ONLINE NONLINEAR CHROMATICITY CORRECTION USING OFF-MOMENTUM TUNE RESPONSE MATRIX*

Y. Luo, W. Fischer, S. Tepikian, D. Trobjevic
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

In this article, we propose a method for the online nonlinear chromaticity correction at store in the Relativistic Heavy Ion Collider (RHIC). With 8 arc sextupole families in each RHIC ring, the nonlinear chromaticities can be minimized online by matching the off-momentum tunes onto the wanted tunes given by the linear chromaticities. The Newton method is used for this multi-dimensional nonlinear optimization, where the off-momentum tune response matrix with respect to sextupole strength changes is adopted. The off-momentum tune response matrix can be calculated with the online accelerator optics model or directly measured with the real beam. This is a report on the correction algorithm for the RHIC. Simulations are also carried out to verify the method. The preliminary results from the beam experiments, taken place in the RHIC 2007 Au run, are reviewed.

INTRODUCTION

There are a total of 144 arc sextupole magnets in each RHIC ring. In previous runs, 12 focusing sextupoles (SFs) and 12 defocusing sextupoles (SDs) in each of the 6 arcs were powered by one SF and one SD power supplies, respectively. All SF power supplies, and all SD power supplies were operated at the same current. This 2 sextupole family correction scheme does not allow nonlinear chromaticity correction.

In the RHIC 2007 run, the number of the arc sextupole power supplies were doubled, that is, from 12 to 24 in each RHIC ring. In 2005, Tepikian proposed a 8-family scheme for the purpose of nonlinear chromaticity correction [1, 2]. In this scheme, each outer or inner arc has 4 sextupole families, and all outer or inner arcs have the same sextupole family patterns.

To online correct the first and second order chromaticities, a simple, fast, and robust correction scheme is required. The online nonlinear chromaticity matching with MAD [3], or other codes based on the online accelerator optics model may be too slow for real operation. Furthermore, for the model-based correction, a better understanding of the linear optics and sextupole components in the arc dipoles are needed.

In this article, instead of directly correcting the second order chromaticities, we propose online matching off-momentum tunes onto their wanted values. The wanted off-momentum tunes are calculated only with the first order chromaticities only. By doing so, all higher order chromaticities will be minimized simultaneously. The off-momentum tune response matrix can be measured with a high resolution phase-locked loop (PLL) tune meter [4].

In the following, we first review the principle of the nonlinear chromaticity correction with off-momentum tune response matrix, followed by the simulation based on the RHIC polarized proton (pp) optics model. Its online implementing and the preliminary results from the beam experiments done in the RHIC 2007 Au run are presented and discussed.

PRINCIPLE

Algorithm

For simplicity, we give an example of how to correct the nonlinear chromaticity by matching 8 off-momentum tunes with the above 8 RHIC sextupole families. The optimization variables are 8 off-momentum tunes at 4 off-momenta,

$$\begin{aligned}\mathbf{q}^T &= (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8)^T \\ &= (q_x(-\delta_1), q_y(-\delta_1), q_x(-\delta_2), q_y(-\delta_2), \\ &\quad q_x(\delta_1), q_y(\delta_1), q_x(\delta_2), q_y(\delta_2))^T.\end{aligned} \tag{1}$$

The measured off-momentum tunes $\mathbf{q}_{meas}$ are

$$\begin{aligned}\mathbf{q}^T_{meas} &= (q_{1,meas}, q_{2,meas}, q_{3,meas}, q_{4,meas}, \\ &\quad q_{5,meas}, q_{6,meas}, q_{7,meas}, q_{8,meas})^T.\end{aligned} \tag{2}$$

Given the wanted first order chromaticities $\xi'_{x,y}$, the wanted off-momentum tunes $\mathbf{q}_{want}$ are

$$\begin{aligned}\mathbf{q}^T_{want} &= (q_{1,want}, q_{2,want}, q_{3,want}, q_{4,want}, \\ &\quad q_{5,want}, q_{6,want}, q_{7,want}, q_{8,want})^T \\ &= (-\xi'_x\delta_1, -\xi'_y\delta_1, -\xi'_x\delta_2, -\xi'_y\delta_2, \\ &\quad \xi'_x\delta_1, \xi'_y\delta_1, \xi'_x\delta_2, \xi'_y\delta_2)^T.\end{aligned} \tag{3}$$

To minimize nonlinear chromaticities, we match online the measured off-momentum tunes $\mathbf{q}_{meas}$ onto the wanted tunes $\mathbf{q}_{want}$. The distance of the measured off-momentum tunes to the wanted ones are

$$\begin{aligned}\Delta\mathbf{q}^T &= (dq_1, dq_2, dq_3, dq_4, dq_5, dq_6, dq_7, dq_8)^T \\ &= \mathbf{q}^T_{meas} - \mathbf{q}^T_{want}.\end{aligned} \tag{4}$$

The error function for the optimization is

$$\chi^2 = |\mathbf{q}^T_{meas} - \mathbf{q}^T_{want}|^2 = \sum_{i=1}^{8}(q_{i,meas} - q_{i,want})^2. \tag{5}$$

* This work was supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy and by a sponsored research grant from Renaissance Technologies Corporation.

In the following, we will use the Newton method to minimize the error function χ^2. In each iteration, we solve the following linear equation with the singular value decomposition (SVD) technique [5],

$$\mathbf{A}\Delta\mathbf{x} = -\Delta\mathbf{q}, \tag{6}$$

where $\Delta\mathbf{x}$ are the correction strengths,

$$\begin{aligned}\Delta\mathbf{x}^T \;=\;& (\Delta(k_2l)_1, \Delta(k_2l)_2, \Delta(k_2l)_3, \Delta(k_2l)_4,\\ & \Delta(k_2l)_5, \Delta(k_2l)_6, \Delta(k_2l)_7, \Delta(k_2l)_8)^T.\end{aligned} \tag{7}$$

$\mathbf{A}$ is the off-momentum tune response matrix with respect to sextupole family strength changes,

$$\mathbf{A}_{i,j} = \frac{\partial q_i}{\partial(k_2l)_j}, i = 1, 2, ..., nconstr, j = 1, 2, ..., nvari. \tag{8}$$

$\mathbf{A}$ is dependent on the choice of $\delta_{1,2}$. It can be numerically calculated with the offline accelerator optics model, or directly measured with the beam.

Simulation

Simulations are carried out to confirm the above nonlinear chromaticity correction algorithm. The store optics for the next polarized proton run at 100 GeV is used. The on-momentum tunes are set to $(Q_{x,0}, Q_{y,0}) = (28.685, 29.695)$. The β-functions at the collision points IP6 and IP8 are 0.9 m. At the other interaction points, the β-functions are set to 5m.

First, with a 2-family correction scheme, we match the first order chromaticities to $+2$. Then, we use the correction algorithm outlined above to match the 8 off-momentum tunes onto their wanted values, which are only calculated with the wanted first order chromaticities $\xi_{x,y} = 2$. In the simulation, we choose $\delta_1 = 0.0007$ and $\delta_2 = 0.0003$. The off-momentum tune response matrix is calculated numerically with the optics model and the Tracy-II code [6].

Tab. 1 lists the chromaticities up to second order in one simulation example. Fig. 1 shows the vertical off-momentum tunes in the simulation. From Table 1, with 2-family correction scheme, the vertical second order chromaticity $\xi_y^{''} = 2440$. After first correction, $\xi_y^{''}$ is reduced to 467. After the second correction, both $\xi_{x,y}^{''}$ are below 100. From the operation experience, the second order chromaticities below 500 are acceptable.

ONLINE IMPLEMENTATION

General Consideration

To online apply the above nonlinear chromaticity minimization, the off-momentum tune response matrix can be calculated from the model or measured with the beam. To measure the off-momentum tune response matrix, from the simulation, the resolution in the off-momentum tune measurement should be better than 10^{-4}.

Table 1: Chromaticities in the simulation.

	2-family	8-family (1 iteration)	8-family (2-iterations)
$\xi_x^{'}$	2.0	2.0	2.0
$\xi_y^{'}$	2.0	2.0	2.0
$\xi_x^{''}$	206	-641	-55
$\xi_y^{''}$	2440	467	24

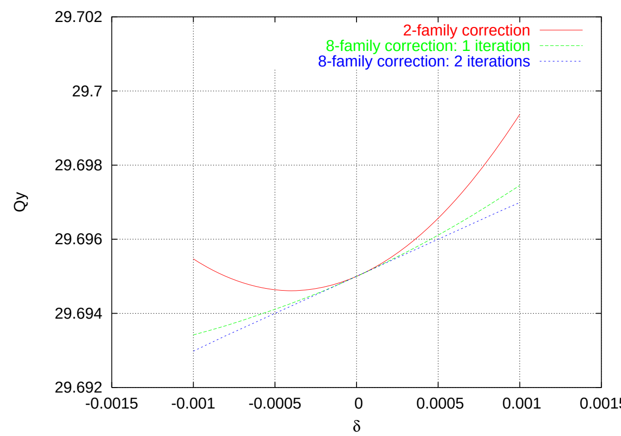

Figure 1: The vertical off-momentum tunes in the chromaticity correction simulation.

To mitigate the requirement to the tune measurement system, we can select to match off-momentum tunes at higher off momenta. And we also can only match the off-momentum tunes at $\pm\delta_1$, which will reduce the number of matching constraints from 8 to 4.

Since Eq. (5) is solved using the SVD technique, one must carefully choose the cut to zero the small singular values, which would yield large correction strengths. The maximum sextupole integral strength at 100 GeV is $(k_2dl)_{max} = 6.33\text{m}^{-2}$.

And in operation it is not required to measure the off-momentum tune response matrix each time when a nonlinear chromaticity correction is applied. We need to measure $\mathbf{A}$ only when the correction using the online optics model does not work well. Even during in the correction iterations, the same matrix $\mathbf{A}$ also can be used.

Beam Experiment

In the RHIC 2007 Au run, a dedicated beam experiment on the nonlinear chromaticity correction with the off-momentum tune response matrix had been set up. In the beam experiment, we measured the off-momentum tune response matrix, and tried nonlinear chromaticity correction with the off-momentum tune response matrix derived from the offline optics model. Before tune data taking, the global betatron coupling is required to be well corrected. Due to sextupole components in the arc main dipoles, the online model was difficult to predict the exact values of linear chromaticities. In Ref. [7], the average integrated sextupole strength in the arc dipoles could be derived with the measured first order chromaticity split at the injection and the store. Based on the updated offline optics with the average integrated sextupole strength in the arc dipoles, the second order chromaticities at the Blue store were well corrected. Before correction, $\xi''_{Blue,x,y} = (-760, -420)$. After correction, $\xi''_{Blue,x,y} = (160, -50)$.

Then we focused on the second order chromaticity correction at the Yellow store. At the Yellow store, the measured linear chromaticities were not well modeled. The discrepancies between the measurements and the predications from the ideal optics model couldn't not be interpreted only with the sextupole components in the arc dipoles. With the 2-family sextupole chromaticity correction scheme, the measured second order chromaticities at the Yellow store were about $\xi''_{Yellow,x,y} \simeq (80, 1600)$. Besides the correction methods with off-momentum tune response matrix, there were several other attempts to online correct the Yellow store second order chromaticities. At least two strength sets for the 8 sextupole families to give acceptable second order chromaticities had been found with other correction methods [8].

First, we measured the off-momentum tune response matrix and compared it to that from the optics model. The high resolution PLL tune meter was used to continuously track the betatron tunes. The off-momentum tunes were measured by offsetting the RF frequency. To maintain an acceptable beam time life, we chose off-momentum tunes at $\pm\delta_{1,2} = \pm 0.00162, \pm 0.00081$ to calculate the off-momentum tune response matrix. The integrated strength change in Eq. (8) for each sextupole family was chosen to $\delta(k_2 dl) = 0.02\text{m}^{-2}$.

During the measurement of off-momentum tune response matrix, changes in betatron tunes were noticed after offsetting single sextupole family's integrated strength. This is due to the feed-down effect from the sextupoles, which introduces extra tune shifts and betatron coupling. Therefore, before calculating the changes in the off-momentum tunes from a single sextupole family's strength change, we always took off the on-momentum tunes from the off-momentum tunes.

The measured off-momentum tune response matrix was compared to that from the optics model. Large discrepancies between them were noticed. And the measured off-momentum tune response matrix didn't repeat well among two measurements taken in two separate beam experiment sessions. The measurement of the off-momentum tune response matrix was limited by the resolution of the PLL tune meter and the feed-down effect of sextupoles.

Then, we tested nonlinear chromaticity correction based on the off-momentum tune response matrix derived from the offline Yellow ring store optics model. Table. 2 shows the measured chromaticities during 2 continuous corrections in one beam test. After each correction, changes in tunes and first order chromaticities were noticed. After the first correction, ξ''_y was reduced by about 850. And the second correction reduced it further by about 200. However, the horizontal second order chromaticities were increased during these corrections.

After analyzing the correction strengths in the above test, we found the signs of the correction strengths for the two sextupole SF families in the inner arcs were suspicious. From simulation, these two SF families with opposite signs will mainly affect the horizontal second order chromaticities. The failed correction in the second order horizontal chromaticity may be due to the wrong order of these two SF families in the simulation code, from which the applied off-momentum tune response matrix was calculated. We planned to knob these two SF families' strengths with opposite signs and to compare the changes in the second order chromaticities to that from the simulation code. Unfortunately, this was not done due to the limited beam experiment time.

Table 2: Chromaticities in the correction test.

	no correction	1st correction	2nd correction
ξ'_x	2.4	3.4	4.9
ξ'_y	2.0	1.7	1.1
ξ''_x	633	882	2245
ξ''_y	2121	1268	1048

CONCLUSION

We have proposed a method for the online nonlinear chromaticity correction using the off-momentum tune response matrix with respect to the sextupole familiy strengths and dedicated beam experiments were carried out in the RHIC 2007 Au run to verify this technique. We found it difficult to directly measure the off-momentum response matrix, due to the feed-down effect of sextupoles and the current tune measurement resolution. With the off-momentum tune response matrix derived from offline optics model, nonlinear chromaticity corrections were also tested. This technique has not been verified in the RHIC 2007 run.

REFERENCES

[1] W. Scandale, S. Tepikian, *Chromatic correction of RHIC when one or two insertions is at beta*=0.5m* , BNL RHIC/AP/45, 1994.

[2] S. Tepikian, private communications, 2005.

[3] MAD homepage, http://mad.home.cern.ch/mad/.

[4] P. Cameron, et al, PRST-AB, 122801, Dec., 2006.

[5] W. Press, S. Teukolsky, W. Vetterling and B. Flannery, *Numerical Recipes in C*, Second Edition, Cambridge University Press, 2002.

[6] J. Bengtsson, private communications, 2006.

[7] Y. Luo, A. Jain, W. Fischer, S. Tepikian, and D. Trbojevic, *Modeling RHIC linear chromaticity with sextupole components in the arc main dipoles*, BNL C-AD AP Note 276, May, 2007.

[8] N. Malitsky, private communications.

