A PROPOSED AGS ACCELERATING CAVITY *

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Introduction. The proposal for the AGS conversion to a higher intensity machine is predicated on 500 MeV injection.

This injection energy raises the space charge limit to about 2×10^{13} protons/pulse. The corresponding rf requirements, if we maintain the twelfth harmonic for acceleration, necessitate a frequency swing from 3.5 mc/sec to 4.45 mc/sec. The proposal includes an additional main magnet power supply which increases the rate of rise of field by a factor of two. This sets the requirement of about twice the volts/turn to be developed by the rf system.

A proposed ferrite accelerating cavity has been investigated which will use ferrite rings which either exist in the present cavities or are within the capability of the manufacturers with regard to both magnetic properties and physical dimensions. This cavity can also serve to accelerate protons in the existing AGS, covering the frequency range from 1.4 mc/sec to 4.45 mc/sec with a voltage 50% greater than we now have.

The methods of computing impedance, losses, beam loading, and saturation characteristics for a number of possibilities of ring size and permeability will be described.

I. General Considerations

The limiting factor in designing a ferrite cavity is the maximum power density which we can allow, since this determines the heating and temperature rise behavior. This, of course, is also coupled to the cooling method selected. Using the present AGS cavity as a guide, a power density of about 0.3 watts/cc was selected as a design level. In our present structure the value of power density is about 0.28 watts/cc at the inside radius of the ferrite. We can allow the value to increase if a more efficient cooling system is provided, using a higher flow of chilled water, for example, to cool the copper plates which interleave the ferrite. In subsequent calculations 0.3 watts/cc at the inside radii of the ferrite rings was used as a selection guide for μ and Q.

The voltage applied to each cavity, at present, is 8 kV peak rf. We wish to reduce the number of cavities from 12 to 10 and with the increased rate of rise the voltage requirement becomes 18 kV peak rf per cavity.

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The effects of beam loading may be considered by looking at the cavity as an impedance driven from two sources. One is the rf power amplifier, the other is an equivalent generator representing the beam. The relative phase of the two generators is known. The P.A. phase is either 30° or 150° (below and above transition) with respect to the center of the proton bunch. The beam equivalent generator is at a phase of 270° with respect to the bunch center, that is, it induces an out of phase voltage across the gap. The sum of these two voltages must equal the required 18 kV. The magnitude of the beam equivalent generator is:

$$V = IZ, \quad I = \frac{N f_{rot}}{6 \times 10^{18}}$$

I = equiv. beam current
N = protons in ring
$f_{rot} = \frac{f_{rf}}{12}$
Z = Impedance/rf station

For 2×10^{13} protons/pulse at $f_{rf} = 4.45$ mc/sec.,

$$I = \frac{2 \times 10^{13} \times 4.45 \times 10^{6}}{12 \times 6 \times 10^{18}} = 1.25 \text{ Amps.}$$

$$V = 1.25 Z_{sta}$$

The corresponding rf P.A. voltage, for Z = 5000Ω would be, adding vectorially for a sum of 18 kV, $V_{pa} = 20.3$ kV.

The cavity, however, still only sees the resultant 18 kV and all calculations for the cavity itself are based on this figure.

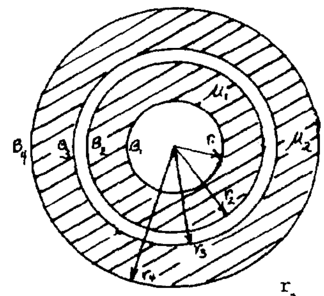
As a first consideration, to reduce the high power levels which would exist if the cavity voltage is raised to 18 kV, we can increase the size of the ferrite rings. The flux density in the rings is inversely proportional to the radius while the volume increases as the square of the radius. This is a very inefficient utilization of the ferrite. One method of equalizing the flux distribution is to have the permeability increase with increasing radius. Unfortunately, this can be done only in discrete steps and a two step arrangement was selected. The outer ring would have a permeability about twice that in the inner ring.

One possible structure would use the existing ferrite, $\mu_o = 400$, air gapped to a $\mu \approx 200$ for the inner rings. The outer rings would maintain a $\mu_o = 400$.

II Flux density and Q determination

The first calculations were done by considering the cavity to be made up of a series of annular rings. The voltage determines B_{rf} vs r, Q vs B has been measured and if we assume a power density, an initial point is determined.

All subsequent calculations assume an axial length of ferrite, t = 80cm per cavity, and two cavities in parallel per station.



From r_1 to r_2, $B = B_1 \frac{r_1}{r}$.

From r_3 to r_4, $B = B_3 \frac{r_3}{r}$.

Also, for a given excitation B is proportional to μ, and inversely proportional to r.

Therefore, $B_3 = \frac{\mu_2}{\mu_1} \frac{r_1}{r_3} B_1$

In general, $V = n\dot{\varphi} = nA\dot{B} = n\omega BA \times 10^{-8}$ volts for sinusoidal excitation, where φ = flux, A = area in sq. cm., and B = rf flux density in gauss. Integrating:

$$V_{tot} = n\omega t \left\{ r_1 B_1 \int_{r_1}^{r_2} \frac{1}{r}\, dr + \frac{\mu_2}{\mu_1} r_1 B_1 \int_{r_3}^{r_4} \frac{1}{r} dr \right\} \times 10^{-8}.$$

But n = 1, therefore

$$V = \omega t \times 10^{-8}\ r_1 B_1 \left(\ln \frac{r_2}{r_1} + \frac{\mu_2}{\mu_1} \ln \frac{r_4}{r_3}\right) \qquad (1)$$

Given V, ω, and the dimensions, B may be calculated.

For the existing rings r_1 = 10cm. The volume of a one centimeter annular ring is 5290cc. For a power density of 0.30 w/cc, P = 1587 watts. The voltage contributed by this segment is:

$V = \omega B_1 A \times 10^{-8}$, where $A = 1 \times t$ sq. cm:

and the impedance for this segment is:

$Z = \frac{V^2}{2P} = \omega LQ$, $L = 4\pi\mu n^2 \frac{A}{\ell} \times 10^{-9}$ hy.

$$\text{Therefore } Q = \frac{V^2}{2\omega LP} \qquad (2)$$

The dependence of Q on B is a measured function and each subsequent annular segment can be calculated. The sum of the impedance values gives the cavity impedance. A similar set of numbers is derived starting at r_3, again using the power density criterion as the starting point.

This is a cumbersome method and requires a great deal of work for any set of assumptions. The first step in simplification was to derive an empirical expression for Q vs B from measured data. A general hyperbola, $(Q + \ell)(B + m) = n$ was fitted at three points and the expression:

$$Q = \frac{3000}{B + 8} + a$$

was found to hold very well. The subsequent procedure is, for a starting point value of B, to evaluate Q from the required power density and the constant "a" is then determined. The major assumption is that the shape of the curve stays constant with Q level changes (as the ferrite is saturated, the Q increases).

The relation of Q to B is determined as follows for the initial point: P.D. = Power density, watts/cc.

$$Q = \frac{V^2}{2\omega PL} = \frac{\omega^2 B^2 A^2 \times 10^{-16}}{2 \mathrm{x vol. x P.D. x} \omega \mathrm{x} 4\pi\mu\frac{A}{\ell} \mathrm{x} 10^{-9}}$$

For a differential section:

$A = t dr$, $\ell = 2\pi r$, volume $= A \times \ell$.

Reducing and using frequency in megacycles

$$Q = \frac{f B^2}{40\mu\ \mathrm{P.D.}} \qquad (3)$$

III Impedance Calculations

At resonance, $Z = \omega LQ$. The contributions to impedance from the inner and outer rings must be computed individually.

From r_1 to r_2, $Z = \omega LQ = 2\pi f \times 4\pi\mu_1 \frac{A}{\ell}\left(\frac{3000}{B+8} + a\right) \times 10^{-9}$

$A = tdr$, $\ell = 2\pi r$, $B = B_1 \frac{r_1}{r}$

Substituting:

$$dZ = K_1 dr \left(\frac{3000}{B_1 r_1 + 8r} + \frac{a}{r}\right), \text{ where } K_1 = 4\pi\mu_1 ft \times 10^{-9}$$

Integrating:

$$Z = \left[\frac{3000}{8} K_1 \ln \frac{B_1 r_1 + 8r_2}{B_1 r_1 + 8r_1} + K_1 a \ln \frac{r_2}{r_1}\right]$$

$$Z_1 = Z \Big|_{r_1}^{r_2} = 120\pi\mu_1 f_{mc} \ln \frac{r_1 B_1 + 8r_2}{r_1 (B_1 + 8)} + .32\pi\mu_1 f_{mc} a \ln \frac{r_2}{r_1} \qquad (4)$$

Similarly:

$$Z_2 = Z\Big|_{r_3}^{r_4} = 120\pi\mu_2 f_{mc} \ln \frac{c+8r_4}{c+8r_3} + .32\pi\mu_2 f_{mc} a \ln \frac{r_4}{r_3} \quad (5)$$

Where $c = \frac{\mu_2}{\mu_1} B_1 r_1$, f in mc/sec

$$Z\ tot/cav = Z_1 + Z_2 \quad (6)$$

$$Z\ tot/sta = \frac{Z_1 + Z_2}{2} \quad (6a)$$

$$Power/sta = \frac{V^2_{tot}}{2Z_{sta}} = \frac{V^2}{Z_1 + Z_2} \quad (6b)$$

The preceding equations are, as yet, completely arbitrary in the selection of a value for Q. To relate the expressions to a known ferrite, some values of μQf must be determined. For Ferroxcube IVH used in the AGS, the average μQ at 1.4 mc/sec as measured on 1000 rings was 6000 or μQf = 8400. From examination of operating power amplifiers the μQf increases by about 70% as the ferrite is saturated and tuned to 4.45 mc/sec. A conservative increase of 50% to a μQf = 12,600 at 4.45 mc/sec has been selected. A linear dependence of μQf vs f is assumed and thus μQ vs f may be plotted. These represent maximum values and since the maximum Q occurs at the outer radii of the inner and outer rings these points are used for initial computations. At any frequency $\mu = \mu_o \left(\frac{1.4}{f_{mc}}\right)^2$, and the maximum Q is determined.

IV Sample Computation

Let $r_1 = 10$ cm, $r_2 = 17.5$ cm, $r_3 = 20$ cm, $r_4 = 25$ cm

V = 18kV t = 80cm f = 3.5 mc/sec (for 500 MeV injection)

At 1.4 mc, $\mu_{2_o} = 400$, $\mu_{1_o} = 200$ (by air gapping).

At 3.5 mc, $\mu_2 = 64$, $\mu_1 = 32$.

From eq. 1 $V = \omega t x 10^{-8}\ r_1 B_1 \left(\ln\frac{r_2}{r_1} + \frac{\mu_2}{\mu_1} \ln \frac{r_4}{r_3}\right)$

$$\ln \frac{r_2}{r_1} = .56,\ \ln \frac{r_4}{r_3} = .23$$

$$B_1 = 100 \text{ gauss} \quad B_2 = 100 \times \frac{10}{17.5} = 57.3 \text{ gauss}$$

$$B_3 = B_1 \frac{\mu_2}{\mu_1} \frac{r_1}{r_3} = 100 \text{ gauss}$$

$$B_4 = 100 \times \frac{20}{25} = 80 \text{ gauss}$$

From the curve of μQ vs f, at 3.5 mc/sec, μQ=3230

At r_2, $Q = \frac{3230}{32} = 101$. At r_4, $Q = 50.5$

At r_2: $a = Q - \frac{3000}{B+8} = 101 - \frac{3000}{57.3+8} = 55.$

From r_1 to r_2: $Q = \frac{3000}{B+8} + 55.$

Then at r_1: B = 100 gauss,

$$Q = \frac{3000}{108} + 55 = 82.8$$

and at r_1 from eq. 3:

$$P.D. = \frac{fB^2}{40\mu Q} = \frac{3.5 \times 100^2}{40x32x82.8} = .33 \text{ watts/cc.}$$

Similarly at r_4: $a = 50.5 - \frac{3000}{88} = 16.3$

At r_3, $Q = \frac{3000}{108} + 16.3 = 44.1$ and

$$P.D. = \frac{3.5 \times 10^2}{40x64x44.1} = .32 \text{ watts/cc.}$$

For the impedance computation using eq. 4 and 5

$$Z_1 = 120\pi x32x3.5 \ln \frac{10x100+140}{10(108)} + .32\pi x32x3.5x55x.56 = 5610\Omega$$

$$Z_2 = 120\pi x64x3.5 \ln \frac{2x100x10+8x25}{2x100x10+8x20} + .32\pi x64x3.5x16.3x.23 = 2370\Omega$$

Z tot/cav = 7980Ω Z station = 3990Ω

$$P\ station = \frac{(18000)^2}{7980} = 40.6\text{kw dissipation}$$

in cavity.

V Saturation Requirements

In general $H = \frac{.4\pi nI}{\ell}$ oersteds if I is in amperes, ℓ in centimeters. In the following discussion H will be expressed as $\frac{nI}{\ell}$ or ampere turns/cm.

From measured data for μ vs H the equation:

$\mu = \frac{400}{H-0.6}$ holds within 5% for H> 2 ampere-turns/cm.

If an air gap, ℓa, is introduced, the equivalent magnetic path length is ℓ eq. $= \mu_f \ell_a + \ell_f$ where μ_f and ℓ_f are the permeability and path length in the ferrite.

Then: $H(r) = \frac{n\,I}{\mu_f \ell_a + 2\pi r}$

$$\mu_{eq} = \frac{\mu_f \ell_f}{\mu_f \ell_a + \ell_f} \qquad \ell_f = 2\pi r.$$

In a differential section

$$dL = 4\pi\, \mu_{eq}\, n^2 \frac{A}{\ell} \times 10^{-3} \text{ microhenries}$$

$A = t\,dr,\ \ell = 2\pi r,\ n=1$

$$L = \int_{r_1}^{r_2} 4\pi\, \mu_{eq} \frac{t\,dr}{2\pi r} \times 10^{-3}$$

$$L = 2t \times 10^{-3} \int_{r_1}^{r_2} \mu_{eq} \frac{dr}{r} = 2t \times 10^{-3} \int_{r_1}^{r_2} \frac{2\pi r \mu_f}{2\pi r \mu_f + \ell_f} \frac{dr}{r}$$

$$L = 4\pi \mu_f t \times 10^{3} \int_{r_1}^{r_2} \frac{dr}{\mu_f \ell_a + 2\pi r}$$

$$L = 2\mu_f t \times 10^{-3} \ln \frac{\mu_f \ell_a + 2\pi r_2}{\mu_f \ell_a + 2\pi r_1}$$

But if we wish $\mu_{eq} = 200$

$$L = 2\mu_{eq} t\, n^2 \ln \frac{r_2}{r_1} \times 10^{-3} \text{ in general.}$$

Equating:

$$L = 2\mu_{eq} t \ln \frac{r_2}{r_1} \times 10^{-3} = 2\mu_f t \times 10^{-3} \ln \frac{\mu_f l_a + 2\pi r_2}{\mu_f l_a + 2\pi r_1}$$

$$\ln \frac{\mu_f l_a + 2\pi r_2}{\mu_f l_a + 2\pi r_1} = \frac{\mu_{eq}}{\mu_f} \ln \frac{r_2}{r_1} \qquad (8)$$

For the numerical case used previously $r_1 = 10$cm, $r_2 = 17.5$cm

$$\ln \frac{r_2}{r_1} = .56, \quad \mu_f = 400, \quad \mu_{eq} = 200$$

$$\ln \frac{400\ \ell a + 35\pi}{400\ \ell a + 20\pi} = \frac{.56}{2}, \quad \frac{400\ \ell a + 35\pi}{400\ \ell a + 20\pi} = 1.323$$

and $\ell a = .208$ cm.

To saturate the outer unsplit ring with $\mu_o = 400$ to an effective permeability of 64 (to tune to 3.5 mc/sec) we can compute the required saturating current. For a differential section, as before:

$$dL = 4\pi\mu \frac{A}{\ell} \times 10^{-3} \ \mu h$$

From eq. 7, $\mu = \frac{400}{H - 0.6} = \frac{400}{\frac{nI}{2\pi r} - 0.6}$ $\qquad n = 1$

$$\mu = \frac{800\pi r}{I - 1.2\pi r} \qquad (9)$$

$$dL = 4\pi \frac{800\pi r}{I - 1.2\pi r} \frac{t\,dr}{2\pi r} \times 10^{-3} \ \mu h$$

$$= 1.6\pi t \frac{dr}{I - 1.2\pi r} \ \mu h$$

$$L = 1.6\pi t \int_{r_3}^{r_4} \frac{dr}{I - 1.2\pi r} = -\frac{1.6\pi t}{1.2\pi} \ln \frac{I - 1.2\pi r_4}{I - 1.2\pi r_3}$$

$$L = -\frac{4}{3} \ln \frac{I - 1.2\pi r_4}{I - 1.2\pi r_3} \ \mu h \qquad (10)$$

But for μ eff = 64

$$L = 2\mu t \ln \frac{r_4}{r_3} \times 10^{-3} = 128\ t \ln \frac{r_4}{r_3} \times 10^{-3}.$$

Equating the two expressions for L,

$$-\frac{4}{3}\ t \ln \frac{I - 1.2\pi r_4}{I - 1.2\pi r_3} = 128 + \ln \frac{r_4}{r_3} \times 10^{-3},$$

$$\ln \frac{r_4}{r_3} = .23$$

$$\ln \frac{I - 1.2\pi r_4}{I - 1.2\pi r_3} = -.02208$$

$$\frac{I - 1.2\pi r_4}{I - 1.2\pi r_3} = .9781 \qquad r_4 = 25\text{cm}, \quad r_3 = 20\text{cm}$$

$I = 942$ amperes

To reach 4.45 cm/sec a saturating current of about 1500 amperes is required.

To check that the split ring saturates in a like ratio we must determine the effective permeability with a saturating current of 942 amps.

In the split ring:

$$H = \frac{I}{\mu_f \ell a + \ell_f} = \frac{I}{\mu_f \ell a + 2\pi r}$$

From eq. 7 $\quad H = \frac{400 + 0.6\mu_f}{\mu_f}$ and therefore

$$\frac{I}{\mu_f \ell a + 2\pi r} = \frac{400 + 0.6\mu_f}{\mu_f}$$

This yields $0.6\,\mu_f^2\,\ell a - (I - 400\ \ell a - 1.2\pi r)\,\mu_f + 800\pi r = 0.$

Solving for μ_f:

$$\mu_f = \frac{K \pm \sqrt{K^2 - 1920\pi r \ell a}}{1.2\ \ell a} \quad \text{where } K = I - 400\ell a - 1.2\pi r$$

If $K^2 >> 1920\pi r \ell a$, $\sqrt{K^2 - 1920\pi r \ell a} \approx K - \frac{1920\pi r \ell a}{2K}$

In the worst case for $\ell a = .208$cm and $r = 17.5$cm we can find the value of I for $1920\pi \ell a = 2\%$ of K^2. i.e. $K^2 = 50 \times 1920\pi \ell a = 9.6 \times 10^4 \pi \ell a = 6.26 \times 10^4$. This represents a negligible error in the approximation.

$$I = 250 + 400\ell a + 1.2\pi r = 250 + 83.2 + 66.1 \approx 400 \text{ amps}.$$

Thus for I>400 amperes the value of μ_f becomes:

$$\mu_f = \frac{K \pm (K - \frac{1920\pi r \ \ell a}{2K})}{1.2 \ \ell a}.$$

Using the negative sign this simplifies to $\mu_f =$

$$\frac{800 \ \pi r}{I - 400\ell a - 1.2\pi r}.$$

The equivalent permeability with an air gap is:

$$\mu \text{ equiv} = \frac{\ell_f}{\frac{\ell_f}{\mu_f} + \ell a}$$

$$\mu \text{ equiv} = \frac{2\pi r}{\frac{2\pi r}{800 \ \pi r / I - 400 \ \ell a - 1.2\pi r} + \ell a}$$

$$\mu \text{ equiv} = \frac{800 \ \pi r}{I - 1.2\pi r.}$$

This is identical to equation 9 for the unsplit ring and the equivalent permeability for a split ring with a small air gap and highly saturated (in this case I>400 amperes) is independent of the air gap length.

We may now find the effective permeability of the entire inner ring by applying equation 10.

$$L = -\frac{4}{3} \ t \ \ell n \frac{I - 1.2\pi r_2}{I - 1.2\pi r_1} \ \mu h$$

$$\text{also } L = 2 \ \mu_{eff} \cdot t \ \ell n \frac{r_2}{r_1} \times 10^{-3} \quad \mu h$$

$$r_2 = 17.5\text{cm}, \ r_1 = 10\text{cm}, \ \ell n \frac{r_2}{r_1} = .56.$$

Solving μ eff = 37.7. If the inner and outer rings saturated and maintained the same ratio of permeability, μ eff should be 32. This indicates that the inner air gapped ring will not saturate as rapidly as the outer ring. The difference is not critical because at the inner radius r = 10cm, where the power density is greatest, the permeability would actually be, for 942 amperes, $\mu = \frac{400}{H-0.6} = 27.8$.

As was mentioned earlier the ideal condition for μ vs r would have the μ increase with r to crowd flux to the outside. As a dc bias is applied just this condition prevails. At r = 10cm, $\mu = 27.8$ in the above case but at r=17.5 cm, $\mu = 50.3$. In the outer ring at r = 20cm, $\mu = 57.5$; at r = 25cm, $\mu = 74$.

VI Injection at 50 MeV

If we inject protons at 50 MeV and if we use the present rate of rise of magnetic field, the frequency range to be covered extends from 1.40mc/sec to 4.45 mc/sec. A set of computations similar to those done above but with a cavity voltage of 12 kV instead of 18 kV yields impedance and power density values consistent with our design criteria. The maximum power density which occurs at the r=10 cm point is 0.27 w/cc and the minimum station impedance is 2700 ohms as compared to about 1800ohms in the existing cavities.

VII Cavity Electrical Length

It is important that the electrical length of the cavity stay well under one quarter wavelength in order to ensure a relatively uniform flux distribution in an axial direction. The unsaturated inductance in the existing cavity is

$$L = 2\mu t \ \ln \frac{r_2}{r_1} \times 10^{-3} = 37.6\mu h$$

For two cavities in parallel L=18.8μh. The electrical length of this cavity is about twenty degrees. In the proposed cavity

$$L = (2\mu_1 t \ \ln \frac{r_2}{r_1} + 2\mu_2 t \ \ln \frac{r_4}{r_3}) \times 10^{-3} = 32.6\mu h$$

The electrical length should be slightly smaller in the new cavity.

VIII Auxiliary Equipment

The proposed cavity appears to be perfectly feasible and could be constructed in a similar way to our present cavities. Rings 50cm O.D. have been fabricated and supplied to the Princeton-Pennsylvania Accelerator and although these had an ID of 30cm, the change in dimension should not be too great a problem.

The saturating supply concept would change. One design would use a large high current supply feeding all the cavity bias circuits in series. A separate transistor bank by-pass circuit at each station would tune the cavity.

The power amplifier would have to be capable of supplying the cavity losses plus the beam loading. A power output rating at each station of 100 kW would be adequate.

IX Conclusion

Further work is necessary to determine whether the selected permeability ratio and dimensions give the maximum performance. However, the numbers selected do yield a workable system and could be used to construct a prototype.