A HIGH FIELD MAGNET FOR THE ALTERNATING-GRADIENT SYNCHROTRON*

George Parzen
Brookhaven National Laboratory
Upton, L.I., N.Y.

I. Introduction

The magnets of alternating-gradient synchrotrons at present obtain the strong gradient by shaping the magnet poles. This paper proposes another way of obtaining the required spatial variation of the magnetic field, which may offer the possibility of going to higher magnetic fields. The design of this magnet is indicated in Fig. 1a.

The magnet shown in Fig. 1a has two poles instead of one, and there are two coils which allow one to vary the relative excitation of the two poles. The coils are excited so that underneath the center of the poles, the field is just that given by the linear variation of a conventional AGS magnet.

The magnet shown in Fig. 1 will be called a Discrete Excited Pole (DEP) magnet. The field variation as a function of r along the azimuthal magnet center, instead of being the usual linear variation, is as shown in Fig. 1b.

The magnetic field in a DEP magnet oscillates about the linear field of the usual AGS magnet. The question immediately arises as to whether such an oscillation in the field would not cause the ν-values to vary more than can be allowed. This question will be treated in the next section.

Assuming for the time being that the DEP magnet will have acceptable orbits, one can see the following advantages:

1) The exact shape of the pole is no longer critical. It is sufficient to adjust the two coils so that the fields under the center of the poles have the desired values.

2) It should be possible to go to higher magnetic fields. The relatively higher saturation of the high field pole can be compensated for by extra excitation of one coil. A rough estimate indicates that one can achieve fields which are about 30% higher than that in conventional AGS magnets.

II. Orbit Results

The magnetic field in a DEP magnet is not a linear field, and the variation in the ν-value with energy will not be acceptable unless something is done to keep the variation in the ν-value down.

If one regards the departure from a linear field in a DEP magnet as a perturbation on the linear field, then one way of reducing the effect of this perturbation on the ν-values is to give the perturbation a rapid variation around the accelerator. This may be accomplished, for example, by moving the DEP magnets in and out around the magnet ring by an amount Δr, such that

$$\Delta r = a \cos M\theta , \qquad (2.1)$$

where $2\pi/M$ is considerably smaller than the fundamental period of the accelerator, $2\pi/N$, which is the period of the alternating gradient. In some sense then, the particles will see on the average just the linear field of the usual AGS. This displacement of the magnets is illustrated by Fig. 2a.

Another arrangement which may have some advantages is to displace the magnets so that the pole faces lie on the logarithmic spirals

$$\frac{\pi R}{b} \ln(1 + x/R) - M\theta = \text{constant} \qquad (2.2)$$

This arrangement is shown in Fig. 2b. Note that the edges of the pole faces have also been made parallel to the logarithmic spiral. The radial separation of the poles is, roughly, 2b.

Logarithmic Displacement of Magnets

For the case where the magnets are displaced so as to lie on a logarithmic spiral, the median plane magnetic field may be approximated by

$$H_z = B_o (1 + K x \cos N\theta) \times (1 + \varepsilon \sin[(\pi R/b)\ln(1 + x/R) - M\theta]) \qquad (2.3)$$

In Eq. (2.3), R is the radius of the central orbit, and the separation of the poles of the DEP magnet is roughly 2b.

Computer runs were done with parameters similar to those of the Brookhaven AGS: K = .1, b = 3 inches, R = 5057.4 inches, N = 60, and M = 270. It is desirable to choose M so that it is not a multiple of N.

In Fig. 3, the changes in ν-values are shown across the vacuum tank for various values of ϵ.

One may note that the variation of ν_r has not been affected. The variation in ν_z, on the other hand, becomes larger with increasing ϵ. In Fig. 3b $\Delta\nu$, which is the total change in ν, as x varies from -3 inches to +3 inches, is plotted against ϵ.

Fig. 3a shows a region near x = 5 inches where the ν_z variation is decreased. However, this gain is partially offset by an increased circumference factor.

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It is difficult to determine the correct value of ϵ without detailed magnet calculations. An estimate of ϵ indicates that $\epsilon \simeq .1$.

Radial Displacement of Magnets

For the case where the magnets are displaced radially according to Eq. (2.1), the median plane magnetic field may be approximated by

$$H_z = B_o\,(1 + K\,x \cos N\,\theta) \times (1 + \epsilon \sin[(\pi/b)(x - a \cos M\,\theta)]) \qquad (2.4)$$

In Eq. (2.4), a is the amount of the radial displacement and the separation of the poles of the DEP magnet is 2b.

The magnet displacement parameter, a, can be chosen so as to minimize the effect of the DEP magnets on the ν-values. One can Fourier analyze the field Eq. (2.5) and demand that the field harmonics for n = 0, N be linear with r, exactly as though the field were linear.

If one writes

$$H_z = B_o\,\{ C_o(x) + C_N(x) \cos N\,\theta + \text{higher harmonics} \ldots \} \qquad (2.5)$$

then one finds for $C_o(x)$, $C_N(x)$,

$$C_o(x) = 1 + \epsilon\,J_o\,(\pi\,a/b)\,\sin\,(\pi\,x/b)\;, \qquad (2.6a)$$

$$C_N(x) = K\,x + \epsilon\,J_o\,(\pi\,a/b)\,\sin\,(\pi\,x/b)\;, \qquad (2.6b)$$

where J_o is the zero order Bessel function. We have used the expression

$$\cos\,(a \cos n\,\theta) = J_o(a) - 2 \sum_{m \geq 1} (-1)^m\,J_{2m}(a) \cos 2\,m\,M\,\theta \qquad (2.7)$$

and we have assumed that N and M are incommensurate.

One sees that to make $C_N(x) = Kx$, one must choose a by the rule

$$\pi\,a/b = \mu\;, \qquad (2.8)$$

where μ is a zero of $J_o(x)$

$$\mu_0 = 2.4048\;, \quad \mu_1 = 5.5201\;, \quad \mu_2 = 8.6537\;, \ldots$$

A computer study was carried out using the magnetic field of Eq. (2.4) and choosing a according to the rule Eq. (2.8), with $\mu = 2.4048$, the lowest zero of the Bessel function.

The parameters of the field were chosen so as to simulate the Brookhaven AGS. The parameters used were N = 60, M = 270, and K = .1. The two poles were separated by 6 inches, which means b = 3 inches, and the optimum choice of a according to Eq. (2.8) is then a = 2.2964 inches.

In Fig. 4a, the changes in ν-values are shown across the vacuum tank for $\epsilon = .1$. In Fig. 4b, $\Delta\nu$, which is the total change in ν as x varies from -3 to +3 inches, is plotted against ϵ.

III. Magnet Design

To illustrate what the DEP magnet might look like, we will design a DEP magnet that might have been used for the Brookhaven AGS.

For simplicity an accelerator will be considered which has no long straight sections. The gradient will alternate with a period of $2\pi/60$.

The logarithmically displaced magnet shown in Fig. 2b will be used. In Fig. 2b, the magnet shown can be considered as the positive gradient magnet. This magnet can be broken up into three magnets, each about 60 inches long and 23 inches apart.

The negative gradient magnet can be obtained by rotating the positive gradient magnet by 180 degrees. The positive gradient magnet is shown in Fig. 5, roughly to scale.

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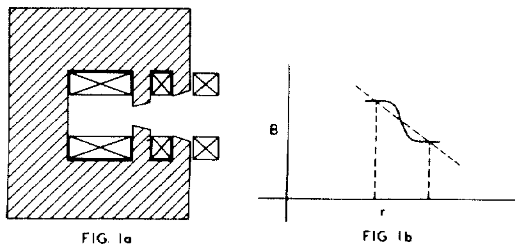

Fig. 1. Cross section and magnetic field of a Discrete Excited Pole (DEP) magnet.

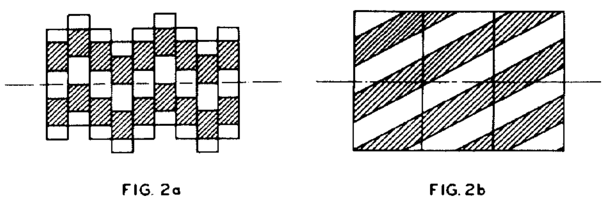

Fig. 2. DEP magnets corresponding to radial and logarithmic displacement of the poles.

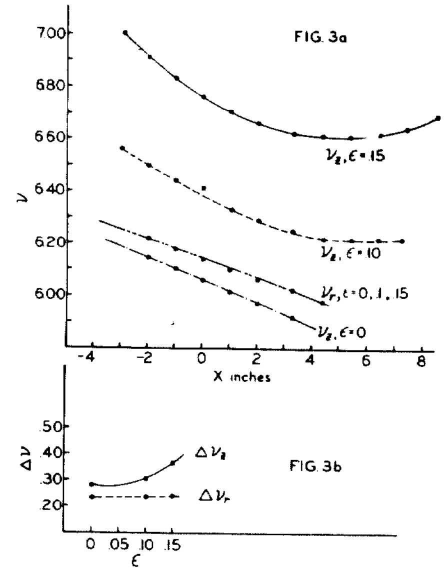

Fig. 3. Computer results for the variation of the ν-values for an accelerator with logarithmically displaced DEP magnets. x is the average orbit radius relative to the center of the vacuum tank. The results for $\varepsilon = 0$ are those for a conventional AGS.

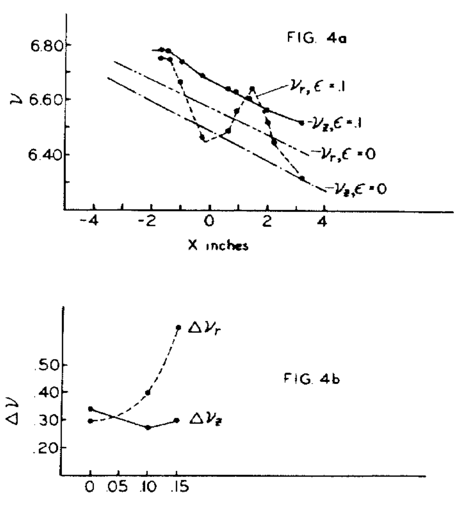

Fig. 4. Computer results for the variation of the ν-values for an accelerator with radially displaced DEP magnets. x is the average orbit radius relative to the center of the vacuum tank. The results for $\varepsilon = 0$ are those for a conventional AGS and are given by the dashed lines.

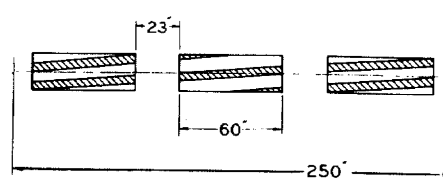

Fig. 5. A positive gradient magnet made up of logarithmically displaced DEP magnets and drawn roughly to scale.