The Extreme Value Theory to estimate beam losses in High Power Linacs

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Outline

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Introduction

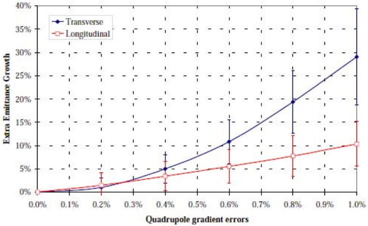
- Once the baseline design of the high power linac (HPL) is achieved, it is necessary to evaluate the effects of imperfect elements on the beam losses to define tolerances for the construction (also a test of the design robustness):

$$X \pm \delta X?$$

- Crandall (LINAC'88) and Raparia (PAC'93) shown how manufacturing errors modeled with multipoles components could induce an emittance growth but the effect of non linear space charge force was not treated in these references. The halo induced by the space charge is then underestimated and the loss prediction becomes distorted.

Background

- To tend to "realistic" simulation of a HPL, it is necessary to perfom start to end transport to be capable to estimate the impact of halo produced at low energy on the beam losses at the high energy part of the accelerator (use of collimators can affect this statement).
- For their error studies, Pichoff (PAC'01) and Ostroumov (PRSTAB, 2004) performed S2E runs taking into account space charge and/or the non linear external fields. These studies used macroparticles to represent the beam distribution and to record the losses at the beam pipe.



N. Pichoff et al. (PAC'01)

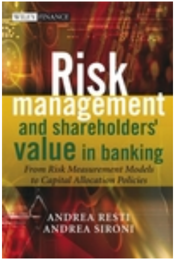
Limit of this direct macroparticular approach

- The discrete recorded losses in the linac allow to build Cumulative Density Function (CDF) to calculate a probability to deposite more than a certain fraction of the beam. But the **discrete** form of this CDF induces that the probability to loose more than the more extreme recorded loss becomes null!
- The Extreme Value Theory provides a firm theoretical foundation to avoid this drawback. Combining this theory with the bootstrap technique, we propose in this contribution, to detail a procedure to compute average probability of occurence of extreme events such a very low beam loss (10^{-5}) including a confidence interval (error bar).

Outline

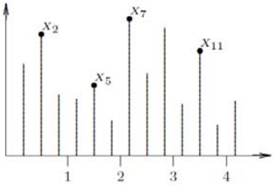
Introduction to EVT

- In many fields of modern science, engineering and insurance, extreme value theory is well established [6,7,8]. When modelling the maxima of a random variable, this theory plays the same fundamental role as the central limit theorem plays when modelling sums of random variables.
- Two approaches can be found in the litterature:
  - the analysis of block maxima with the GEV,
  - the analysis of the Peaks Over Threshold (POT).
- Only the first one is developped here.



Distribution of Maxima (GEV)

The limit law of the block maxima, which we denote by M_n, with n the size of the subsample (block), is given by the following theorem:



Theorem (Fisher and Tipett (1928), Gnedenko (1943))

Let (X_n) be a sequence of random variables. If there exists two series of real constants $C_n > 0 \ \forall \ n$ and d_n and some non-degenerate distribution function H such that

$$\frac{M_n - d_n}{c_n} \longrightarrow H,$$

then H belongs to one of the three standart extreme value distributions: Fréchet, Weibull or Gumbel.

Generalized Extreme Value distribution (GEV)

- Jenkison and von Mises suggested to represent these three distributions with the following representation:

Definition

$$H_{\xi\sigma\mu}(p) = exp\left(-\left(1+\xi\frac{p-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right) \quad (1)$$

with μ, the location parameter, σ, the scale parameter and ξ a form parameter.

- When $\xi \to 0$, the GEV tends to the Gumbel distribution.
- This general representation is very useful as at the beginning of the treatement, we don't know in advance the limiting distribution type of the sample.

Bootstrap technique [9]

- The bootstrap technique allows to construct confidence intervals (precision) for each fitted parameters of the GEV or a particular return level.
- The most fundamental idea of the bootstrap method is that we compute measures of our inference uncertainty from that estimated sampling distribution.
- In practical application, the bootstrap means using a *resampling* with *replacement* from the actual data X to generate B bootstrap samples X^*.
- Properties expected from the replicate real sample are inferred from the bootstrap samples by analysing each bootstrap sample exactly as we first analyzed the real data sample. From the set of results of sample size B, we measure our inference uncertainties (variance).

Outline

The data acquisition strategy (1/2)

- Usually, EVT is used to analyse real events (measurements) but, to study the construction tolerances of a high power linac, it would correspond to the analysis of a huge number of similar built linacs:

Data set

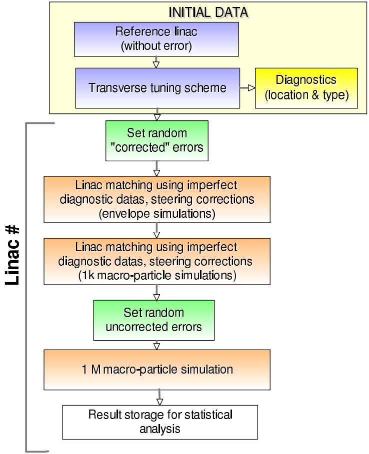
$$SNS_1 + SNS_2 + SNS_3 + \ldots + SNS_n$$

with n a huge number!

- For obvious reasons, it is necessary to produce the data set with virtual accelerators.
- One limit of this strategy is the resolution which can be achieved with the simulation tools. Differently, it has to be shown that the relevant physics is present in the codes.

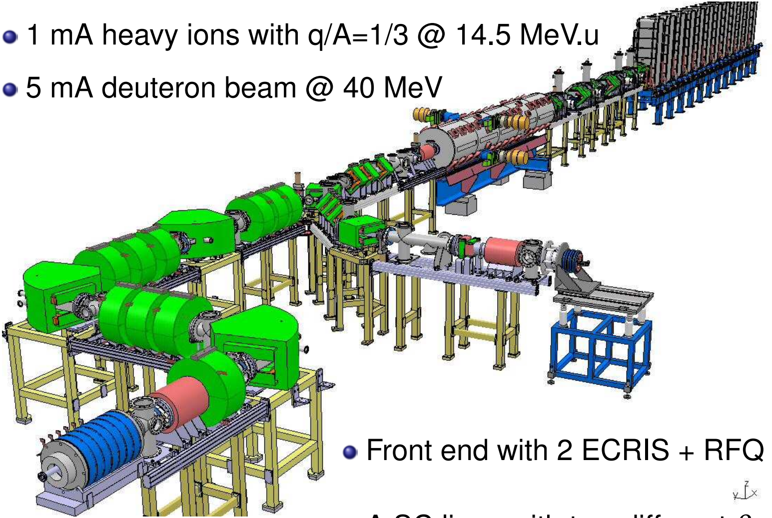
The data acquisition strategy (2/2)

- The correction scheme is characteristic of the linac.
- The envelops for the merged errors can be obtained after cross-checks with engineers and/or preliminar computations in which each defect is studied separately.
- Errors on the diagnostics are required.
- The error envelops are discretized to find the working point.



A test case: the SPIRAL 2 driver

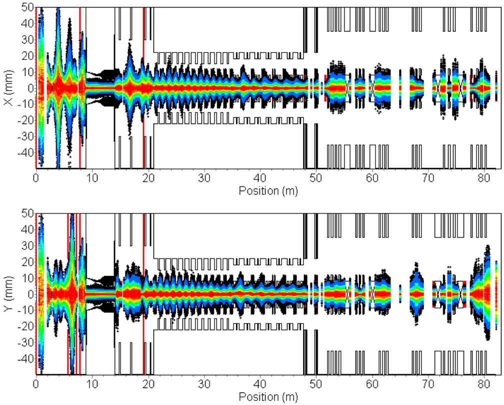
- 1 mA heavy ions with q/A=1/3 @ 14.5 MeV.u
- 5 mA deuteron beam @ 40 MeV



- Front end with 2 ECRIS + RFQ
- A SC linac with two different β_g

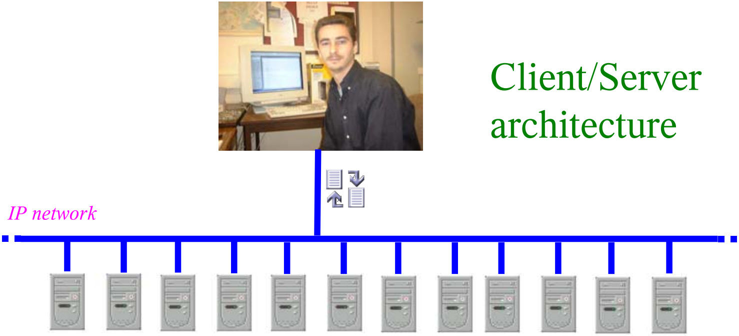
Start to end simulation

- Simulated with Saclay codes (space charge with 3D PIC routine, field maps for the external fields).
- 1,3 Mega macroparticles to allow a resolution lower than 1 watt in the whole linac.



Software for Monte Carlo

Each run can be performed on a single PC but a lot of runs is necessary to get a good statistic.



Multi parameters scheme is matched for beam dynamics errors studies

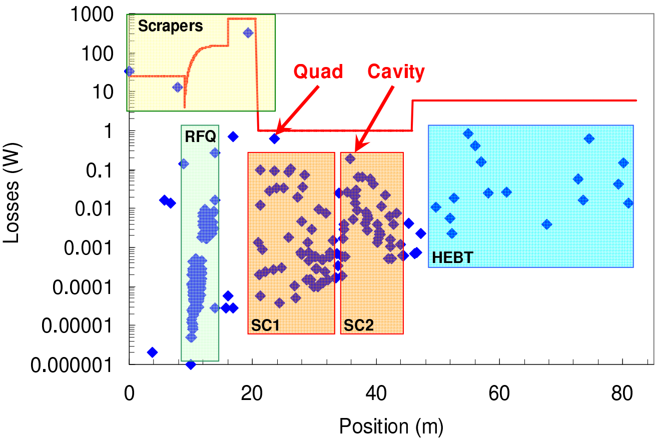
($\neq$ parallel computations)

Strategy for the hot spot analysis

1. Scan the mean deposited power for each element of the accelerator to detect the most critical components.
2. Fit the data with the Generalized Extreme Value (GEV) distribution.
3. Estimate confidence intervals for value of interest with the bootstrap method.

Average beam losses

(341 linacs)



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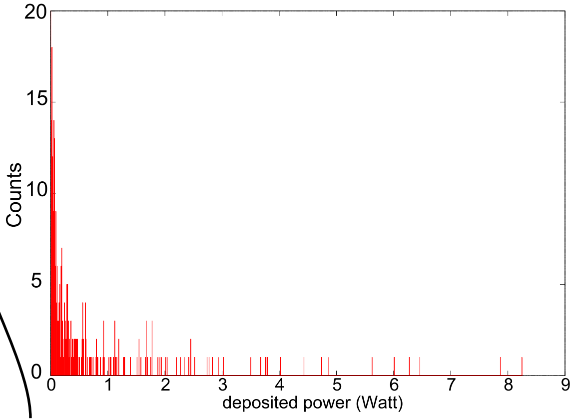
Build a CDF from the recorded losses (QUAD losses)

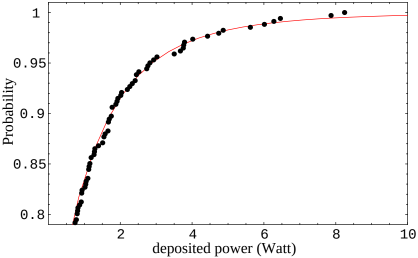


$$F_n\left(x_i^n\right) = \frac{i}{n} \text{ for } i = 1, \ldots, n$$

CDF (Cumulative Density Function)

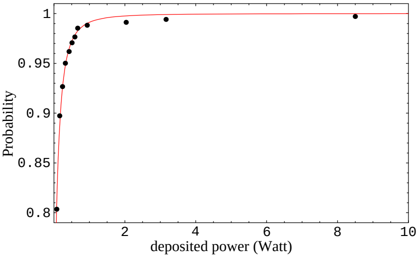
GEV distribution

QUAD



$$\text{P(losses} > 4\text{ W)} = 3 \cdot 10^{-2}$$

CAVITY

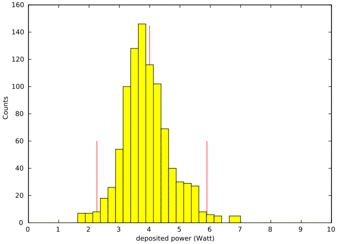


$$\text{P(losses} > 1\text{ W)} = 10^{-2}$$

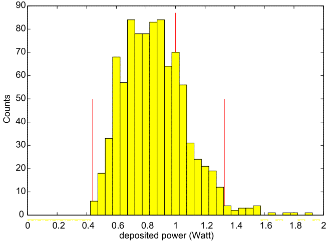
Confidence intervals at $\pm 2\sigma$

(1000 resampled loss distributions for a fixed probability)

QUAD



CAVITY



	CDF @ PE	Lower bound	Point estimate (PE)	Upper bound
Quadrupole	$3 \cdot 10^{-2}$	2.3 W	4 W	5.9 W
Cavity	10^{-2}	0.44 W	1 W	1.33 W
Quadrupole	10^{-4}	20 W	36 W	52 W

Conclusions

- This application of the Extreme Value Theory to beam losses estimates in the SPIRAL2 linac based on large scale Monte Carlo computations allowed us to provide low losses probability.
- We used the bootstrap technique to estimate the precision of this prediction.
- To go further to "realistic" estimates of the beam losses, a more faithful modelisation of the linac is required (resolution).
- For instance, the output beam distribution of the particle source is necessary to enhance the start to end modelisations and the beam interaction with the residual gas (neutralisation) has to be taken into account to simulate more accurately the space charge force especially at low energy in a high power ion linac.

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