A New Code for Orbit Response Matrix Analysis

Scaled Levenberg-Marquardt Algorithm

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Motivation

Why use **Orbit Response Matrix** ?

- To understand the lattice, how much different from design ?
  - BPM gains, tilt angles.
  - Orbit corrector strength.
  - Quadrupole strength, tilt angles.
- Correct errors of magnets.
- Symmetrize the lattice.

Why a new code ?

- Better convergence property.
- More robust, solve larger models with more parameters.
- Solve the coupling of parameters[1].
- Proven by three different lattices.

Orbit Response Matrix

The Orbit Response Matrix(ORM) is defined by

$$G_{ij} = \frac{dy_i}{d\theta_j^y} \qquad y \text{ is horiz. or vert.} \tag{1}$$

- y_i is closed orbit deviation in horizontal or vertical direction at the i^{th} BPM.
- θ_j^y is horizontal or vertical bump angle at the j^{th} corrector.

The full matrix for m BPMs and n corrector is

$$\begin{pmatrix} x \\ z \end{pmatrix}_{2m} = \begin{pmatrix} G^{xx} & G^{xz} \\ G^{zx} & G^{zz} \end{pmatrix}_{2m\times 2n} \begin{pmatrix} \theta^x \\ \theta^z \end{pmatrix}_{2n} \tag{2}$$

where $G_{ij}^{xz} = dx_i/d\theta_j^z$, which is actually a Green function between j^{th} corrector and i^{th} BPM[2].

G^{xz} and G^{zx} will be zero, if no coupling presents.

χ^2-Minimization

We can model the accelerator by minimizing the χ^2 of our model and experiment measurement*.

$$\chi^2(\mathbf{b}) = \sum_{ij} \left(\frac{G_{ij}(\mathbf{b})^{\{x,z\},\text{model}} - G_{ij}^{\{x,z\},\text{meas}}}{\sigma_{M_{ij}}} \right)^2 + \sum_{x,z} \left(\frac{\nu_{x,z}(\mathbf{b})^{\text{model}} - \nu_{x,z}^{\text{meas}}}{\sigma_{\nu_{x,z}}} \right)^2$$
$$+ \sum_{k} \left(\frac{\phi_k(\mathbf{b})^{\{x,z\},\text{model}} - \phi_k^{\{x,z\},\text{meas}}}{\sigma_{\phi_{ij}}} \right)^2 \quad (3)$$
$$\equiv \sum_{i} \left(\frac{f_i(\mathbf{b})^{\text{model}} - f_i^{\text{meas}}}{\sigma_{f_i}} \right)^2$$

b is the parameters of our model, which can quadrupole strength, titlt angle and BPM gain, BPM tilt angle.

*ICA measurement can give the phase advance[1]

Steepest-descent(Gradient) method

Follow the inverse-gradient direction: **Increment vector for Steepest-descent method**

$$\boldsymbol{\delta_s} = -(\frac{\partial \chi^2}{\partial b_1}, \frac{\partial \chi^2}{\partial b_1}, ..., \frac{\partial \chi^2}{\partial b_k})^T \tag{4}$$

and the increment of **b** in iteration n^{th} step is

$$b_i^{(n+1)} = b_i^{(n)} + (\boldsymbol{\delta_s})_i \Delta_i \tag{5}$$

Δ_i is the step size for b_i, which can be a constant or adaptive.

Gauss-Newton(Taylor Series) method

$$\chi^2(\mathbf{b}) \equiv \sum_{i=1}^{n} \left(\frac{f_i^{\text{model}} - f_i^{\text{meas}}}{\sigma_i} \right)^2 \tag{6}$$

f_i can be M_{ij}, ν_x, ν_y, ϕ_x, ϕ_y and other quantities measured or determined by other analysis.

$$f_i \approx f_i^0 + \sum_{j=1}^{m} \frac{\partial f_i}{\partial b_j} (\boldsymbol{\delta_g})_j \equiv f_i^0 + \sum_{j=1}^{m} J_{ij} (\boldsymbol{\delta_g})_j \tag{7}$$

Increment vector for Gauss-Newton method

$$\mathbf{J}^T \mathbf{J} \boldsymbol{\delta_g} = \mathbf{J}^T (\mathbf{f}^{\text{meas}} - \mathbf{f^0}) \equiv \mathbf{g} \tag{8}$$

Scaled Levenberg-Marquardt method

Increment vector for scaled Levenberg-Marquardt method[3]

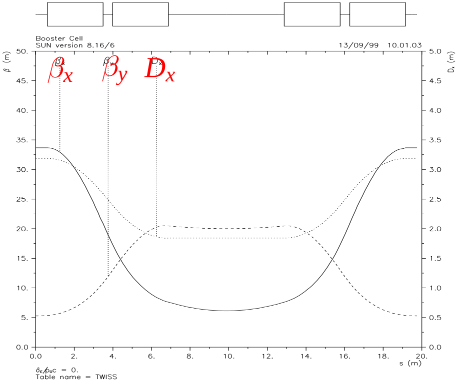
$$(\mathbf{J}^T\mathbf{J} + \lambda\mathbf{D}^T\mathbf{D})\boldsymbol{\delta} = \mathbf{J}^T(\mathbf{f}^{\text{meas}} - \mathbf{f}^0) = \mathbf{g} \qquad (9)$$

The main improvement is

1. $\lambda = 0$, it's Gauss-Newton method. $\theta(\boldsymbol{\delta_{lm}}, \boldsymbol{\delta_g}) = 0$
2. $\lambda \to \infty$, it's Steepest-descent (Gradient) method. $\theta(\boldsymbol{\delta_{lm}}, \boldsymbol{\delta_s}) = 0$
3. λ is an interpolation between Gauss-Newton and Gradient method.
4. $\mathbf{D}$ is a diagonal although can be any nonsingular matrix which takes into account the scaling of the problem.
5. A step bound variable Δ is introduced to limit the length of increment vectors: $\|\mathbf{D}\boldsymbol{\delta}\| \leq \Delta$
6. The scheme for update Δ keeps the ratio of actual reduction of χ^2 and expected reduction at a resonalble level
7. More detailed consideration when solve the equation of $\boldsymbol{\delta}$

Strategy of D_k, Δ and λ.

Lattice of Fermi Lab Booster



Fermilab Booster (12/05/2002)

- Circum., 474.2 [m]
- Lattice, (FOFODODO)*24
- L(FU,...), 2.8896$[m]$.
- Drift, 6/1.2/0.5 $[m]$
- β_x, 33.67/6.12 m
- β_y, 20.46/5.27 m
- D_x, 3.19/1.84 m
- ϕ/cell, $\phi_{x,y} = 100.5, 102$
- Tune, $\nu_x = 6.7$, $\nu_y = 6.8$
- $K_1(F/D)$, 0.0542, -0.0577 $[m^{-2}]$.

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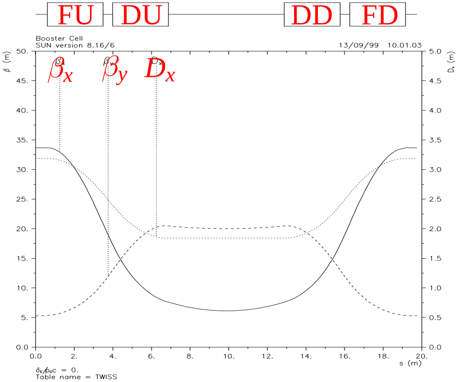

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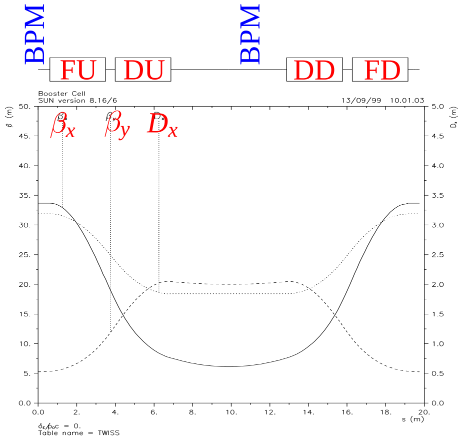

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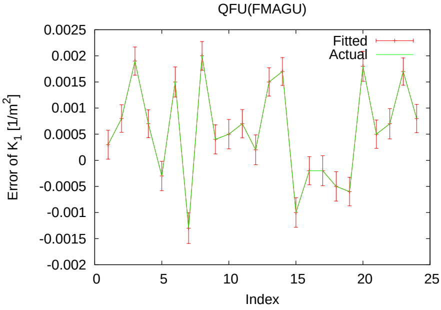
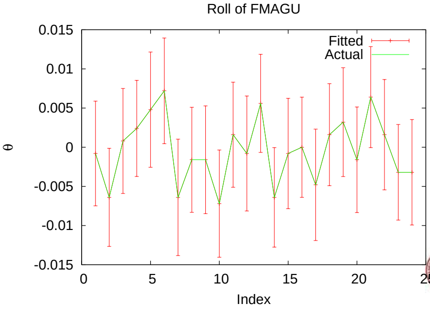
Fermilab Booster: A model of 192 parameters, Quads. K_1 and θ

- Only show K_1 and θ_q for FU.
- the other fittings has similar accuracy.
- Probable uncertainties in the estimation of K_1 and θ_q

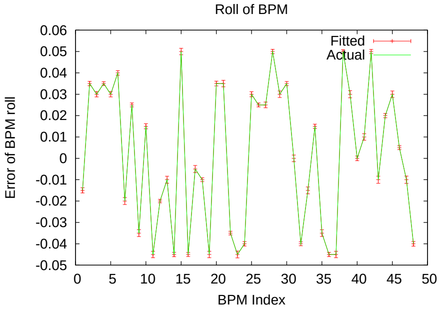
$$\sigma_{b_k}^2 = \sum_{i=1}^{n} \left(\frac{\partial b_k}{\partial f_i} \right)^2 \sigma_{f_i}^2 \qquad (10)$$

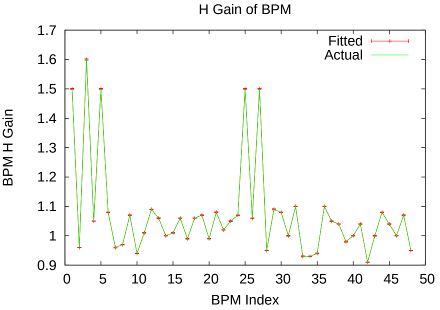




When BPM errors are considered, BPM b_x, b_y, θ_b

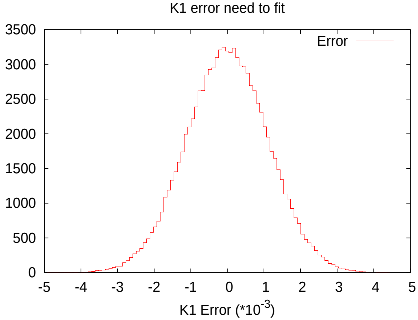
- Model has 192+144 parameters, Quads and BPM gain, roll.
- Same accuracy for K_1 and θ_q.

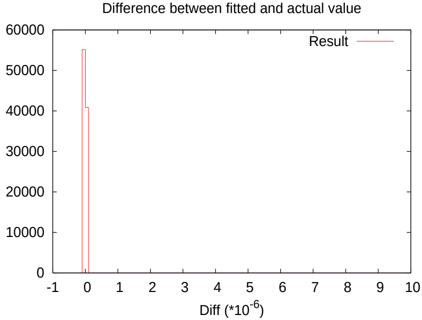




Statistics: Gaussian Distribution

- Model has 96 parameters, i.e. only K_1 errors.
- K_1^{actual} has Gaussian distribution, $\sigma \approx 2\% K_1(F)$
- 1002 random parameter sets fitted.





$$\frac{\left|K_1^{\text{fit}} - K_1^{\text{actual}}\right|}{K_1(F)} < 10 \times 10^{-6}/0.05 = 0.02\%$$

BPM noise

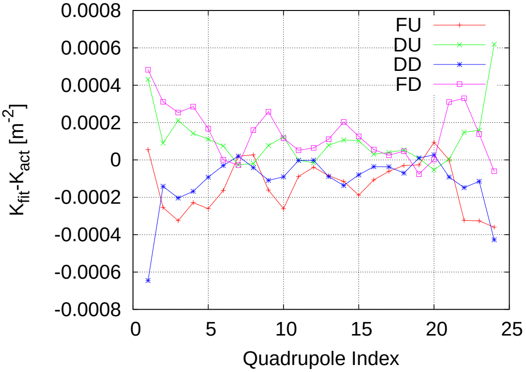


Figure: An example of $\delta_n = 10^{-2}$, $10\mu m/1mrad$. The new algorithm breaks the coupling between nearby quadrupoles in [1] and give the right converged solution.

Design Lattice of Taiwan Photon Source

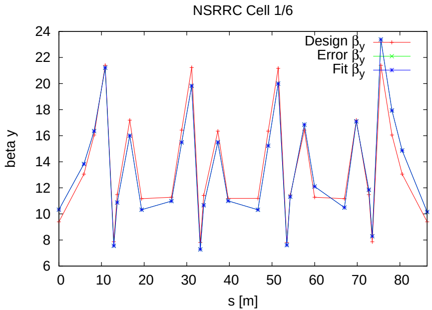


Figure: Restore betatron amplitude function β_x

24 Superperiod, 518.4m. (24p18K1). $\nu_x = 26.226$, $\nu_y = 12.280$. 48 types of quadrupoles, 168 BPMs. 600 parameters: quadrupole gradient and roll, BPM gains and roll. 112,898 terms in χ^2

Source of Vertical Orbit Oscillation

What is the source of vertical orbit oscillation ?

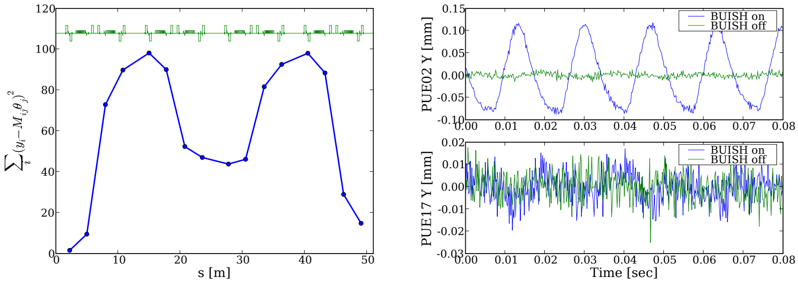


Figure: NSLS VUV ring, vertical orbit oscillation, (FDBK off).

After turning off the BUISH, the vertical orbit oscillation disappeared.

Conclusion

1. Quadrupole and BPM errors are fitted successfully for Fermilab Booster lattice, Taiwan Photon Source design lattice.
2. The coupling of parameters for nearby quadrupoles[**?**] can be resolved by new code.
3. New code can fit BPM data with noise.
4. It can used for beam diagnosis.

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