COMPUTATIONAL REQUIREMENTS FOR GREEN'S FUNCTION BASED PHOTOCATHODE SOURCE SIMULATIONS*

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Abstract

We demonstrate the computational requirements for a Green's function based photocathode simulation code called IRPSS. In particular, we show the necessary conditions, e.g. eigenmode number and integration time-step, for accurately computing the space-charge fields in IRPSS to less than 1 % error. We also illustrate how numerical filtering methods can be applied to IRPSS in conjunction with a multislice approach, for dramatically improving computational efficiency of electromagnetic field calculations.

INTRODUCTION

The rf photoinjector is widely used as an electron source in many electron accelerators to generate high-brightness beams. Since the electron beam is emitted from the cathode with very low energy, the space-charge forces play an important role in beam dynamics. Therefore, it is necessary to accurately analyze the space-charge fields in the rf photoinjector. There a wide variety of simulation codes used for modeling photoinjectors. Examples of these are electrostatic codes, such as PARMELA [1], electromagnetic Lienard-Wiechert codes, such as TREDI [2], and electromagnetic Yee/PIC codes, such as MAFIA [3]. Amongst the electromagnetic codes, the Yee/PIC method is the most popular. Unfortunately, there exist non-negligible numerical errors in the Yee/PIC method, such as numerical grid dispersion and numerical Cherenkov radiation which can inhibit the simulation. [4] We have developed a fully electromagnetic simulation code called IRPSS (Indiana RF Photocathode Source Simulator) for computing the electromagnetic space-charge fields based on Green's function methods [5, 6, 7]. The Green's function method in IRPSS enables it to calculate electromagnetic space-charge fields to very high-accuracy ($< 1\%$ field error). In the present paper, we outline the numerical requirements on the Greens function method for achieving such high-accuracy. Our paper is organized as follows. In Sec. 2, we review the Greens function methodology used in IRPSS. In Sec. 3, we illustrate the numerical requirements on IRPSS by using simulation parameters corresponding to two photocathode source experiments: the BNL 2.856 GHz experiment [8] and ANL AWA 1.3 GHz experiment [9]. In Sec. 4, we give a summary of the paper.

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THEORETICAL ELECTROMAGNETIC FIELDS CALCULATIONS

The electromagnetic space-charge potentials with conductor boundary conditions can be solved using Green's functions. [6] For the cavity of arbitrary pipe cross-section with a flat cathode located at $z = 0$ the electromagnetic potentials are written as

$$\phi(\mathbf{r},t) = \frac{1}{\epsilon_0}\int_{-\infty}^{t}\int G_\phi(\mathbf{r},t;\mathbf{r}',t')\rho(\mathbf{r}',t')d^3\mathbf{r}'dt'$$

$$A_z(\mathbf{r},t) = \mu_0\int_{-\infty}^{t}\int G_z(\mathbf{r},t;\mathbf{r}',t')J_z(\mathbf{r}',t')d^3\mathbf{r}'dt'$$

where $G_{\phi,z}$ are Green's functions constructed from the wave equations with the delta function sources, and ρ and J_z are beam charge and current densities. The Green's functions may have different forms for different geometries and sources. We assume in this paper, the beam distribution function is

$$\rho(\mathbf{r},t) = \sum_i \frac{2Q}{\pi r_{b,i}^2}\Theta(r_{b,i}-r)\left(1-\frac{r^2}{r_{b,i}^2}\right)\delta[z-z_i''(t)] \tag{1}$$

For a cylindrically symmetric half-pipe cavity, the Green's functions are given by

$$G_{\phi,z} = \frac{c}{2}\sum_n \psi_{mn}(\mathbf{r}_\perp)\psi_{mn}(\mathbf{r}'_\perp) \times\left[J_m\left(j_{mn}\frac{\lambda_-}{a}\right)\Theta(\lambda_-^2) \mp J_m\left(j_{mn}\frac{\lambda_+}{a}\right)\Theta(\lambda_+^2)\right]$$

where ψ_{mn} are transverse eigenfunctions of the Helmholtz equation and $\lambda_\pm = c^2(t-t')^2 - [z \pm z''(t')]^2$.

NUMERICAL CALCULATIONS OF THE ELECTROMAGNETIC FIELDS

In the previous paper, we demonsrated the accuracy of this method using an analytical benchmark. [6] The numerical calculations required are transverse eigenmode summations, a time integration, spatial integrations over three dimensional coordinates, and multisliced summations for the bunched beam. In order to accurately simulate the space-charge fields within less than 1% error, however, one needs to consider the computational requirements for these numerical implementations. For the given ρ in Eq. 1, one can perform spatial integration analytically leaving only the radial eigen mode summation and time integration for numerical evaluations.

We assume that the beam is moving through a prescribed trajectory, $z''(t)$, which is calculated in the presence of the external rf field. In these simulations, the current is only in the longitudinal direction.

Eigenmode Summations

We can estimate the required mode numbers from the expansion of the charge density of the beam, in which each Bessel function can be expressed in an asymptotic form of

$$J_0(j_{0n}r/a) \approx \sqrt{\frac{2a}{\pi j_{0n}r}}\cos\left(j_{0n}\frac{r}{a}-\frac{\pi}{4}\right)$$

where $j_{0n}r/a \gg 1$. In order to resolve the behavior of the electromagnetic fields in the transverse direction, the argument of the asymptotic form must be larger than at least one period. Since we are especially interested in the region around the beam, r can be replaced by the beam radius, r_b. Thus, we can get the inequality,

$$j_{0M}\frac{r_b}{a} \gg 2\pi, \qquad (2)$$

where M is the minimum required radial mode number. From this inequality relation, the appropriate transverse eigenmode numbers can be estimated.

In Fig. 1, the radial electric fields are plotted as a function of r for 100, 500, 1000, and 2000 eigenmodes using beam parameters corresponding to the BNL 2.856 GHz photocathode source experiment, with $r_b/a = 0.02433$, $ct/a = 0.25$, and $z/a = 0.225$. [8] The number of transverse eigenmodes necessary for accurately determining the fields is inversely proportional to the transverse size of the beam. In order to model the fields within 1% accuracy, Eq. 2 indicates that it is necessary to sum over at least 2000 modes.

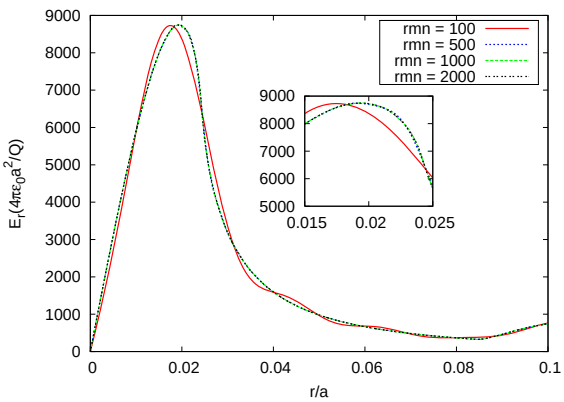

Figure 1: E_r vs. r/a for different eigenmode number.

Numerical Time Integrations

The formula for the space-charge fields using the Green's function method contain time integration factors, which include the trajectory history of the bunch. Due to the complexities of the integrands, however, it is necessary to integrate these numerically, and to consider the relation between the integration time step and the eigenmode number. Each field equation includes similar Bessel function type arguments: $J_0(j_{0n}\lambda_\pm/a)\theta(\lambda_\pm^2)$. The different signs correspond to the real(minus) and image(plus) charges. The step functions represent the causality conditions of the field, thus these factors enable rapid convergence of the numerical calculation, when we simulate the region in the front of the bunch.

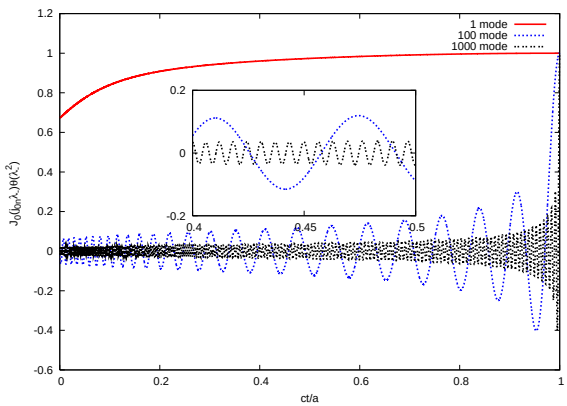

Figure 2: $J_0(j_{0n}\lambda_-)\Theta(\lambda_-^2)$ for 1, 100, 1000 eigenmode numbers.

Fig. 2 shows $J_0(j_{0n}\lambda_-/a)\theta(\lambda_-^2)$ as functions of t' for different transverse mode number, n, when t is fixed. As n increases, $J_0(j_{0n}\lambda_-/a)\theta(\lambda_-^2)$ oscillates with time, and these oscillations are smooth. But there are discontinuities at certain values of t and z, which are generated by the step function. These discontinuities generate numerical time integration errors, so that the time step, $\Delta t'$, must be small enough to reduce errors around these points. This imposes a maximum limit on the time step.

The time step must be chosen sufficiently small such that

$$j_{0n}\frac{c\Delta t'}{a} << 2\pi,$$

so that time integration over the Bessel functions are accurate. Using Eq. 2, we find

$$\Delta t' << \frac{0.01 r_b}{c}.$$

From Fig. 2, however, the oscillation time is very short for the large eigenmode number, thus we need much smaller time step.

For the E_z calculations, the longitudinal field strongly depends on the beam trajectory. Fig. 3 shows a plot of E_z vs. z/a along the axis of symmetry for different time steps. The oscillation periods are determined by the transverse eigenmode number, n, and the integration step size, $\Delta t'$. A smaller $\Delta t'$ reduces the amplitude of the oscillation. We can see that $c\Delta t'/a = 10^{-4}$ is not small enough

to resolve the fields, even though it is about factor of 200 times smaller than r_b/a.

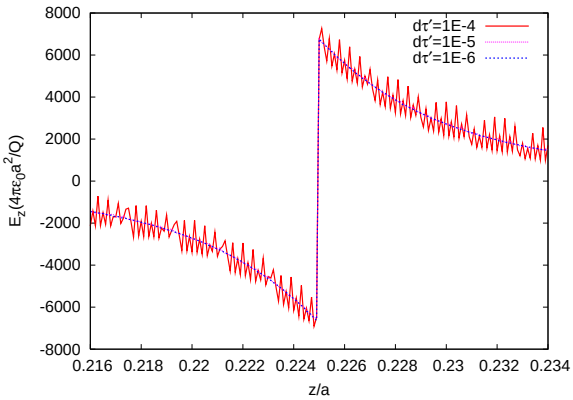

Figure 3: E_z vs. z/a for different numerical integration time steps: $cdt'/a = 10^{-4}, 10^{-5}$, and 10^{-6}.

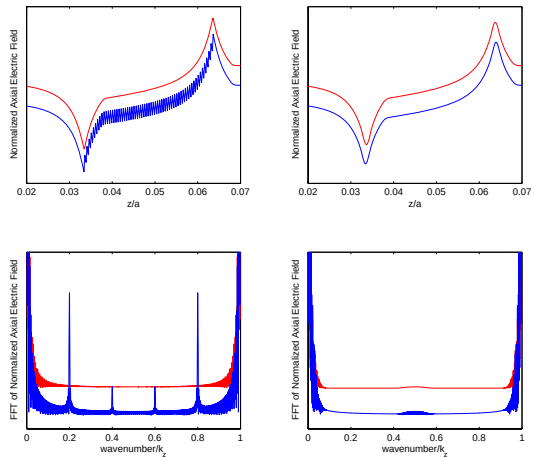

Figure 4: FFT and Filtering results for different values of slice spacing: $\Delta t'_s = 1 \times 10^{-4}$(upper) and $\Delta t'_s = 5 \times 10^{-4}$(lower).

Multislice Bunch Simulation

Since the bunched beam has finite size in the longitudinal direction, a single slice beam is not adequate for modeling the real beam. Thus the multi-sliced bunch model is introduced to make it realistic. [7] We set up uniformly spaced (in time), equally charged slices, which form one complete bunch. In this section, the ANL AWA rf photoinjector gun parameters with a bunch length of 9 ps are used. [9]

The required number of slices increases the computational time. In order to reduce the errors and computational time, we use a stopband filtering method for a small number of slices. Fig. 4 shows plots of E_z, Filtered E_z, and Fast Fourier Transforms of each, along the axis and the wavenumber, for different slice spacings. The filtering method is a promising technique for reducing errors in a multislice approach.

SUMMARY

In this work, we presented the numerical requirements to computing the electromagnetic space-charge fields with a Green's function method within less than 1% error. We estimated the required eigenmode number for a given charge distribution. For example, at least 2000 eigenmodes are needed to get accurate results for the BNL 2.856 GHz rf photoinjector. Numerical time integrations are conducted using the trajectory of the beam. The numerical simulation showed that the time integration step, $c\Delta t'$, must be much less than $0.01r_b$. To generalize the finite size beam, the multislice beam model was studied with the filtering method. For 9 ps bunch length of the AWA gun, the filtering method reduces the required number of slices. We plan to continue investigating improvements of code perfomance as well as simulating photocathode source experiments.

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