MUON ACCELERATORS

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Abstract

A muon accelerator is required to have very large acceptance and very quick acceleration. Recent studies show that a Fixed Field Alternating Gradient (in particular a non-scaling FFAG) is one of the most promising candidates for a muon accelerator as a building block for a neutrino factory. There are, however, some unresolved problems which should be studied in more detail. We will talk about why a FFAG draws more attention than other types of accelerators such as a linear accelerator or a RLA as a muon accelerator. Present R&D status and beam dynamics issues will be also discussed.

INTRODUCTION

Both a muon collider and a neutrino factory require acceleration of a muon beam. Since a muon beam has a short lifetime of 2.2 μs in the rest frame, acceleration has to be quick as possible, especially at the early stage where the Lorenz boost is relatively small. Although cooling of a muon beam is anticipated right after the production target, the emittance of a muon beam is still large: a few tens of thousand π mm mrad, compared with a conventional beam in an accelerator such as an electron or proton beam: normally of the order of one to hundred π mm mrad. This is because a muon is a tertiary particle. Therefore, a muon accelerator has to have a large acceptance.

For the last few years, intensive efforts have been made, first in the US and later worldwide, to design a machine complex for a muon collider and a neutrino factory. Among them, the so called Feasibility Study II [1] and Study 2-A [2], and most recently the International Scoping Study (ISS) [3] specifically proposed a detailed machine complex of a neutrino factory.

At the beginning, the muon accelerator part comprised a linear accelerator (Linac) and a Re-circulating Linear Accelerator (RLA) because it seemed that they were the only available options that satisfied the above requirements. Then, the use of Fixed Field Alternating Gradient (FFAG) accelerator as an alternative was suggested in the US [4,5,6] and later a whole machine complex with several FFAGs was proposed by a Japanese group [7]. Since then, the R&Ds and design work on FFAGs became one of the major activities in the accelerator community. We will describe the progress of muon acceleration design and related developments on FFAGs.

FROM FEASIBILITY STUDY II TO 2-A

As shown in Fig. 1, the accelerator complex described in Study II comprises RLA as a main accelerator and Linac as an injector. It could be all Linac to the final energy. It is, however, more expensive because the rf acceleration system dominates the cost. The specifications of the rf cavity are a gradient of 15 MV/m and an acceptance of 7,500 π mm-mrad normalized. Multiple use of the rf acceleration system with beam transport which make a beam back to the same rf system is one solution to minimize the cost, i.e. a RLA. In addition, a later study shows that phase mixing in a RLA turns out to be advantageous over a Linac (and a FFAG). This will be discussed later. The cost of the muon accelerator was, however, still the major part of the machine complex.

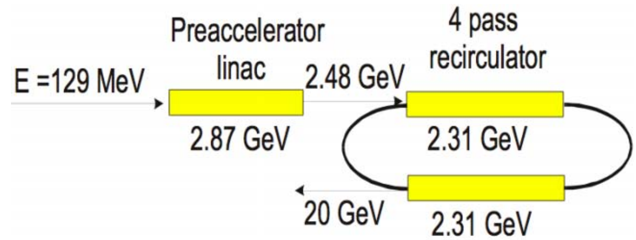

Figure 1: Acceleration scheme in Study II (from ref. [1]).

The use of a FFAG had been proposed as an alternative to a RLA in the US in 1997 even before Study II [4,5,6]. In fact, it was not the conventional type of FFAG (invented in 1950's [8,9,10]). The basic idea here was that if one could make an arc with a very small dispersion function of the order of a cm, one could replace the multiple arcs of a RLA by a single arc. That could be made possible by strong quadrupole focusing in a high periodicity lattice. The rf cavities would then be distributed all around the ring instead of being lumped together in a RLA. From the accelerator point of view, that is nothing but an ordinary storage ring. It has, however, rf cavities and a particle will gain net energy. This was the origin of what is now known as a nonscaling FFAG.

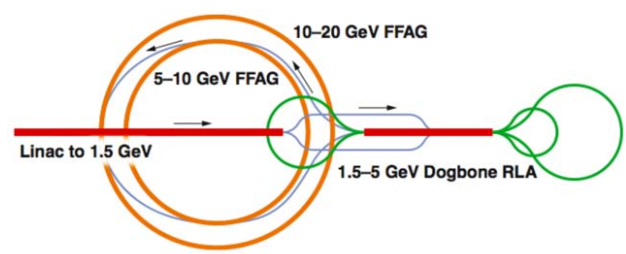

Figure 2: Acceleration scheme in Study 2-A (from ref. [2]).

It seemed that the idea of using a FFAG as a muon accelerator was not much appreciated until a Japanese group demonstrated proton acceleration by a conventional type of FFAG [11], which we call a scaling FFAG, and a neutrino factory proposal solely based on a FFAG appeared [7]. It is also interesting that the US and European groups started the development of a nonscaling FFAG from the

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beginning although the demonstration in Japan was on a scaling FFAG.

As shown in Fig. 2, the high energy part of the accelerator was replaced by FFAG in Study 2-A. The Linac and RLA were retained, but output energy of the latter was reduced to 5 GeV. In any case, it showed the shift of interest from RLA to FFAG.

NONSCALING FFAG

FFAG adopted in Study 2-A was not the conventional scaling FFAG. It consists of dipoles and quadrupoles just as in an ordinary synchrotron. In fact, there is no ramping of magnetic fields so that it is nothing but a storage ring. The distinguishing feature from a storage ring is, however, that the dispersion function is kept small and therefore the machine has a large momentum acceptance, typically +/-33% or more [12,13]. In that way, acceleration in a "storage ring" becomes possible. The ratio of injection and extraction momenta can be a factor of two.

Since there are only linear elements, namely dipoles and quadrupoles, dynamic aperture is expected to be large, which satisfies one of the requirements. The magnetic fields are constant and acceleration is only governed by the rf voltage so that the quick acceleration is possible, which satisfies the other requirement. Acceleration of muons with a few to 10 turns in a FFAG seemed the most economical way of using the rf system compared to a Linac and a RLA.

Acceleration was, however, an issue. In a RLA, the path length of each arc transporting fixed momentum beams can be adjusted so that the rf wave has the right phase when a beam comes to the entrance of a Linac. In addition, the phase relation between a beam and the rf wave is independent of momentum since a beam traverses Linac with the speed of light. In a nonscaling FFAG, however, there is a finite though small dispersion function. That introduces a variation of revolution frequency or a variation of time of flight from cell to cell which depends on beam momentum. What is worse, it is not possible to modulate the rf frequency in a time scale of 10 turns, especially considering a high Q cavity with frequency of a few hundreds MHz.

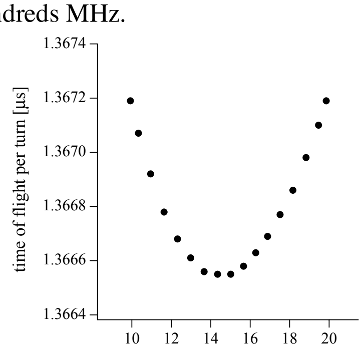

Figure 3: Time of flight as a function of momentum in 10 to 20 GeV muon ring.

One solution is to make the lattice as isochronous as possible [14]. That is in fact the same requirement as that of a smaller dispersion function. Revolution time scales with the total path length since the beam always travels with the speed of light. Thus, when the variation of revolution time is minimized, the variation of the path length is minimized as well. Therefore the overall orbit deviation which depends on momentum, which is nothing but the dispersion function, becomes small. Figure 3 shows the revolution time of the 10 to 20 GeV muon FFAG as a function of momentum. It has the minimum value at the middle of acceleration and become longer on both injection and extraction sides.

Now, suppose that the rf frequency is fixed to the inverse of the minimum revolution time (times harmonic number in practice). Then a beam at injection momentum experiences a slight phase slip with respect to an rf crest. That continues until the beam gets to the momentum where the revolution time is minimum. Around that momentum, the beam is almost synchronized with rf and its phase stays constant. As the beam is further accelerated, the phase slip gradually increases in the same direction as before. Therefore, if the total phase slip is less than π and one chooses the initial phase such that the beam reaches at the top of an rf crest at the middle of acceleration, almost isochronous acceleration can be achieved. That is called an out of bucket acceleration or "gutter acceleration" because a beam follows a flow between two rf buckets in longitudinal phase space.

Actually, the rf frequency is not necessarily fixed to the minimum revolution time. If the frequency is slightly lower, phase slip disappears before a beam reaches the middle momentum and then the beam phase moves in the opposite direction. Since the time of flight has an almost symmetrical parabola shape, phase motion freezes once again before the beam is accelerated to the extraction momentum and starts moving in the original direction. In that case, trajectory in the longitudinal phase space has a serpentine shape. From the phase slip point of view, the lower the frequency, the smaller the phase slip will be. The rf voltage can be, however, higher with a higher frequency. The time structure of the beam created in upstream components should match the rf frequency of the FFAG, too. The baseline frequency is determined at 200 MHz in Study 2-A.

DEVELOPMENTS IN JAPAN

In late 1990's, a successful development of a new type of rf cavity with Magnetic Alloy (MA) [15] enabled Mori and colleagues to start the design and construction of a FFAG accelerator in Japan. That was just right after the idea of using a FFAG as a muon accelerator was proposed in the US. The machine that the Japanese group chose was a conventional, scaling FFAG, because their aim was not only a muon accelerator, but a high intensity proton driver and accelerators for other applications.

A scaling FFAG uses a magnet that has the field profile of,

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$$B_z = B_0 \left(\frac{r}{r_0} \right)^k F(\phi),$$

where k is the field index and F describes the azimuthal dependence. The magnets occupy the same fraction in azimuthal direction independent of radius. Either by switching the sign of the field alternately from magnet to magnet or by introducing an edge focusing for the vertical plane by a spiral shape magnet, alternating gradient focusing is realized [16]. Also, orbits of different momenta have a similar shape though they lie on different radii. A major advantage of this focusing structure is that the transverse tunes are constant throughout acceleration. That follows from the fact that orbits are similar and focal length is proportional to the radius. In a accelerator terminology, chromaticity is corrected to zero in the whole acceleration range by the nonlinearities of the field profile.

Success of a 500 keV proof of principle (POP) model [11], which was the first proton FFAG with a MA cavity, initiated many programmes aiming at applications of a scaling FFAG. A muon accelerator was among them. A neutrino factory proposal in Japan uses only a series of scaling FFAGs from the beginning to the top energy without a beam cooling section [7]. That is possible because of the large acceptance both in the longitudinal and in the transverse planes. Although there are nonlinear forces from the field profile, dynamic aperture is still huge and, for reasons unknown, comparable to that of a nonscaling FFAG. Another interesting feature is in the momentum compaction factor. Because it is exactly $1/(k+1)$ and the slippage factor only depends on a kinematic term $1/\gamma^2$, the rf bucket can be made as large as the rf voltage allows. Accordingly, acceleration of a muon beam in a large fixed frequency bucket was proposed. A beam is injected into the bottom side of a bucket and extracted from the top side of a bucket after less than a half synchrotron oscillation [7].

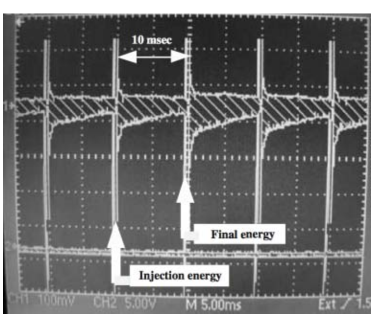

Figure 4: Beam signal of 150 MeV FFAG with 100 Hz operation (from ref. [19]).

Apart from the application to muon acceleration, there are several construction projects going on. One obvious target is a medical use with high intensity operation and high repetition rate. That makes a new form of treatment possible, namely spot scanning. The same Japanese group started construction of 150 MeV FFAG as a prototype for medical use [17]. In 2005, beams were extracted with an efficiency of more than 90%. The repetition rate of 100 Hz was also demonstrated as shown in Fig. 4 [18,19].

BEAM DYNAMICS ISSUES

Transverse Tune Excursion

Although the lattice of a nonscaling FFAG is designed such that the orbit excursion becomes minimum for the whole momentum range, there is no chromaticity control of transverse tunes. As a result, transverse tunes decrease significantly during acceleration as shown in Fig. 5. Although the use of nonlinear magnets to correct chromaticity was studied for some time, it turned out that dynamic aperture decreased considerably, becoming too small to accommodate the large muon emittance.

As shown in Fig. 5, transverse tunes are in fact carefully chosen so that the structure integer and half-integer tunes will not be crossed. Integer and half-integer tunes that may cause resonances with random misalignments and fabrication errors of magnets are, however, not avoided in the course of acceleration. One plausible argument is that the crossing of those tunes is not a problem if the tune change is very fast. In fact, the 10 to 20 GeV muon ring, for example, is supposed to finish acceleration in 17 turns. Within one turn, the total tune changes an order of one and a particle may not even see the periodicity of a ring. With tracking simulation, we studied distortion of orbit and optics due to alignment and magnet errors.

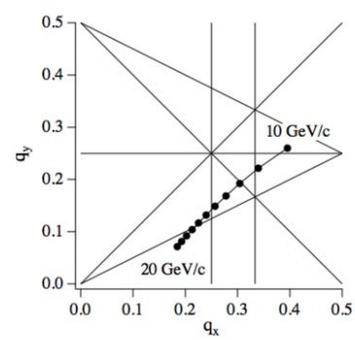

Figure 5: Cell tune diagram of 10 to 20 GeV FFAG.

As shown in Fig. 6, alignment errors introduce orbit distortion all the way from injection to extraction. We have alignment errors of 0.1 mm (rms) and the maximum errors are twice as much. It, however, does not show any structure that depends on the total transverse tunes. Distortion is not necessarily excited when the tune cross integer values.

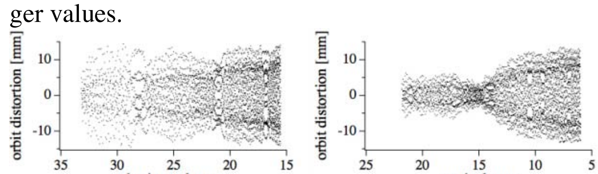

Figure 6: Horizontal (left) and vertical (right) orbit distortion as a function of total tune.

A similar exercise has been carried out for optics distortion. We prepare a beam with emittance of 3 π mm mrad (horizontal only) and introduced gradient errors of $1x10^{-3}$. It is matched to the optics calculated for injection momentum. Without gradient errors, optics function β and α calculated from the ensemble of macro particles vary over the tune range with no oscillations. With errors, those functions start oscillating because of beam tumbling as shown in Fig. 7. The modulation, however, does not have any explicit correlation with integer and half-integer tunes.

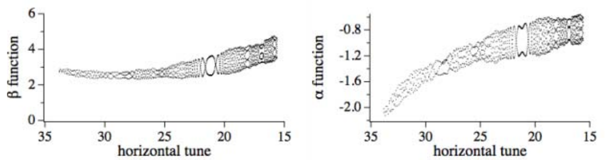

Figure 7: Distortion of β (left) and α (right) functions as a function of total tune.

These results suggest that distortion is not due to a crossing of integer or half-integer "resonance". Rather, it is caused by random error kicks of dipoles and quadrupoles [20]. The question is, then, the magnitude of distortion when there are realistic errors in a lattice. By using random seeds, we tested 501 different error patterns and calculated the amplification factor, which is defined as the ratio of maximum and rms orbit distortion vs. rms alignment errors. They are 143 and 53.3 for the horizontal plane and 110 and 40.9 for the vertical one, respectively. Similarly, for gradient errors, a growth factor, which is the growth rate of maximum single particle emittance is defined. It becomes 0.2 when the rms gradient error is $1x10^{-3}$. Both factors scale linearly with the magnitude of errors.

Path Length Dependence on Amplitude

In a FFAG, the path length changes as a beam is accelerated. In addition, the path length also depends on betatron amplitude. Suppose that there are two particles with zero and finite betatron amplitude. A zero amplitude particle follows a path determined only by momentum, whereas a finite amplitude one takes a longer path to complete one revolution. In fact, Berg pointed out that the path length difference is proportional to chromaticity and betatron amplitude [21]. Therefore it is only a problem in a nonscaling FFAG.

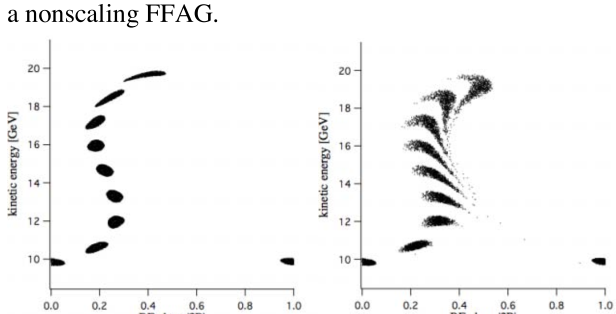

Figure 8: Longitudinal emittance evolution without (left) and with (right) finite transverse amplitude.

Figure 8 illustrates the longitudinal emittance evolution with and without finite transverse emittance. We assume that the 100% transverse emittance (normalized) is 30,000 π mm mrad for the right-hand figure. Large transverse amplitude particles have more phase slip and result in a larger spread in phase space. Some particles even get to the deceleration phase and cannot be accelerated to the top energy. That can be cured partially by applying higher voltage or making an rf crest flatter with higher harmonic rf components. The former simply finishes acceleration before large phase slip occurs and the latter gives more uniform energy gain for extended phase area. These remedies work well if there is only a single FFAG. As shown in Fig. 2, however, the baseline scenario of Study 2-A was a FFAG cascade and we need to find the optimum solution for the cascade system. Figure 9 is the best result we have achieved to date [22].

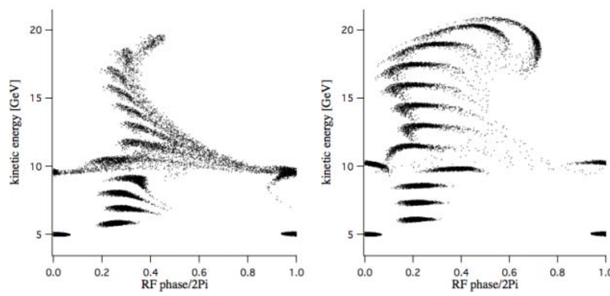

Figure 9: Longitudinal emittance evolution of a two FFAG cascade with 100% emittance of 30,000 π mm mrad. Left figure uses nominal rf voltage and no higher harmonic rf. Right one uses second harmonic rf with 10% increase in fundamental rf voltage.

FROM STUDY 2-A TO ISS AND BEYOND

Those beam dynamics issues have been studied during the ISS period. Basically, problems are all related to the finite chromaticity that a nonscaling FFAG possesses. Therefore, replacing a nonscaling FFAG by a scaling FFAG is one solution. Although there are other new problems in a scaling FFAG such as the requirements of larger magnets, choice of rf frequency, etc., a study in this direction continues [23].

On the other hand, it is interesting to see if similar problems exist in a Linac or a RLA, especially path length dependence on amplitude. A Linac should have the same problem and this has been confirmed by tracking simulation [24]. Fortunately, a RLA has an additional knob that can be used to cure the problem, at least partially. In a RLA, in each arc a desirable value of m_{56} matrix element can be set (coefficient relates time of flight with momentum). This means that a large transverse amplitude particle that has a larger phase slip can catch up with an rf crest after going through the arc. In other words, synchrotron oscillations wipe out the undesirable phase relation. Although a realistic design of the arc is not yet done from this point of view, an optimization with this knob, m_{56}, improves the beam behavior as shown in Fig. 10.

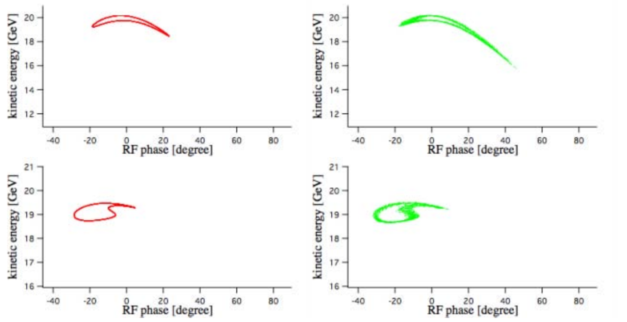

Figure 10: Bunch shape at the top energy of RLA. With zero transverse emittance (red) and finite emittance of 30,000 π mm mrad (green). Upper figures shows when a beam is on a crest and m_{56}=0. Lower figure shows when a beam is off crest and m_{56}=1 [m].

According to those findings, the accelerator complex is slightly modified in ISS report as shown in Fig. 11. The importance of the RLA is increased and only a single FFAG with a possible extension to a second FFAG of higher energy remains. Further optimization study continues.

Finally, no nonscaling FFAG exist. Feasibility and also its beam dynamics issues have yet to be demonstrated experimentally. The project of constructing an electron model has been approved and started this year at Daresbury Laboratory in U.K. [25].

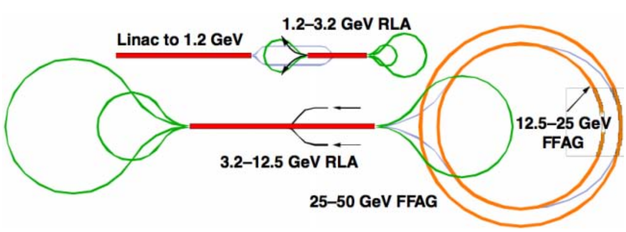

Figure 11: Acceleration scheme in ISS (from ref. [3]).

REFERENCES

[1] "The Neutrino Factory and Muon Collider Collaboration Feasibility Study II", http://www.cap.bnl.gov/mumu/studyii/

[2] "The Neutrino Factory and Muon Collider Collaboration Study 2-A", http://www.cap.bnl.gov/mumu/study2a/

[3] "International scoping study of a future Neutrino Factory and super-beam facility", http://www.hep.ph.ic.ac.uk/iss/

[4] F. Mills, "Linear Orbit Recirculators", Muon Collaboration Meeting, p.693, San Francisco, December 13, 1997.

[5] C. Johnstone, "FFAG design", Muon Collaboration Meeting, p.696, San Francisco, December 13, 1997.

[6] C. Johnstone, et. al., "Fixed field circular accelerator designs", PAC'99, New York, March 1999, p.3068.

[7] S. Machida, "Neutrino factory design based on FFAG", Nucl. Instrum. Methods Phys. Res., Sect. A **503**, (2003) 41 and http://www-prism.kek.jp/nufactj/index.html

[8] T. Ohkawa, Bulletin of Physical Society of Japan, 1953.

[9] K. R. Symon, et. al., "Fixed-Field Alternating Gradient particle accelerators", Phys. Rev. Vol. 103, **6**, p.1837, 1956.

[10] F. T. Cole, "O Camelot! A Memoir of the MURA years", http://accelconf.web.cern.ch/AccelConf/c01/cyc2001/extra/Cole.pdf

[11] M. Aiba, et. al., "Development of a FFAG proton synchrotron", EPAC'00, Vienna, 2000, p.581, http://www.jacow.org.

[12] E. Keil and A. M. Sessler, "Muon acceleration in FFAG rings", Nucl. Instrum. Methods Phys. Res. Sect. A **538** (2005) 159.

[13] D. Trbojevic, et. al., "Design of a nonscaling fixed field alternating gradient accelerator", Phys. Rev. ST Accel Beams **8**, 050101 (2005).

[14] S. Koscielniak and C. Johnstone, "Mechanisms for nonlinear acceleration in FFAGs with fixed RF", Nucl. Instrum. Methods Phys. Res. Sect. A **523**, (2004) 25.

[15] Y. Mori, et. al., "A new type of rf cavity for high intensity proton synchrotron using high permeability magnetic alloy", EPAC'98, Stockholm, 1998, p.299, http://www.jacow.org.

[16] A. A. Kolomensky and A. N. Lebedev, *Theory of Cyclic Accelerators* (North-Holland, Amsterdam, 1966).

[17] M. Aiba, et. al., "150-MeV fixed field alternating gradient accelerator and return-yoke free magnet", EPAC'02, Paris, 2002, p.1028, http://www.jacow.org.

[18] M. Aiba, et. al., "Beam extraction of 150 MeV FFAG", EPAC'06, Edinburgh, 2006, p.1672, http://www.jacow.org.

[19] Y. Yonemura, et. al., "Development of RF acceleration system for 150 MeV FFAG accelerator", Nucl. Instrum. Methods Phys. Res. Sect. A **576** (2007) 294.

[20] S. Machida, "Orbit and optics distortion in fixed field alternating gradient muon accelerators", submitted to Phys. Rev. ST Accel. Beams.

[21] J. S. Berg, "Amplitude dependence of time of flight and its connection to chromaticity", Nucl. Instrum. Methods Phys. Res. Sect. A **570** (2007) 15.

[22] S. Machida, "Longitudinal emittance blowup in fixed field alternating gradient muon accelerators", Phys. Rev. ST Accel. Beams 9, 104002 (2006).

[23] Y. Mori, et. al., "Muon acceleration with scaling FFAG using harmonic number jump", Nufact06, Irvine, 2006, http://nufact06.physics.uci.edu/Workshop/Program/WG3/Default.aspx

[24] S. Machida, "Muon acceleration system", Nufact06, Irvine, 2006, http://nufact06.physics.uci.edu/Workshop/Program/WG3/Default.aspx

[25] R. Edgecock, "EMMA – the world's first nonscaling FFAG", in this proceedings.