ACCELERATING CAVITIES FOR AN 800 MEV SOC*

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Summary

The minimum required energy gain per turn for the 800-MeV Separated Orbit Cyclotron increases by a factor of about five from the injection radius to maximum orbit radius. Ordinary rectangular cavities operating in the TM_{110} mode can be used to provide the accelerating voltage; however, the cavity length is then about twice the distance between the inner and outer orbits (~ 18 ft). This length, and the rf power loss, can be reduced by shaping the cavity to excite in addition the TM_{210} and TM_{310} modes. Inclusion of these higher order modes shifts the maximum voltage from the midpoint of the cavity out toward the end, resulting in a shorter cavity and lower losses. The performance of a 1/4-scale model of the "shaped rectangular cavity" was found to agree quite well with theory. "Wedge-shaped" cavities were also investigated. In this cavity, which is a sector of a cylinder, the length of the accelerating gap increases with machine radius. The upper and lower boundaries of either type of cavity can be shaped to excite higher order modes.

Introduction

One of the distinguishing features of a separated orbit cyclotron (SOC) is, as the name implies, the relatively large distance between adjacent orbits of the particles. The orbits may be separated by placing the particles in a three-dimensional spiral path[1]; however, with sufficient acceleration per turn the orbits may remain in a plane. For an 800-MeV machine the latter approach appears to be more economical. A conceptual model of such a machine is shown in Fig. 1.

For a plane SOC the basic specifications for the accelerating system can be determined from the machine diameter, the injected and maximum kinetic energies of the particles (T_i and T_o), and the minimum allowable spacing (Δr) between adjacent orbits. The operating frequency of the cavities may be determined from $\omega_{rf} = n\,\omega_p$, where n is an integer and ω_p is the angular velocity of the particles. Since $\omega_p = v_p/r = \beta c/r$, then $\omega_{rf} = n\beta c/r$ or $\lambda = 2\pi r/(n\beta)$. In practice the operating wavelength λ and n are selected to produce a maximum orbit radius r_o nearly equal to the desired value at maximum energy. In other words, $r_o = n\beta_o\lambda/(2\pi)$. The injection radius is then $r_i = n\beta_i\lambda/(2\pi)$. The required energy gain per turn may be determined from $\Delta T \approx (\Delta r)(T)[2 + (T/E_o)(3 + T/E_o)]/r$, if $\Delta r << r$. Then the minimum cavity gap voltage is $V_{min} = \Delta T/(mF\cos\varphi_s)$, where m = number of accelerating gaps per turn, F = transit time factor and φ_s = phase-stable angle. A curve of V_{min} as a function of radius is shown in Fig. 2 for a typical case. Here the minimum voltage increases by a factor of almost five between injection and maximum energy, and the radial distance ($r_o - r_i$) over which a specified voltage must be maintained is 210 in. or 0.875 λ. The actual voltage developed by the cavities as a function of radius may have any shape, provided it is always greater than the minimum value.

Several types of cavities appear to be useful in SOC's. These may be divided into two general groups--coaxial cavities and TM cavities in which the rf magnetic field is transverse to the direction of particle motion. Coaxial cavities operating strictly in the TEM mode would, of course, provide an accelerating voltage constant with radius. The distance between gap centers must be at least $\beta\lambda/2$, however. At the higher energies this distance, combined with the large radial extent of the cavity, makes the coaxial system unattractive. Since this paper is concerned primarily with an accelerating system for a machine having a maximum energy of 800 MeV and an injected energy of 200 to 350 MeV, the coaxial system will not be considered further.

Rectangular Cavities

The simplest cavity which could be used in an 800-MeV SOC appears to be a rectangular cavity operating in the TM_{110} mode. The electric field in such a cavity is given by $E_z = E_m \sin(\pi x/\ell)(\cos by)$ where

$b = (2\pi/\lambda)[1 - (\lambda/2\ell)^2]^{\frac{1}{2}}$, and ℓ = cavity length in the radial direction. The gap voltage (where the gap coincides with y = 0) is then $V_g = V_m \sin(\pi x/\ell)$, with $V_m = -gE_m$. To provide the required Δr at r_i and r_o

$$V_m \sin(\pi a/\ell) = V_i,$$

$$V_m \sin[\pi(a + r_o - r_i)/\ell] = V_o,$$

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or

$$V_m^2 = V_i^2 + \left[\frac{V_o - V_i \cos \pi(r_o - r_i)/\ell}{\sin \pi(r_o - r_i)/\ell}\right]^2,$$

where V_i and V_o are the required gap voltages at r_i and r_o. Since cavity power loss is a function of both V_m and ℓ, there is an optimum length for the cavity. In general the optimum length is such that r_o coincides with a point on the gap somewhat greater than $\ell/2$, in other words $V_m > V_o$.

If other TM_{n10} modes are excited in a cavity it is possible to produce a non-sinusoidal gap voltage as a function of distance along the gap. Consider the following field expansion.

$$E_z(x, y) = \sum_{n=1}^{p} C_n \sin(n\pi x/\ell) \cos b_n y,$$

where $b_n = (2\pi/\lambda)\sqrt{1 - (n\lambda/2\ell)^2}$. Given an unlimited number of terms in this expansion, the gap voltage can be shaped to any desired function of x. In a cavity of finite length this result cannot be achieved since the y boundary of the cavity y_o will exist only if the C_n's fall within a limited range. It is obvious from the definition of b_n that $\cos b_n y$ will become $\cosh b_n y$ for terms with $n > 2\ell/\lambda$. If y_o is to be finite under this condition then C_n must be very small compared with C_1; therefore, such terms can have very little effect on the shape of the gap voltage.

For an 800-MeV SOC the distance between the inner and orbits will be about 0.7 to 1.0 λ if λ = 240 in., and the cavity length may be assumed to be about 1.5 λ. Under this assumption about three terms could be included in the field expansions. Where the cavity dimensions are defined in terms of wavelength, $X = x/\ell$, $Y = 2y/\lambda$, $L = 2\ell/\lambda$, and $B_n = b_n\lambda/2$ the cavity fields can be written as

$$E_z = \sum_{n=1}^{p} C_n \sin n\pi X \cos B_n Y,$$

$$H_x = -(j/\pi\eta) \sum_{n=1}^{p} C_n B_n \sin n\pi X \sin B_n Y,$$

$$H_y = -(j/L\eta) \sum_{n=1}^{p} nC_n \cos n\pi X \cos B_n Y.$$

In designing cavities to produce these fields, the coefficients C_n and the length ℓ are selected to give a gap voltage which will meet the minimum energy gain requirements. The boundary $Y_o(X)$ and the power loss may then be computed. A plot of $Y_o(X)$ and $E_g(X)$ for a typical case (for a three-term expansion) is shown in Fig. 3. A computer program was written to perform the rather lengthy computation of pertinent parameters.

The power loss in the walls (z = constant) of either the simple or the "shaped" rectangular cavity is a function of the electric field in the cavity but the power loss in the perimeter (top, bottom, and ends) is a function of electric field and gap length. For a fixed azimutal cavity space in an SOC it can be shown that there is an optimum number of cavities for minimum total power loss in the machine. The rf power loss in all cavities may be expressed as

$$P_T = \left(\frac{K_o \theta}{m G \sin(\theta/m)}\right)^2 (mp_w + Gp_p)$$

where $K_o = (\Delta T)_o/\cos\varphi_s$, m = total number of cavities, G = total azimutal distance available for cavities, $\theta/m = \pi G/m\beta_o\lambda$ = transit time angle at r_o, p_w = power loss in cavity walls for unit electric field at r_o, and p_p = power loss per unit gap length in perimeter of cavity for unit electric field at r_o. This equation is plotted in Fig. 4, along with the power loss per cavity P_L and the transit time factors F_i and F_o for K_o = 30 MeV, T_i = 200 MeV, T_o = 800 MeV, G = 1200 in., $p_w = 1.39 \times 10^{-4}$ and $p_p = 2.04 \times 10^{-6}$. The values of p_w and p_p are those calculated for the cavity of Fig. 3.

Tapered-Gap Cavities

Since the magnets in an SOC are approximately sectorial in shape, the space available for cavities increases with machine radius. The gap length of rectangular cavities is fixed by the space available at r_i. If the gap length is increased with machine radius the cavity volume can be increased, with a possible reduction in power loss. The electric field in tapered gap cavities can be expressed as

$$E_\varphi(\rho, y) = \sum_{n=1}^{p} C_n Z_1(a_n\rho) \cos b_n y,$$

where

$$Z_1(a_n\rho) = J_1(a_n\rho) - \frac{J_1(a_n\rho_2)}{N_1(a_n\rho_2)} N_1(a_n\rho),$$

$$b_n = (2\pi/\lambda)\sqrt{1 - (a_n\lambda/2\pi)^2}.$$

The y has been used in place of the conventional z, and ρ is the radial coordinate whose origin may or may not coincide with the machine center. If the ends of the cavity are located at ρ_1 and ρ_2 then the "a_n's" may be determined from

$$J_1(U_n) - \frac{J_1(kU_n)}{N_1(kU_n)} N_1(U_n) = 0,$$

where $U_n = a_n\rho_1$, and $k = \rho_2/\rho_1$. Roots of this equation are easily obtained by a computer routine since $U_n(k-1) \approx n\pi$. As was the case with rectangular cavities, the number of terms which can be practically included in the field expansion is about $2(\rho_2 - \rho_1)/\lambda$. The field equations are simplified somewhat if the following substitutions are made: $R = 2\rho/\lambda$, $A_n = a_n\lambda/2$, $Y = 2y/\lambda$, and $B_n = b_n\lambda/2$. Then

$$E_\varphi = \sum_{n=1}^{p} C_n Z_1(A_n R)\cos B_n Y,$$

$$H_R = (j/\pi\eta) \sum_{n=1}^{p} C_n B_n Z_1(A_n R)\sin B_n Y,$$

$$H_Y = (j/\pi\eta) \sum_{n=1}^{p} C_n A_n Z_o(A_n R)\cos B_n Y,$$

$$V_g = -(R\varphi_o\lambda/2) \sum_{n=1}^{p} C_n Z_1(A_n R),$$

$$\text{where } Z_o(A_n R) = J_o(A_n R) - \frac{J_1(A_n R_2)}{N_1(A_n R_2)} N_o(A_n R),$$

and φ_o = angle between cavity walls. Again, the coefficients C_n and the cavity length $\rho_2 - \rho_1$ can be chosen to produce an acceptable gap voltage. $Y_o(R)$ and power loss may then be calculated with a computer routine. Curves of $Y_o(R)$, $E_g(R)$, and the normalized gap voltage, $V_g(R)$, are shown in Fig. 5 for a typical case.

The total rf power loss in an SOC using tapered gap cavities may be expressed as

$$P_T = \left[\frac{K_o\theta_o}{m\psi\rho_o \sin(\theta_o/m)}\right]^2 (mp_w + \psi p_\varphi),$$

where $\psi = m\varphi_o$ = total angle available for cavities, $\theta_o/m = (\pi\rho_o\psi)/(m\beta_o\lambda)$ = transit time angle, and p_φ = power loss per radian in cavity perimeter for unit electric field at ρ_o. The optimum number of cavities m_b may be determined from the equation

$$\tan(\theta_o/m_b) = (2\theta_o/m_b)\frac{1 + \psi p_\varphi/m_b p_w}{1 + 2\psi p_\varphi/m_b p_w} .$$

Comparison of Cavity Types

To compare the three types of cavities which have been considered, the following SOC parameters are assumed: λ = 240 in., T_i = 200 MeV, T_o = 800 MeV, $n = \omega_{rf}/\omega_p = 20$, Δr = 4.5 in., G = 990 in., and ψ = 2.287 radians. Then $\beta_i = 0.5662$, $\beta_o = 0.8418$, $r_i = (n\beta_i\lambda)/(2\pi)$= 433 in., r_o = 643 in., $(\Delta T)_i \approx 5.58$ MeV, $(\Delta T)_o \approx 29.6$ MeV, and $V_iF_i/V_oF_o \geq (\Delta T)_i/(\Delta T)_o \approx 0.1886$. If $F_i/F_o \approx 0.9$ for rectangular cavities, then $V_i/V_o \geq 0.21$. Where $F_i/F_o = 1$ for the tapered gap cavity, $V_i/V_o \geq 0.1886$. If r_o coincides with X = 0.68 in the shaped cavity of Fig. 3 then r_i would correspond to X = 0.68 - $2(r_o - r_i)/(3.1\lambda)$, or X = 0.116 and $V_i/V_o = 0.258$. The cavity then provides the required Δr when the maximum voltage is properly adjusted. By the same reasoning it can be shown that the tapered gap cavity of Fig. 5 also provides the required Δr.

Table I provides a comparison of the three cavity types for the assumed machine. The length given for the simple rectangular cavity is nearly optimum. For each case the number of cavities is optimized.

TABLE I - Comparison of Cavities

Type*	Number	Radial Length (in.)	Max. Height (in.)	Total RF Power Loss (MW)
R	18	408	125.5	9
S	18	372	151	6.93
T	24	372	149.5	5.06

*R-simple rectangular, S-shaped rectangular, T-tapered gap.

Experimental

A one-quarter scale model of the shaped rectangular cavity, shown in Fig. 6, was used to check theory, fabrication tolerances, and tuning methods. Computed and measured characteristics of the model are given in Table II.

TABLE II - Model Cavity Parameters

	Resonant Frequency (Mc/s)	Q	Effective Shunt Resistance* (MΩ)
Calc.	196.83	22,000	0.922
Meas.	197.49	21,000	0.81

$*R_e = (V_{g\,max})^2/(2P_L)$

The effective shunt resistance R_e and the relative gap field were measured by perturbation techniques. The measured values are for the cavity as received from the fabricator. Fig. 7 provides a comparison between measured values and theoretical values of the relative electric field along the accelerating gap of this cavity. The first "higher-order mode" observed in the model occurred at a frequency of 220 Mc/s.

Conclusion

Shaping of SOC cavities to produce a non-sinusoidal variation of voltage with machine radius can significantly reduce rf power requirements for machines spanning a wide energy range. The advantage of shaped cavities disappears, however, as the energy range is decreased. For example, preliminary calculations indicate that cavity shaping would be uneconomical in a 350 to 800 MeV machine.

Reference

1. F. M. Russell, Nucl. Instr. and Meth. 23, 229 (1963).

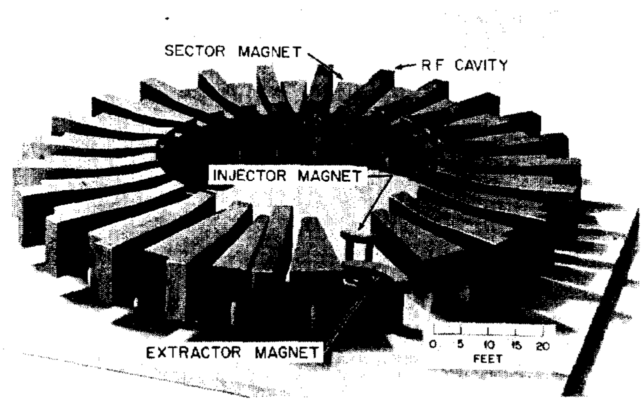

Fig. 1 - Conceptual Model of an SOC

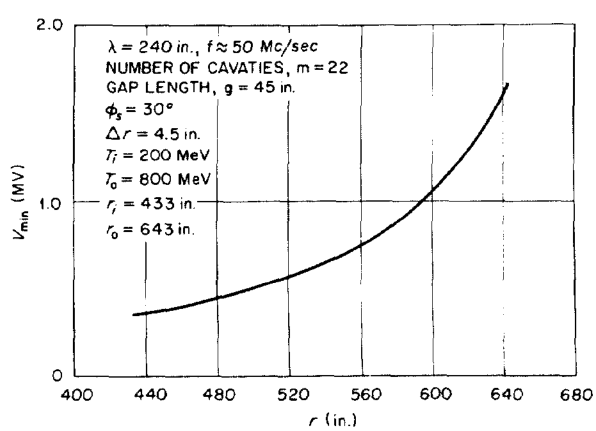

Fig. 2 - Minimum Accelerating Voltage as a Function of Radius

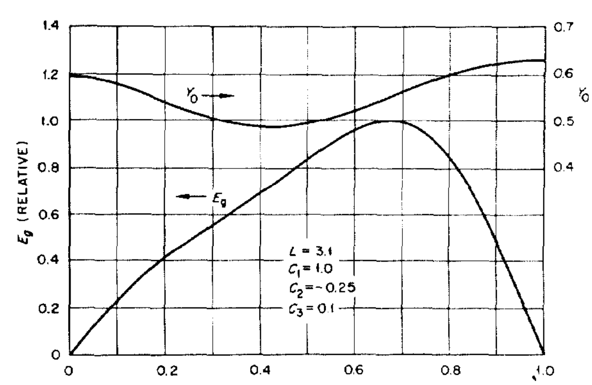

Fig. 3 - Height and Gap Field for a Shaped Rectangular Cavity

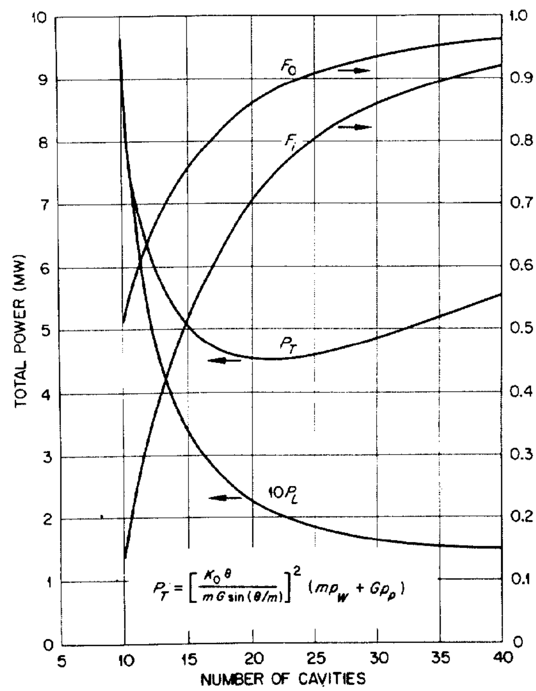

Fig. 4 - RF Power Loss vs Number of Cavities for a Typical SOC

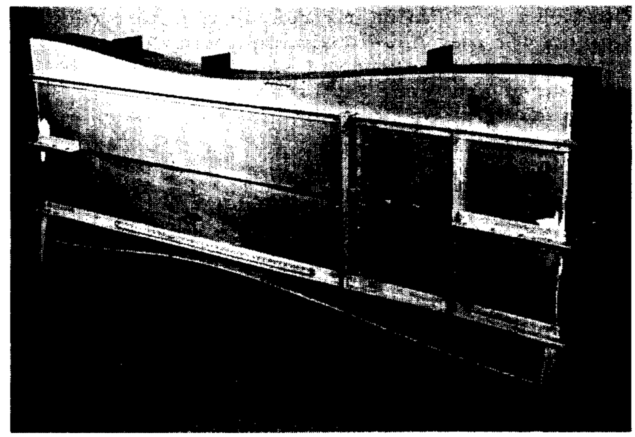

Fig. 6 - Scale Model Cavity

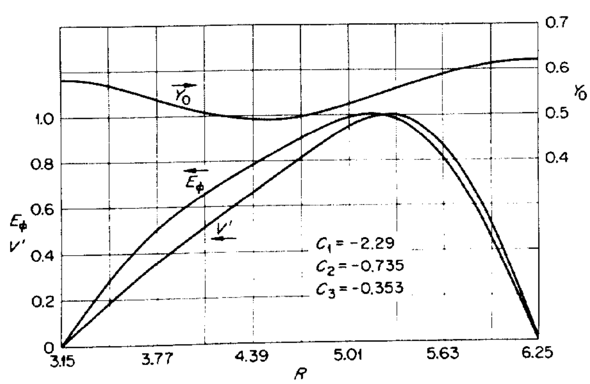

Fig. 5 - Height, Gap Field, and Voltage for a Tapered-Gap Cavity

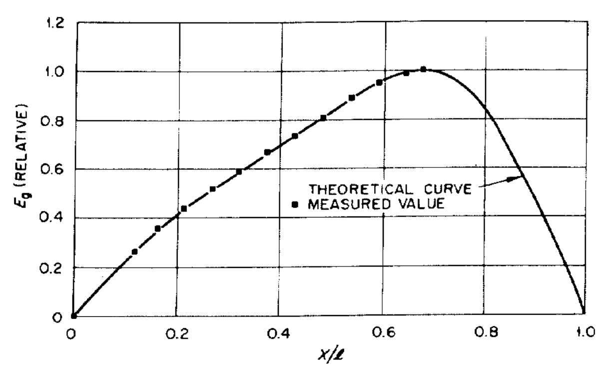

Fig. 7 - Measured Field in Gap of Model Cavity