NUMERICAL STUDIES OF THE SHAPES OF DRIFT TUBES AND LINAC CAVITIES

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Summary

At the Los Alamos Scientific Laboratory, a computer program for calculating the magnetic and electric fields in resonant cavities has been developed. This program, LALA, will compute the fields in any cavity having cylindrical symmetry; arbitrary shapes for both drift tubes and external boundaries can be treated. Both π-mode and 2π-mode cavities can be handled. Calculations of several π-mode cavities at about 800 Mc/sec frequency have been carried out. At this frequency, for the π-mode, and in a cavity containing a drift tube, an external conducting boundary having a circular cross-section gives about a ten percent increase in ZT^2 from the more normal rectangular external boundary.

Description of Program

The resonant cavities which are to be considered have cylindrical symmetry. Furthermore, we shall consider only cavities for which the boundaries of the region over which the calculation is performed are (1) the axis of symmetry and (2) either conducting surfaces or symmetry boundaries. The electric and magnetic fields are determined by Maxwell's Equations (1) and the boundary conditions (2). Figure 1 illustrates the region used when symmetry permits only half a cavity to be considered.

$$\left.\begin{aligned} \nabla\cdot\bar{D} &= 0, & \nabla\times\bar{E} &= -\frac{\partial\bar{B}}{\partial t}, \\ \nabla\cdot\bar{B} &= 0, & \nabla\times\bar{H} &= \frac{\partial\bar{D}}{\partial t}. \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} &E_r = 0 \text{ at } r = 0, \\ &\bar{n}\times\bar{E} = 0 \text{ on rest of boundary.} \end{aligned}\right\} \qquad (2)$$

The electric field $\bar{E}$ in our cavity should have only r- and z-components. Equations (1) then say that the magnetic field $\bar{H}$ should have only a Θ-component. We also wish to state that H_Θ can be written as H exp(jωt), where ω = 2πf and f is the frequency of the cavity. Now the wave equation, which can be found from equations (1), becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial H}{\partial r}\right) - \frac{H}{r^2} + \frac{\partial^2 H}{\partial z^2} + \frac{\omega^2}{c^2}H = 0 \qquad (3)$$

For numerical calculations it is convenient to let F = rH and $\lambda^2 = (\omega^2/c^2)$; then eq. (3) becomes

$$\frac{\partial^2 F}{\partial r^2} - \frac{1}{r}\frac{\partial F}{\partial r} + \frac{\partial^2 F}{\partial z^2} + \lambda^2 F = 0 \qquad (4)$$

In terms of F the boundary conditions (2) are

$$\left.\begin{aligned} &F = 0 \text{ at } r = 0, \\ &\frac{\partial F}{\partial n} = 0 \text{ on rest of boundary,} \end{aligned}\right\} \qquad (5)$$

where $\partial F/\partial n$ is the derivative of F normal to the boundary. Lines of constant F give tubes of electric flux and thus correspond to electric field lines.

The numerical method used is point-by-point relaxation; the program has provision for over-relaxation. A square mesh is used. The difference equation for the interior points of the mesh is derived from a variational formulation of the problem. If we take the integral

$$I = \frac{1}{2}\int\!\!\int\frac{1}{r^2}\left[\left(\frac{\partial F}{\partial r}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2 - \lambda^2 F^2\right] r\,dr\,dz \qquad (6)$$

and require that the variation

$$\delta I = 0, \qquad (7)$$

we obtain the differential equation (4). If we substitute from Maxwell's Equations (1) into (6), we find that

$$I \propto \int(\epsilon E^2 + \mu H^2)\,dV, \qquad (8)$$

and so δI = 0 says that the energy of the field in the cavity must be a minimum. The difference equation for interior points is obtained by substituting difference approximations in (6), then performing the process of taking the variation and setting it equal to zero. The result is a nine-point difference equation. Equations (6) and (7) also can be used for determining the difference equations for boundary points; these difference equations, however, satisfy the boundary condition $\partial F/\partial n = 0$ on an approximation to the boundary rather than on the actual boundary. Consequently, a different set of boundary difference equations, satisfying $\partial F/\partial n = 0$ on the actual boundary, have been used.

The LALA program uses a zig-zag approximation to the actual boundary. This zig-zag boundary lies either on or outside the actual boundary (see Fig. 1). The calculation is carried out for the region inside the zig-zag boundary. The equations used to compute the values of F for the zig-zag boundary points are derived so as to satisfy physical boundary conditions on the actual boundary. It is the use of the zig-zag boundary which permits us to calculate for arbitrary boundary shapes. This ability to consider any boundary shape makes the LALA program considerably more versatile than other cavity mesh calculations.

The problem stated by equations (4) and (5) is an eigenvalue problem. In addition to computing the fields (F-values) we must determine the frequency of the cavity. This is computed by means of the following relation:

$$\lambda^2 = \frac{\iint \frac{1}{r^2}\left[\left(\frac{\partial F}{\partial r}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2\right] r\,dr\,dz}{\iint \left(\frac{F^2}{r^2}\right) r\,dr\,dz} \tag{9}$$

Calculational procedure involves making a guess as to the F-values and λ^2, iterating until a satisfactory new set of F-values is obtained, computing a new λ^2, and then repeating the process. Convergence is determined by a requirement that both F-values and λ^2 have settled down to certain small variations. Upon determination of convergence, those quantities of interest to the designer are calculated. These are voltage, V, across the cavity, transit time factor, T, shunt impedance per unit length, Z, Q, ZT^2, and the power loss, P. The surface integral for Z is computed over the actual boundary. A number of other quantities also are calculated. Plots of electric field lines and of E_z on the axis are automatically produced.

The program can be used to calculate over a full cavity or a half-cavity. Both π-mode and 2π-mode cavities can be treated. Because of the method for handling boundaries, cavities with iris structures also may be treated.

Results of Calculations

The LALA program has been used to carry out some design studies for π-mode cavities operating at frequencies near 800 Mc/sec. The examples considered actually have frequencies near 836 Mc/sec; scaling the results will give values for 800 Mc/sec if these are required. The object of the study has been to find ways of increasing the value of ZT^2 for cavities operating near 800 Mc/sec. Since

$$ZT^2 \propto \frac{(\Delta\mathcal{E})^2}{P}, \tag{10}$$

where $\Delta\mathcal{E}$ is the energy gain of a particle traveling across the cavity and P is the power loss in the cavity. Increasing ZT^2 is similar to increasing the energy gain per unit power expended. For all of the cases shown the length of the cavity is 11.43 cm, and the energy of the proton being accelerated is about 280 Mev. We have designed for an energy gain of 1.25 Mev/meter. The cavities are separated by conducting walls; the only opening between adjacent cavities is the hole in the drift tube.

The results of the calculations are shown in Figures 2a, b, c, d. In Fig. 2a, the cavity is a right circular cylinder containing a drift tube. The radius of the cylinder equals its length. The outer radius of the drift tube is twice the radius of the hole. The drift tube length is just under half the length of the cavity. The value of ZT^2 is 36.6 megohms/meter. The result of decreasing the size of the hole in the drift tube is shown in Fig. 2b. The length of the drift tube has been increased slightly to keep the frequency constant. The value of ZT^2 has increased to 40.1 megohms/meter. In Fig. 2c the outer wall of the cavity has been curved; this was accomplished by drawing a circular arc with its center on the symmetry boundary. The drift tube of Fig. 2c has the same outer radius and hole radius as that of Fig. 2b; the length is very slightly different. The value of ZT^2 for this configuration is 44.2 megohms/meter, about ten percent greater than that of Fig. 2b. The final configuration is given in Fig. 2d. Here the outer surface of the drift tube has been changed from that of Fig. 2a to look more like a cone. The value of ZT^2 is 32.4 megohms/meter, about a ten percent decrease from that of Fig. 2a.

As a result of these calculations one can conclude that ZT^2 can be increased significantly by curving the outer wall of the cavity. The LALA program provides a means of investigating these and other boundary changes much more easily than by experiment.

Acknowledgments

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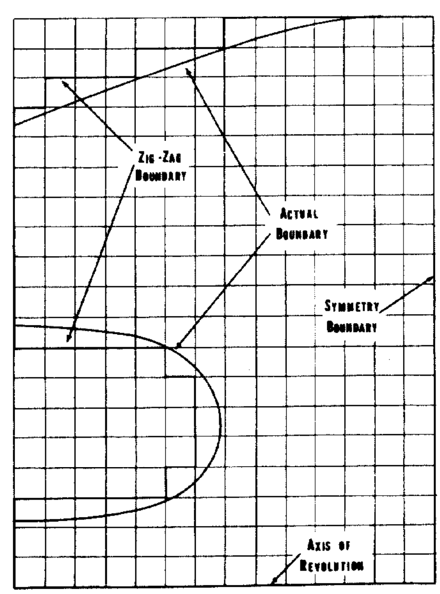

Fig. 1. Mesh and Boundary Configurations.

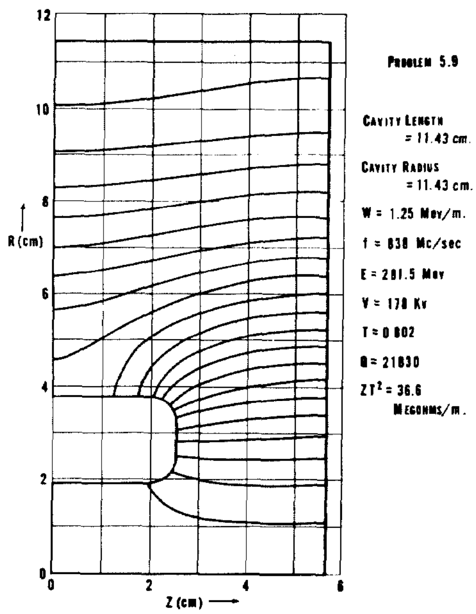

Figure 2a.

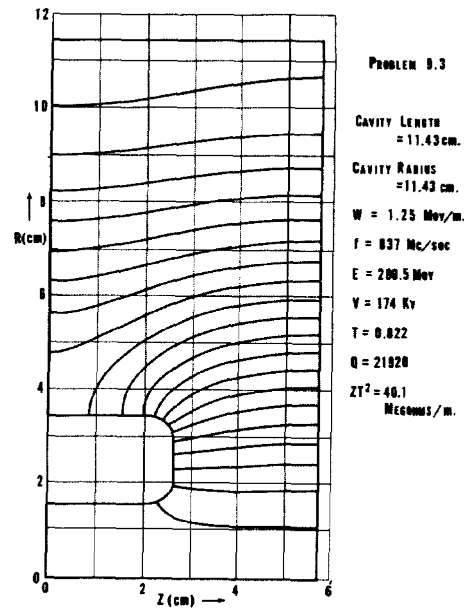

Figure 2b.

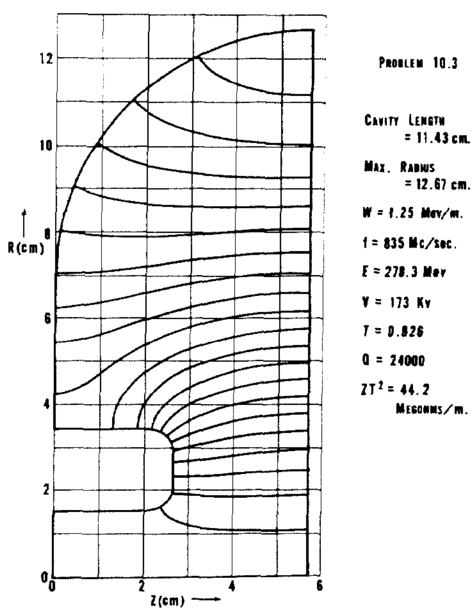

Figure 2c.

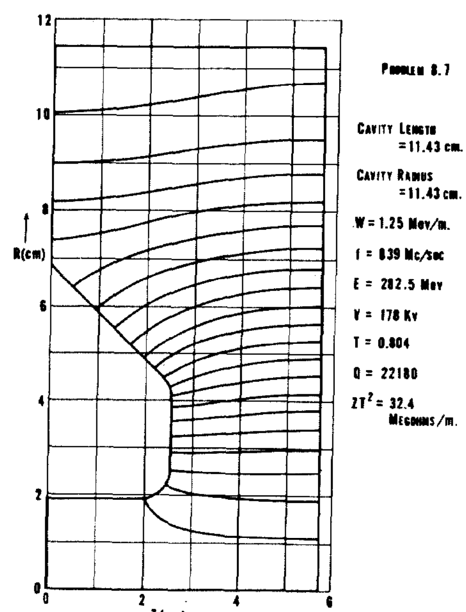

Figure 2d.