STABILIZATION OF ACCELERATING VOLTAGE UNDER HIGH-INTENSITY BEAM LOADING

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Summary

The problem of fluctuating beam load in high-intensity cascade synchrotrons is discussed. A comparison is made of various methods of controlling rf system amplitude and phase fluctuations, with emphasis on feedback control as an economical method of stabilization. Equations describing performance are given, together with typical values for the 200 BeV machine at 3 x 10^{13} protons/pulse.

Description of Problem

Consideration of the factors affecting rf system performance of the 200 BeV synchrotron presently being studied at UCLRL discloses two central points. First, the beam intensity is high enough to become a dominating factor in design; present plans call for a circulating beam of 0.33 amperes, with a future possibility of one ampere. Second, the use of an 8 BeV rapid-cycling synchrotron as an injector to the 200 BeV ring brings up the problem of rapid, 100% fluctuations in beam loading with a basic recurrence rate of 68 kc, the circulation frequency in the main ring.

It appears that the cascade synchrotron principle will continue to be used, and that design beam intensity in future accelerators may well climb higher. For these reasons, we discuss the problem of stabilizing rf cavity voltage amplitude and phase under high-intensity, fluctuating beam loads, using as an example the quantities associated with the 200 BeV machine, discussed elsewhere in this symposium.[1]

Figure 1 helps to illustrate how these points relate to system design. The circular portion of the figure on the right represents the ring; the blocks at the left, the rf hardware. In our case, bunch-to-bucket injection from a rapid-cycling synchrotron one-seventh the diameter of the main ring implies that there will not at first be a full turn of beam circulating in the main ring. Instead, there will be one-seventh of a turn, 108 bunches (the harmonic number of the small machine). Every one-eighteenth second, 108 bunches will be added, until seven sets have been injected.

The fast injection requirements, however, make it unlikely that each of the seven sets will contain a full 108 bunches; there will, in all probability, be gaps. Also, we may expect set-to-set intensity variations. The net result is that the rf cavities of Figure 1 will see a beam load of 1.2 megawatts when beam is crossing a cavity, and zero load when beam is absent. Briefly, we may say that the azimuthal charge is highly non-uniform in distribution, and impressive in magnitude.

Alternative Solutions

One common approach to the problem of fluctuating beam load is to endure it, as follows: One sets the amplifier output to achieve the average cavity voltage and phase desired, and allows the instantaneous voltage and phase fluctuations to be what they will. Where the beam intensity is small, the approach is adequate. Energy storage in the cavity helps to limit the magnitude of the disturbance. Where the load is quite large, however, a more deliberate reduction of disturbance is indicated.

The control of disturbances due to fluctuating loads (the missing bunch effect) can take various forms. Damping or swamping resistive loads may be employed, but these waste considerable power. Load isolators will prevent the rf generator from being affected by changing loads, but will not stabilize voltage at the load, and in addition, are costly at our frequency and power levels (50 Mc/sec and 1.2 MW cw). The most straightforward and economical solution is stabilization of a high standing-wave ratio cavity by fast negative feedback. A basic argument in favor of this approach is that much of the circuitry has to be present for other reasons anyway; the addition of negative feedback does not represent a major cost.

Basic Relationship of Beam Power, Stored Energy, and Response Time

The cascade-synchrotron injection method introduces the beam as a pulsed load on the main accelerating structure. The azimuthal discontinuities in the segmented bunch train excite a cavity voltage disturbance, which is periodic at the circulation frequency, and proportional to the beam intensity.

If the accelerating voltage is not stabilized against this disturbance, there is a threshold for intensity above which there will be loss of beam.

In the high-energy ring the circulating charge is constant; the current, as seen by a stationary monitor, is $q\beta c$, changing only with β. The change in β is small if injection energy is high compared to the particle rest energy, e.g., in the Berkeley design, the injection kinetic energy is 8 BeV, and the fractional change in β (and beam current) is only 0.55%. In this situation, it is valid to regard the beam as a current load on the rf system. The load transitions associated with an injected segment occur within the rf period. The modulation of the load envelope approaches an ideal step, or pulse.

Under this type of transient loading, the rf cavities provide an energy storage bank, or buffer, to control the magnitude of the disturbance during the time it takes (regulator response-time) for the power amplifiers to adjust to the beam kVA demand following a load transition. (In accelerators, the beam load is highly reactive--purely so in the beam storage mode.) For a system with a fixed response time, δt, a minimum stored energy, U, can be specified once the incremental (step) demand P, and the tolerable limits on voltage (and phase) disturbances have been ascertained. U is proportional to V^2, and from the conservation of energy,

$$\frac{\delta V}{V} = \frac{P}{2U}\ \delta t \qquad (1)$$

Taking P to be the kVA demand, and δV to include the effects of amplitude and phase shift disturbance, equation (1) is true for each frequency component in the beam-cavity interaction.

Economic Limits on Maximum Energy Storage in the RF Structure

Examination of equation (1) shows that the cavity voltage disturbance $\frac{\delta V}{V}$ (which is to be kept small) could in principle be minimized by selecting an arbitrarily large value for U_{rf}, the stored energy. There is no practical difficulty in designing low-impedance cavities which will store considerable energy. The essential difficulty with a large value for U_{rf} is that it increases the cost of the rf system. The argument is as follows: First, some of the stored energy is continuously lost by joule heating in the walls of the rf structure; this power loss is

$$P_{copper} = \frac{\omega U_{rf}}{Q_{copper}}$$

As U_{rf} goes up, P_{copper} also goes up, and Q_{copper} at best stays constant, and may even go down. The first result: Cavities of large stored energy will have relatively large power loss in the walls. Second, the necessity of tuning the cavity (refer to Figure 1 for synchronous acceleration conditions) means that the tuner power will increase as the cavity stored energy increases; the tuner must control a fraction $2\frac{\Delta f}{f}$ of the cavity stored energy.

Thus, the tuner stored energy

$$U_{tuner} \approx 2\,\frac{\Delta f}{f}\,U_{rf} \quad .$$

From this fact, and the fact that the tuner has some loss,

$$P_{tuner} = \frac{\omega U_{tuner}}{Q_{tuner}} = 2\ \frac{\Delta f}{f}\ \frac{U_{rf}}{Q_{tuner}} ,$$

we have the second result--the tuner stored energy and the tuner power loss go up as the cavity stored energy goes up. Third, the cavity phase control necessary for beam steering (not discussed in the present paper) requires appreciable rf power for the duration of the rephasing interval, which may be tens of microseconds. As a specific case, rephasing the cavities $\frac{\pi}{2}$ radians in 60 microseconds (4 turns of the beam) takes rf power of the following order:

$$P_{steering} \approx \frac{U_{rf}}{60 \times 10^{-6}\ \text{sec.}} \quad .$$

The steering power should be compared to the cavity copper loss; comparing the earlier equations:

$$\frac{P_{steering}}{P_{copper}} = \frac{Q_{copper}}{\omega \times 60\ \text{usec.}}$$

For the 200 GeV machine, $Q_{copper} \approx 15{,}000$, $\omega = 2\pi \times 52$ Mc, and $P_{steering}/P_{copper} \approx 1$, independent of the value chosen for cavity stored energy. The third result is that the relative difficulty of steering is constant, and steering can be done with the available power.

The first result is made more interesting by comparing specific cases. Using feedback control, we may put U_{rf} = 18 joules, Q = 15,000, whence P_{copper} = 0.4 MW, about one-third the power required for accelerating the beam.

If we were to rely entirely on cavity energy storage (no feedback control), it would be necessary to increase U_{rf} about 20 times to secure the same stabilization, whereupon the P_{copper} = 8 MW, about seven times the beam power. The tuner rf power loss would increase the total further. Power supply and rf tube power cost would be high, since the copper loss is a continuous power drain.

To summarize, one may say that some form of feedback control to achieve adequate stabilization is less costly than simply increasing the stored energy.

Allowable Limits on Cavity Voltage Disturbance

The beam-induced disturbance envelope is periodic at the circulation frequency of the beam. This is to say that each bunch (referring to bunch center-of-charge throughout this section) is offered a distinct accelerating voltage, different from that given to other bunches in the segment. Nevertheless, the average energy gain of all bunches will be equalized to the synchronous value by independent coherent synchrotron oscillations of each bunch. We can determine the relationship between accelerating voltage error and synchrotron oscillation amplitude, assign limits on the latter, and thus derive the necessary voltage stability $\frac{\delta V}{V}$ under beam loading.

In the following, V is the total rf cavity voltage (per turn), and the angles, ϕ_s, ϕ, refer to the rf phase with respect to voltage zero at which the bunch centers-of-charge cross the accelerating gap. (Subscript s denotes synchronous quantities).

The energy-gain per turn for the ideal synchronous particle is,

$$eV_{bs} = eV \sin \phi_s \; (= 2\pi R\rho e\dot{B}) \qquad (2)$$

Due to beam loading disturbances, among other errors, the accelerating voltage for a particular bunch is given at any instant by,

$$V_b = V_{bs} + \Delta V_b = V \sin \phi = V \sin (\phi_s + \Delta\phi) \qquad (3)$$

On the average, $\Delta V_b = 0$ by phase oscillation of the bunch center-of-charge:

$$\Delta\phi = \Delta\phi_m \cos \Omega t.$$

This linearized form assumes that $\Delta\phi_m$ is small (e.g., 0.1 radian). The angular frequency of these small phase oscillations is given by:

$$\Omega = \frac{c}{R} \left[\frac{(1 - \beta^2 - \hat{\alpha})\, heV \cos \phi_s}{2\pi\beta E} \right]^{\frac{1}{2}} \qquad (4)$$

By expanding (3) at $\Delta\phi = \Delta\phi_m$, and expressing the result as a fraction of (2), we obtain the phase oscillation amplitude associated with a fractional deviation in accelerating voltage from the synchronous value.

$$\Delta\phi_m = \frac{\Delta V_b}{V_{bs}} \tan \phi_s \qquad (5)$$

The amplitude of the corresponding radial (energy) oscillations of the bunch about the synchronous orbit in the vacuum chamber is given by,

$$\Delta R_{max} = \frac{\hat{\alpha}\, ceV \cos \phi_s}{2\pi\beta E\Omega} \Delta\phi_m \qquad (6)$$

From (5) and (6), we find the allowable cavity voltage disturbance,

$$\frac{\Delta V_b}{V_{bs}} = \frac{2\pi\beta E\Omega}{\hat{\alpha} ceV \sin \phi_s} \Delta R_{max} \qquad (7)$$

We can specify that ΔR_{max}, due to beam loading effects, be kept a small fraction of the chamber width to aid in accurate beam steering and targeting.

In the 200 BeV design, for example, we substitute the following values:

ΔR_{max} = 1 mm

V = 7 x 10^6 volts/turn

ϕ_s = 30°

$\Omega/2\pi$ = 1.6 kc at start of acceleration

E = 8.94 BeV total energy

β = .9946...

$\hat{\alpha}$ = .004, machine average momentum compaction factor, $\frac{\Delta R}{R} / \frac{\Delta P}{P}$

and find,

$$\frac{\Delta V_b}{V_{bs}} = 0.13,$$

i.e., transients in the accelerating voltage due to beam loading must be 13% or less to limit the consequent radial oscillation amplitude to 1 mm.

As a matter of interest, we mention that equations (4) through (7) may be deduced from Figure 1.

Discussion of δt, the Response Time, and τ_d, the Regulator Delay Time

Regulation is improved as the response time δt is reduced, but there is a lower limit on achievable response time. As will be seen, the response time, δt, is 3/2 the delay time τ_d, in a correctly designed negative feedback regulator loop.

At this point, an important indirect consequence of high-beam intensity affects the design. Shielding the radiation that attends high-energy acceleration at high beam intensity requires that several meters of concrete or earth be interposed between the cavities in the beam tunnel and areas occupiable by personnel. In the interest of maintainability and reliability, the bulk of the electronics equipment should be located in occupiable areas. There is therefore a round-trip signal transmission delay through the shielding. For the system of Figure 2, the total delay τ_d can be estimated. A tabulation of the contributions of various parts of the system in the Berkeley 200 BeV design appears in Table I.

TABLE I

CAVITY STABILIZATION LOOP DELAY τ_d

Transmission delay	75 nsec
RF Driver delay	75 nsec
Amplitude and phase modulator	50 nsec
RF Comparator (detection, amplification)	50 nsec
Total τ_d	250 nsec

The delay contributions listed in Table I include propagation delays and bandwidth time constants associated with energy storage at various localities, e.g., the driver delay listed in Table I represents the group delay, plus rise-time associated with a 5 Mc bandwidth coupling network between driver and final amplifier.

Where a number of such small time constants are involved, and the magnitude of each is small compared with the total delay, it is valid to simply add the time constants to the propagation delays to obtain the total delay.[2] So far as the servo analysis is concerned, this is a worst-case since it ignores the signal attenuation that accompanies phase shift in the lumped energy storage elements, or poles. We adopt this worst-case for the sake of generality.

The servo loop cannot make any essential distinction between pure delay and delay mixed with the phase lag (discrete pole) effects. However, these effects are basically different in their response to predictive or programmed control. Pure delay can be compensated to the limits of prediction accuracy while bandwidth time constants can only be compensated to the limits of available forcing power.

The Cavity Voltage Regulator

We have seen that the loading disturbance is periodic at the circulation frequency, and the amplitude of the consequent phase oscillation depends on the fractional deviation of the accelerating voltage from the synchronous value. The rf system stabilization is that which keeps the synchrotron oscillation amplitude within prescribed limits.

The loading transient, as will be shown, is short compared to the duration of an injected segment. This is desirable in order that the disturbance may settle or damp out between consecutive on and off transitions associated with an injected beam segment. Thus, the first bunches of a segment (which encounter the unloaded voltage) will experience the maximum departure from synchronous voltage, while following bunches (arriving after the cavity voltage has settled to its loaded value) will meet nearly synchronous conditions, i.e., the cavity voltage averaged over the duration of the segment satisfies equation (2).

The cavity voltage regulator is shown in Figure 2. It is composed of the cavity (serving as output energy storage bank), rf power amplifier chain, and comparator, which measures the rf cavity voltage waveform with respect to input amplitude and phase references. The negative feedback error output of the comparator modulates the rf driving signal. One may describe the cavity voltage regulator as a negative feedback loop having a single dominant time-constant and distributed delay.

It is easy to see that the steady-state gain limit, $K_{o\,max}$, for stability in such a loop is:[2,3]

$$K_{o\,max} = \frac{\pi}{2}\,\frac{\tau_c}{\tau_d} \qquad (8)$$

where τ_c is the cavity time constant and τ_d the distributed delay in the loop. We may express (8) in terms of maximum permissible gain bandwidth, K_{max}, of the amplifier-cavity combination,

$$K_{max} = \frac{\pi}{2\tau_d} \qquad (9)$$

where K has the dimensions of time^{-1}.

For stability under transient disturbances, the gain must be reduced to about half.[2] An acceptable figure for maximum gain and maximum gain-bandwidth is:

$$K_o = \frac{2}{3}\,\frac{\tau_c}{\tau_d}\ ;\ K = \frac{2}{3\tau_d}\ . \qquad (10)$$

This value gives a slightly underdamped response with less than 20% overshoot. Damping of the oscillatory component proceeds as

$$\exp\left(-\frac{\pi t}{4\tau_d}\right)\ .$$

Since a beam segment is about 2 usec (8 τ_d) long, there is a factor of 500 attenuation between consecutive transients.

Considering only amplitude effects for a moment, and assuming that the amplifiers are linear over the load current range, we obtain the steady state beam load disturbance that will be allowed by the regulator for $K_o \gg 1$.

$$\delta V = \frac{2\,I_B\,R_{SH}}{1 + K_o} \approx \frac{3}{2}\,\frac{(2\,I_B\,R_{SH})}{\tau_c}\,\tau_d$$

$$= \frac{3}{2}\,\frac{P_b}{2U}\,V\,\tau_d$$

The fractional amplitude disturbance is

$$\frac{\delta V}{V} = \frac{3}{2}\ \frac{P_b}{2U}\ \tau_d, \qquad (11)$$

where I_B is the average beam current ($2\ I_B$ is the peak fundamental component), R_{SH} is the cavity shunt resistance, U the cavity stored energy, and P_b is the power absorbed by the beam. The nominal value of P_b is,

$$P_b = Ne\ \frac{\beta c}{2\pi R}\ V \sin\phi_s$$

N = number of protons per pulse.

The reactive component of the beam load is

$$P_x = Ne\ \frac{\beta c}{2\pi R}\ V \cos\phi_s\ \frac{P_b}{\tan\phi_s}$$

It produces a reactive or quadrature voltage disturbance δV_x which can be viewed as a phase error:

$$\delta\phi = \frac{\delta V_x}{V} = \frac{3}{2}\ \frac{P_b}{2\ U \tan\phi_s}\ \tau_d \qquad (12)$$

From equations (1) and (2), we have

$$\frac{\delta V_b}{V_{bs}} = \frac{\delta V}{V} + \frac{\delta\phi}{\tan\phi_s} \qquad (13)$$

Substituting (11) and (12) in (13) we obtain the fractional accelerating voltage disturbance (for the storage ring case, one uses equation (15)):

$$\frac{\delta V_b}{V_b} = \frac{3}{2}\ \frac{P_b}{2U}\ [1 + \frac{1}{\tan^2\phi_s}]\tau_d \qquad (14)$$

Now, we can numerically evaluate the minimum stored energy per unit beam power to hold the consequent radial synchrotron oscillation amplitude within the limits earlier prescribed, i.e.,

$$\Delta R_{max} = 1\ \text{mm for}\ \frac{\delta V_b}{V_b} = 0.13.$$

For $\phi_s = 30°$, $\tau_d = 250$ nsec.

We obtain

$$\frac{U}{P_b} = 10\ \text{joules/MW minimum.}$$

In the Berkeley design, the total rf energy stored in the accelerating structure is 18 joules during acceleration. At 1.2 MW beam power, this is 15 joules per megawatt.

In the beam storage mode, the voltage may be reduced to one-half the accelerating value; the stored energy then is 4.5 joules. In this mode, however, the disturbance is smaller. P_b and V_b are zero, and the phase shift component is given by

$$\delta\phi = \frac{3}{2}\frac{P_x}{2U}\ \tau_d = \Delta\phi_m \qquad (15)$$

$$V_{storage} = 1/2\ V_{accel}$$

$$P_x = Ne\ \frac{\beta c}{2\pi R}\ \frac{V_{accel}}{2} = 1.2\ \text{MVAR}\ .$$

Additional Design Factors Influenced by Beam Loading

Power Tube Efficiency

The beam load introduces a reactive load on the cavity. To compensate for the reactive load, the tuner acts slowly to tune out the average reactance, while the power tubes act more rapidly to counteract the transient reactance. In selecting power tubes, we recognize this reactive current, and plan accordingly for somewhat less than normal rf power tube efficiency (43% rather than 60%).

Harmonic Voltages

The beam current pulses and the rf power tube pulses are narrow compared to an rf cycle. Under the influence of this strong harmonic current excitation, the cavity-transmission line system could develop appreciable harmonic voltages; such voltages if large are undesirable because they cause lower tube efficiency, may precipitate sparking at unexpected locations, and they alter the bucket shape seriously.

For the transmission line-cavity system, considered as a 4-terminal network, we are making the driving point impedance a maximum at the fundamental frequency ω, and (as nearly as possible) minimum at integer multiples of ω. The physical arrangement of transmission line, loop, tube, and cavity is being adjusted to avoid integer resonances. We are studying the use of a periodic non-uniform transmission line in shifting the location of the network's poles and zeros; it appears that the harmonic distortion of the accelerating voltage waveform can be kept within a few percent without adding damping devices.

References

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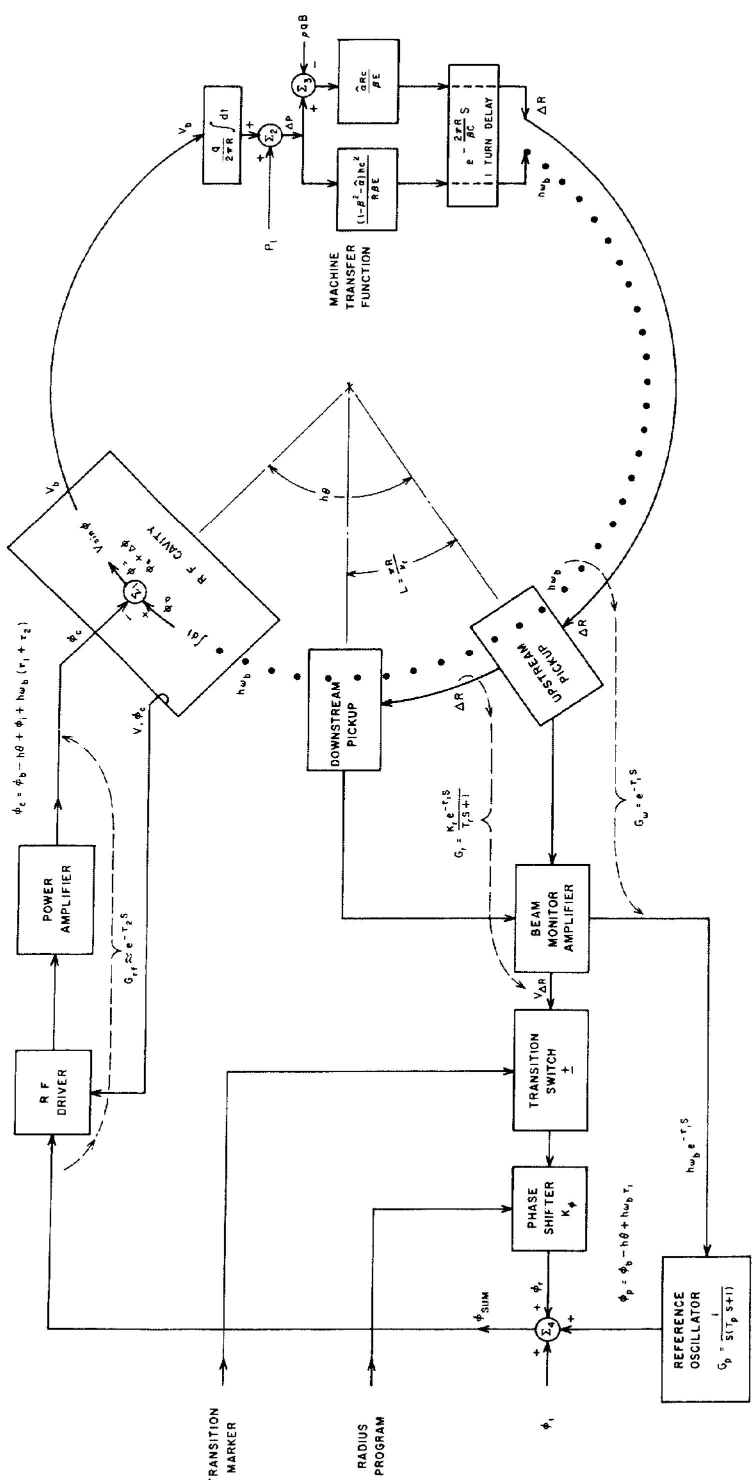

Figure 1. Beam Control System.

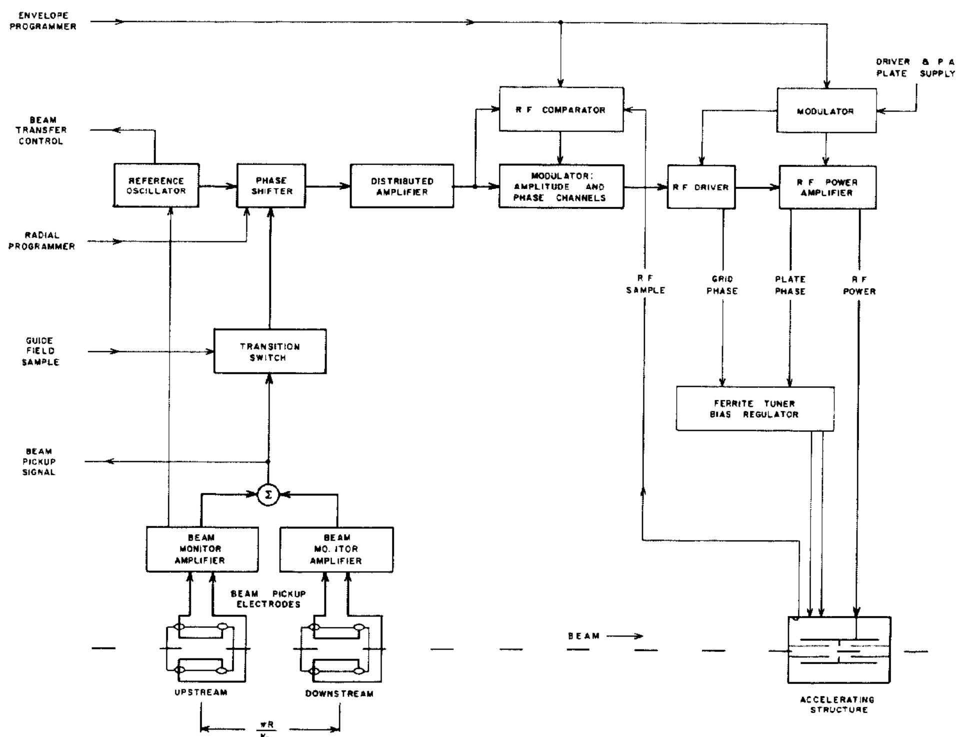

Figure 2. Main Synchrotron Acceleration System.