MEASURING SINGLE PARTICLE AMPLITUDES WITH MICE

M. Apollonio, Sub-Department of Particle Physics and John Adams Institute,
University of Oxford, UK (for the MICE Collaboration)

Abstract

Ionisation Cooling will be an essential element of a future Neutrino Factory [1]. The Muon Ionization Cooling Experiment, MICE, being built at RAL (UK) will be the first apparatus to demonstrate the feasibility of this technique. MICE will be unique in being able to make single-particle measurements: it will be possible to measure the amplitude of each muon in 6D phase space. It is shown how amplitude measurements can be used to quantify the transmission of the cooling channel and the increase in central phase space density due to cooling.

INTRODUCTION

MICE is designed to demonstrate the feasibility of a cooling section for a Neutrino Factory, measuring the amount of cooling with muon beams in a variety of optical configurations. Emittance reduction is expected to be quite modest (10 to 15%) which requires a precise determination of such quantity in order to extrapolate the performance of a full cooling channel (accuracy of 10^{-3}, absolute, an unprecedented precision).

Muons passing through MICE one at a time will be tracked by two fiber based spectrometers placed upstream and downstream of the cooling section which is made of three liquid hydrogen absorbers interleaved by two RF sections. A view of the experiment layout is shown in fig. 1.

Normalized Transverse Emittance can be determined from the beam covariance matrix V, after measuring a statistically meaningful ensemble of phase space vectors (x, p_x, y, p_y) characterizing each muon of the beam:

$$\epsilon_N^T = \frac{1}{m_\mu c}\sqrt[4]{det(V)}, \tag{1}$$

where,

$$V = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{p_x x} & \sigma_{p_x p_x} & \sigma_{p_x y} & \sigma_{p_x p_y} \\ \sigma_{yx} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{p_y x} & \sigma_{p_y p_x} & \sigma_{p_y y} & \sigma_{p_y p_y} \end{pmatrix}$$

Comparison of emittances before and after the cooling section is the standard way to show evidence of cooling.

This approach exploits only partially the important feature of MICE as a single particle experiment. The alternative approach described here is based on the use of a single particle feature: its (normalised) amplitude.

SINGLE PARTICLE AMPLITUDE

The amplitude (sometime called single particle emittance) is an invariant of the motion associated to any specific particle of the beam: if no dissipation (or cooling) is acting, amplitude must be conserved.

In a simple 2-dimensional world our knowledge of the kinematical terms (x, x') and of the optical functions defines completely the amplitude of a particle by means of the equation (the Courant-Snyder invariant):

$$A = \gamma x^2 + 2\alpha x x' + \beta (x')^2. \tag{2}$$

The normalised amplitude is:

$$A_N = \frac{P}{mc}A \simeq \frac{1}{Pmc}\left(\gamma(xP_z)^2 + 2\alpha(xP_z)P_x + \beta(P_x)^2\right) \tag{3}$$

where the paraxial approximation $P_z \simeq P$ has been used.

In 4 dimensions we have to consider the coupling between horizontal and vertical components and the definition of amplitude must take into account these correlations. Following Penn [2] an extension of formulae (2) and (3) leads to the definition of transverse amplitude,

$$\begin{aligned} A_\perp &\simeq \frac{x^2+y^2}{\beta_\perp} \\ &+ \beta_\perp\left(x' - \kappa y + \frac{\mathcal{L}}{\beta_\perp}y + \frac{\alpha_\perp}{\beta_\perp}x\right)^2 \\ &+ \beta_\perp\left(y' + \kappa x - \frac{\mathcal{L}}{\beta_\perp}x + \frac{\alpha_\perp}{\beta_\perp}y\right)^2 \end{aligned} \tag{4}$$

$\mathcal{L} \simeq \frac{<L_{canon}>}{2mc\epsilon_N}$ is the ratio between the canonical angular momentum and the normalized transverse emittance of the beam. $\alpha_\perp$, $\beta_\perp$ and $\gamma_\perp$ are the optical functions defining properties of the overall beam and are determined as [1]:

$$\alpha_\perp = -\frac{(\sigma_{xp_x} + \sigma_{yp_y})}{2m\epsilon_N^T} \tag{5}$$

$$\beta_\perp = \frac{(\sigma_{xx} + \sigma_{yy}) < P_Z >}{2m\epsilon_N^T} \tag{6}$$

$$\gamma_\perp = \frac{(\sigma_{p_x p_x} + \sigma_{p_y p_y})}{2m\epsilon_N^T < P_Z >}. \tag{7}$$

κ is the linearized focussing term,

$$\kappa(z) \simeq \frac{qB_z(r=0;z)}{2P_z} \simeq \frac{0.15B[\mathrm{T}]}{P_z[\mathrm{GeV/c}]}\mathrm{m}^{-1} \tag{8}$$

[1] These formulae are implemented in the code ECALC9 [5] and used to compute the optical functions.

1-4244-0917-9/07/$25.00

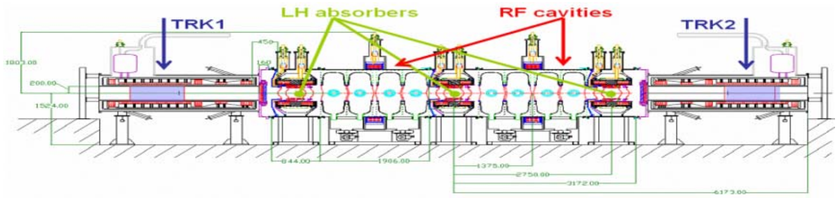

Figure 1: The MICE experiment: upstream and downstream the two spectrometer solenoids with the fiber trackers. In the middle clearly visible are the RF sections and the three liquid hydrogen absorbers used to cool the beam. Particle Identification Devices (TOF, Cherenkov and electron calorimeter) are located at the edges of the overall structure.

and for a solenoid where β is flat and constant (*i.e.* $\alpha = 0$) one gets $\beta\kappa \simeq 1$. The normalised form of (4) is easily shown to be:

$$A_{\perp N} = \frac{1}{m}[\frac{\beta_\perp}{< P_z >}(P_x^2 + P_y^2) + \gamma_\perp < P_z > (x^2 + y^2) +$$

$$2\alpha_\perp(xP_x + yP_y) + 2(\beta_\perp\kappa - \mathcal{L})(xP_y - yP_x)] \quad (9)$$

This definition of amplitude is also implemented in ECALC9 and is the quantity used in the present analysis. It contains variables related to beam optics (and beam properties) and terms describing the kinematical properties of the muons. A determination of the amplitude for any muon in MICE is possible.

An ICOOL [3] simulation of muons passing through MICE has been used to assess the performance of the cooling channel. In order to make calculations more realistic, momentum resolution for reconstructed tracks has been parametrized according to [4]. In what follows only transverse normalized emittances (and amplitudes) will be considered. Initial beams are generated with different transverse emittances and a gaussian distribution for $P_z = 207$ MeV/c with $\sigma_{P_z} = 20$ MeV/c.

TRANSMISSION

One of MICE goals is informing future projects about the transmission capabilities of the cooling channel. Stating the fraction of surviving muons as a function of amplitude can provide a more powerful insight of channel performance. A measurement can be made with empty absorbers and a small amount of RF, in order to recover the small momentum loss through the thin absorber windows. In this study windows have been removed (for sake of simplicity) with no need for RF. The result is illustrated in fig. 2 where a beam of 20 mm rad initial emittance is considered.

Amplitude is computed for every muon in the first spectrometer and this value is used downstream where only surviving muons are counted. The ratio of the two plots is the specific transmission of the channel (bottom plot).

COOLING

Muon amplitude as defined by (9) allows to look at cooling in a different way. Fig. 3 shows a scatter plot of muon

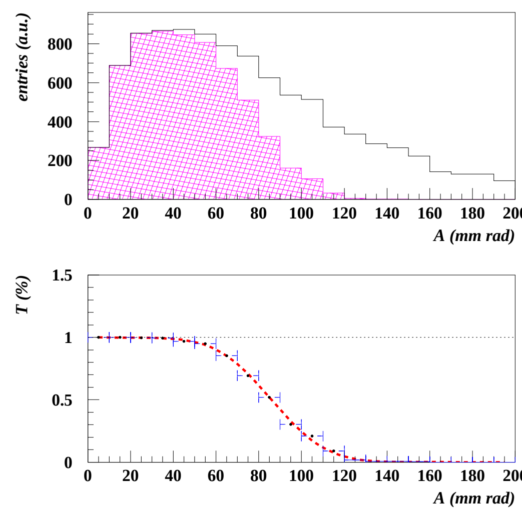

Figure 2: (top) amplitude distributions for muons in the first spectrometer (solid line) and muons surviving after passing the cooling section (hatched histogram). The ratio of the two distributions is the specific transmission of the channel.

amplitudes, for a 6 mm rad beam, as measured in the downstream tracker versus the ones measured in the upstream device. Higher amplitudes are shifted below the dashed line (where points would lie in case of amplitude conservation) by about 13%, as expected from energy loss in liquid hydrogen. Another important feature is the heating at low amplitudes where the multiple scattering dominates on cooling, as it is shown in closer detail in the top magnified insertion.

Fig. 4 (top) shows a comparison of two amplitude distributions as taken before (solid line) and after (hatched plot) the MICE cooling section for a 20 mm rad beam. This approach evidences the distinctive feature of cooling, which reshapes initial distributions by depleting higher amplitudes enhancing phase space density at lower values. Albeit somewhat higher than typical values for MICE, this initial transverse emittance helps understand the power of the single particle analysis.

The effect of transmission is superimposed on cooling, depleting higher amplitudes muons by scraping them off due to the physical aperture of the channel.

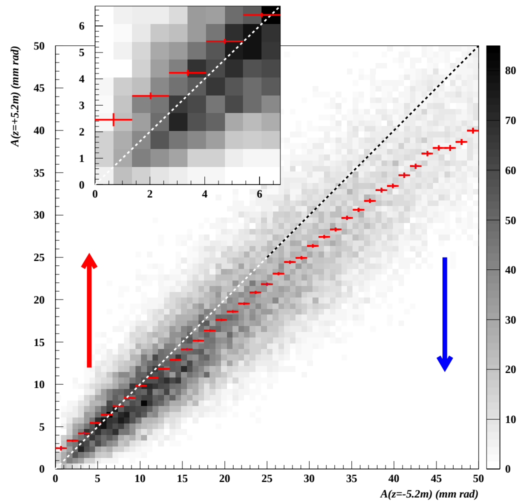

Figure 3: Cooling of an $\epsilon_N^T = 6$ mm rad muon beam. The scatter plot displays amplitudes evaluated after the channel in the downstream tracker (z=+5.2 m) versus the one computed in the first tracker (z=-5.2 m). Higher amplitudes are cooled down while smaller amplitudes are heated due to multiple scattering (see zoomed inset).

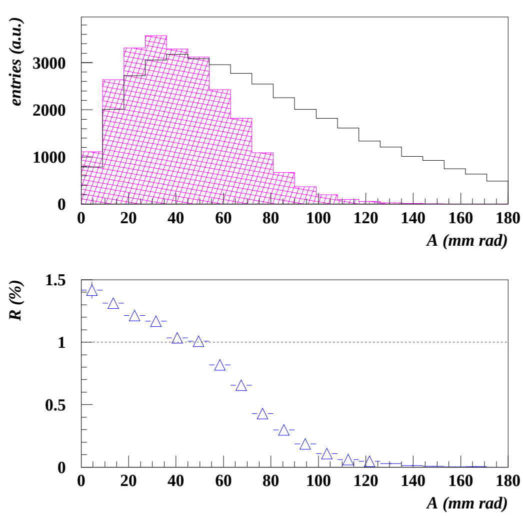

Figure 4: (top) amplitude distributions before (z=-5.2 m, solid line) and after (z=+5.2 m, hatched histogram) the cooling channel. The effect of cooling is clearly visibile as an increase of the lower amplitudes accompanied by a depletion of higher ones. (bottom) ratio of the above histograms.

Finally the ratio of the two distributions is calculated which gives a dramatic and unambiguos demonstration of cooling for any specific amplitude bin. This is illustrated in fig. 4 (bottom) for the aforementioned case.

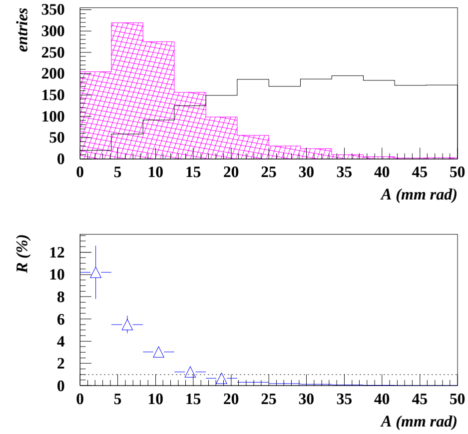

Figure 5: Cooling and transmission acting together on a 20 mm rad beam injected in a long channel a la Neutrino Factory (the meaning of the plots is the same as in fig. 4).

COOLING AND TRANSMISSION FOR A NEUTRINO FACTORY

The analysis shown so far can be used to demonstrate the cooling in a realistic system made of 16 MICE cells (corresponding to about 90 m). Fig. 5 illustrates how amplitudes are modified increasing the central phase space density by a factor of about 10.

CONCLUSIONS

A study of muon cooling and transmission within the MICE apparatus is shown which is based on the concept of single particle amplitude. Cooling appears as a re-shaping of a distribution, which contains more information than the bare all-beam emittance. Transmission is revisited too becoming a fraction related to the amplitude of every muon. This approach, which exploits the capabilities of MICE trackers, will help understand in finer detail the real performance of the first muon cooling channel.

REFERENCES

[1] J. S. Berg *et al.* "A Cost-effective design for a neutrino factory", Phys.Rev.ST Accel.Beams 9:011001,2006.

[2] G. Penn "Beam Envelope Equations in a Solenoidal Field", MUC-NOTE-COOL-THEORY-0071, January 2000.

[3] R. C. Fernow et al., "ICOOL Reference Manual v2.96", Brookhaven National Laboratory, 10 April 2006.

[4] A. Khan *et al.*, "MICE Scintillating Fibre Tracker Prototype", MICE-NOTE-DET-0090, 2 February 2005.

[5] R. Fernow, "Physics Analysis performed by ECALC9", MUC-NOTE-COOL-THEORY-280, September 2003.