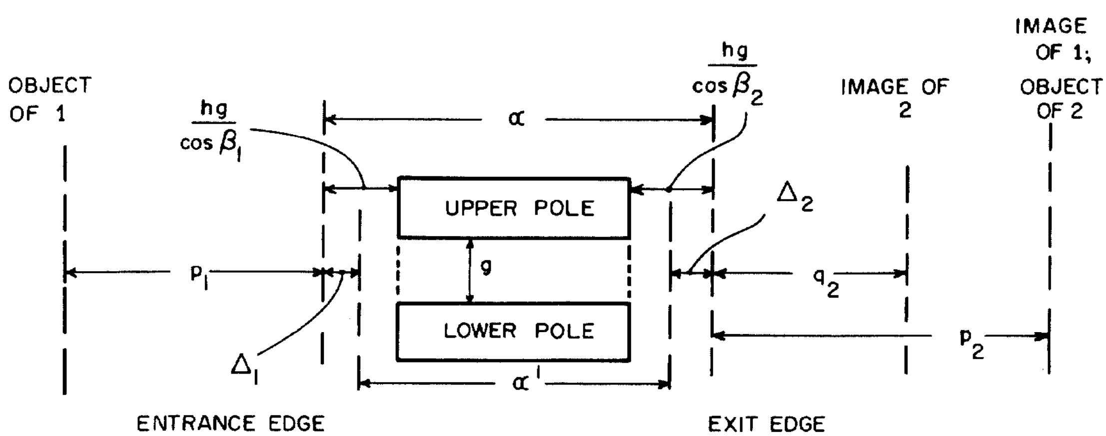

Fig. 2. The geometry used for transverse focusing calculations. The thin lenses used for transverse focusing are shown by the vertical dashed lines separated by α'. The effective field boundaries are shown by the vertical dashed lines separated by α.