TWO-BEAM RESISTIVE-WALL WAKE FIELD

F. Zimmermann, CERN, Geneva, Switzerland

Abstract

In storage-ring colliders, two beams propagating in opposite direction share a common beam pipe over parts or all of the ring circumference. The resistive-wall wake field coupling bunches of these two beams is different from the conventional single-beam wake field. The magnetic force and the longitudinal electric force experienced by a probe bunch invert their sign, while the transverse electric force does not. In addition, the distance between driving and probing bunches is not constant, but the net wake field must be obtained via an integration of the force experienced over the drive-probe distance. We here derive the two-beam resistive-wall wake field for a round beam pipe.

INTRODUCTION

The Large Hadron Collider, now under construction at CERN, will accelerate, store and collide two counter-rotating proton beams of 2808 bunches and with an average beam current of about 0.54 A each. The beams are injected at 450 GeV during about 25 minutes, and then accelerated to the nominal collision energy of 7 TeV per beam. The resistive-wall impedance of the cold and warm section, as well as of the more than 100 collimators is known to represent the largest source of impedance, which can render the beam unstable, especially at coupled-bunch modes of low frequency.

In the vicinity of the four interaction regions, where particle-physics detectors are located, the two LHC beams share a common beam pipe, which includes a number of tertiary collimators that are installed to protect the low-beta quadrupoles against beam loss. In this region the resistive-wall wake fields induced by one of the two beams can excite oscillations in the other beam. This type of resistive-wall wake field acting between bunches propagating in opposite direction could be important not only for the LHC but for all storage-ring colliders or even for the interaction regions of linear colliders, whenever the two colliding beams pass through a common region of beam pipe.

Longitudinal instabilities involving the coupling of two counter-propagating oppositely charged beams via wake fields were first analyzed by A. Renieri and C. Pellegrini at Frascati in 1974 [1]. Later, similar longitudinal and transverse two-beam instabilities were studied for LEP by C. Pellegrini [2] and J.M. Wang [3], respectively. The wake field these authors considered modeled the resonator impedance of a circular rf cavity symmetric in z. For such resonator, the transverse impedance describing the wake field excited by a particle of one beam and acting on a particle in the other beam has exactly the same magnitude

as for the wake field acting on a particle travelling in the same direction, while the sign of the two-beam coupling impedance can be either the same or opposite, depending on the cavity mode [3].

The case of the resistive-wall impedance is different. The longitudinal electric field excited by a bunch moving in the opposite direction changes sign with respect to that of a bunch moving in the same direction. In particular, as the two bunches move apart towards larger distances the field excited by the other bunch becomes decelerating instead of accelerating. The total wake-field effect on a "probe" particle (or "probe" bunch) must be obtained by integrating over the distance z from the "driving" particle (or "driving" bunch) in the opposite beam, instead of considering a constant value of z as is common for single-beam wake-field calculations. As a consequence of the change in sign for the longitudinal electric field, the coupled-beam longitudinal resistive-wall impedance also changes sign.

The transverse resistive-wall wake field acting between counter-propagating beams or particles has an electric and a magnetic component. The transverse electric force of the two-beam coupling wake stays the same as for the conventional single-beam wake field, but the magnetic Lorentz force inverts its sign due to the change of direction. To simplify our subsequent discussion, we will assume that both beams contain the same species of positively charged particles, as is the case of the LHC.

TWO-BEAM RW WAKE FIELD

To compute the two-beam transverse resistive-wall wake field, we follow the procedure of Chao [4] for the single-beam wake (which itself may go back to Morton, Neil and Sessler [5]), and solve the fields for the $m = 1$ dipole moment by Fourier transform

$$\left(\begin{array}{c} E_r(z_1) \\ B_\theta(z_1) \end{array} \right) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikz_1} \left(\begin{array}{c} \tilde{E}_r(k) \\ \tilde{B}_\theta(k) \end{array} \right) , \quad (1)$$

which yields (see [4], p. 52, for details of the calculation)

$$\tilde{E}_r = -\frac{ikA}{4}r^2 + \frac{1}{2}\left(-\frac{iA}{k} + B \right) , \tilde{B}_\theta = \tilde{E}_r + \frac{iA}{k} \quad (2)$$

where A and B are constants, and we have dropped the direct space-charge terms unrelated to the wall resistivity.

The transverse single-beam wake function $W_{\perp,\text{cl}}^{1 \to 1}$ [4] is proportional to $-(E_r - cB_\theta)$, namely

$$W^{1 \to 1} \propto \left(\tilde{E}_r - \tilde{B}_\theta \right) = -\frac{iA}{k} . \quad (3)$$

05 Beam Dynamics and Electromagnetic Fields

D04 Instabilities - Processes, Impedances, Countermeasures

For the two-beam wake, the magnetic Lorentz force changes sign and we have

$$W^{1\to2} \propto \left(\tilde{E}_r + \tilde{B}_\theta\right) = -\frac{ikA}{2}r^2 + B \approx B . \quad (4)$$

The coefficients A and B follow from field matching at the chamber surface. We now make the same approximations as Chao [4] and require that the skin depth is much shorter than both the chamber radius b and the thickness t of the beam pipe, or $|\lambda| \gg 1/b$ and $|\lambda| \gg 1/t$, where

$$\lambda \equiv \sqrt{\frac{2\pi\sigma|k|}{c}} \left[i + \text{sgn}(k)\right] . \quad (5)$$

With these approximations A and B become [4]

$$A = \frac{4qa}{b^3 \left(\frac{ikb}{2} - \frac{\lambda}{k} - \frac{i}{kb}\right)} ; \quad (6)$$

$$B = -\frac{\lambda}{k}bA = -\frac{4qa}{b^2 \left(\frac{ik^2b}{2\lambda} - 1 - \frac{i}{\lambda b}\right)} , \quad (7)$$

where q is the charge of the driving particle, and a its transverse displacement from the center of the beam pipe (or, in other words, the product qa equals the dipole moment driving the wake), using the notation of Chao [4].

Bane and Sands [6] neglected the last term in the denominator of (6), when they computed the extremely short-range wake field for a single beam at $z \leq s_0$. Chao's approximation for the typical range of interest, $\chi^{1/3}b \ll |z| \ll b/\chi$ (with $\chi \equiv c/(4\pi\sigma b)$), goes further and keeps only the single term λ/k in the denominator [4]. In the following we derive the two-beam resistive-wall wake field under an approximation similar to the one used by Chao for the single-beam wake.

Namely, the first term in the denominator of (7) can be dropped if $k^2 \ll 2/b^2$, i.e. roughly speaking for frequencies below the pipe cutoff. This term indeed is small for typical parameters and frequencies of interest. For example, considering an object of particular interest, an LHC graphite collimator, with a resistivity $\sigma = 9.0 \times 10^{14}$ s^{-1}, and a half gap $b = 1.5$ mm, at a frequency of 100 MHz, we find $k^2b/(2\lambda) \approx 4 \times 10^{-7}$ m^{-2}, which is much smaller than $1/(\lambda b) \approx 0.02$ m^{-2}. Only at very high frequencies, corresponding to the extremely short-range wake field, does the last term in the denominator dominate over the first. At 10 GHz $k^2b/(2\lambda) \approx 0.01$ m^{-2} is larger than $1/(\lambda b) \approx 0.002$ m^{-2}. It is worth noting here that the frequency spectrum of LHC bunches extends only up to $f_{\text{bunch}} \approx c/(2\pi\sigma_z) \approx 0.6$ GHz, with $\sigma_z \approx 7.5$ cm the nominal rms bunch length at 7 TeV beam energy.

Abbreviating $X \equiv \sqrt{2\pi\sigma/c}$, we therefore approximate (7) as

$$B \approx \frac{4qa}{b^2}\left(\frac{1}{1 + \frac{i}{\lambda b}}\right) = \frac{4qa}{b^2}\left(\frac{k^{1/2}}{k^{1/2} + \frac{1+i}{2bX}}\right) . \quad (8)$$

To perform calculations in the complex k plane we consider the function $w = k^{1/2}$, take k to be complex and

write it as $k = re^{i\theta}$ with positive real amplitude r. In an analogous way, we express w as $w = le^{i\phi}$ As the angle θ varies between 0 and 4π, the angle ϕ of the image varies between 0 and 2π. The w plane accommodates two Riemann sheets corresponding to the image of k. These two Riemann sheets are shown in Fig. 1, separated by branch cuts, together with their partner Riemann sheets in the original k plane. The latter are spread out over two complex planes. Also indicated is the single pole of the function B from (8), which is located in the w plane, at

$$w_{\text{pole}} = -\frac{1+i}{2bX} . \quad (9)$$

In the k coordinate only one pole exists. It is obtained by squaring w_{pole}, and located on the second k Riemann sheet.
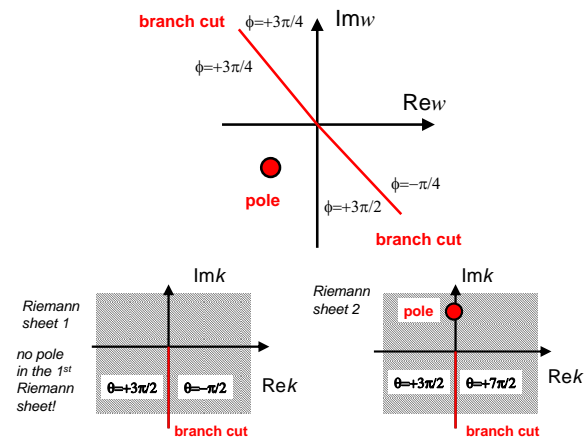

Figure 1: The complex w plane and the two k-plane Riemann sheets with branch cuts corresponding to the complex function $w = k^{1/2}$, and the pole of the function $k^{1/2}/(k^{1/2} + (1+i)/(2bX))$.

Since we found the coefficient B by analytical continuation of $\sqrt{|k|}(1+i)$, and replaced the latter with the general complex variable k, we need to operate in the first Riemann sheet of the k plane, limited by $-\pi/2 < \theta < 3\pi/2$. There are no poles on this Riemann sheet, as illustrated in Fig. 1, and the wake force is simply given by the integral over the right-hand and left-hand sides of the branch cut ('b.c.'). $F(z, r = 0)/e = \int_{-\infty}^{\infty} dk/(2\pi)e^{ikz}B = -\int_{\text{b.c.}} \frac{dk}{2\pi}\left(e^{ikz}B\right)$. Calculating this integral we obtain

$$\int_{\text{b.c.}} = \frac{4qa}{b^2}\int_0^\infty \frac{du}{2\pi}e^{uz}\left(\frac{u^{1/2}\frac{\sqrt{2}}{bX}}{u + \frac{1}{2b^2X^2}}\right)$$

$$= \frac{4\sqrt{2}qa}{b^3X}\frac{1}{2\pi}\left[\frac{\sqrt{\pi}}{\sqrt{|z|}}\right.$$

$$\left. + e^{z/(2b^2X^2)}\frac{\pi}{\sqrt{2}bX}\left(\text{Erf}\left(\frac{\sqrt{|z|}}{\sqrt{2}bX}\right) - 1\right)\right] .$$

The transverse wake function per unit length at a distance z from the driving particle is defined as

$$W_\perp(z) = -\frac{F(z, r=0)}{eqa} \ . \tag{10}$$

For large distances $|z| \gg \chi b$, the two-beam resistive-wall wake function becomes

$$W_{\perp,\text{cl}}^{1\to2} \approx \frac{2}{\pi b^3} \sqrt{\frac{c}{\sigma}} \frac{1}{\sqrt{|z|}} \ . \tag{11}$$

Except for the sign, this formula equals the corresponding expression for the classical single-beam resistive wake function in the range $|z| \gg \chi^{1/3}b$ (see [4], p. 59).

DISCUSSION

Since the longitudinal electric field also differs by the sign only, the equality of the transverse single-beam and two-beam wake fields, except for the sign, can be attributed to the Panofsky-Wenzel theorem [7], which relates the transverse and longitudinal wake fields as $\partial W_\perp/(\partial z) = \partial W_{||}/(\partial r)$, and which should hold for any force that can be derived from a Hamiltonian [8].

Though we found that the magnitude of the Green-function wake field for a fixed large distance is the same as the one for a single beam, there still remains an important difference when we want to use this Green-function wake field for calculating the impedance coupling between two bunches. For a single beam the total net wake effect is obtained by multiplying the wake field with the length L of the resistive beam pipe considered,

$$\bar{W}_\perp^{1\to1}(z) = W_\perp^{1\to1}(z)\, L \ , \tag{12}$$

where z refers to the distance e.g. between the source bunch and a later bunch moing in the same direction, which does not change. In the case of the two-beam wake field, we must integrate with respect to z, and compute expressions of the form

$$\bar{W}_\perp^{1\to2}(0) = \int_0^L W_\perp^{1\to2}(z)\, dz = \frac{4}{\pi b^3} \sqrt{\frac{cL}{\sigma}} \ . \tag{13}$$

The total net force acting during the passage of two bunches moving in opposite direction consists of the resistive-wall component and the direct long-range beam-beam force. Consider the situation that two proton beams are offset from the center of a round vacuum chamber, by $\pm a$, so that the beams are transversely separated by $2a$. The total beam-beam deflection experienced by a proton at the center of one beam after passing a bunch of the second beam and then traversing a common circular vacuum chamber of conductivity σ and length L, equals

$$\Delta p_\perp = -\frac{4aN_b e^2}{\pi b^3} \sqrt{\frac{L}{c\sigma}} + \frac{N_b e^2}{ca} \ , \tag{14}$$

where N_b denotes the number of protons per bunch, and e is the elementary charge. The second term represents the

long-range beam-beam force in free space. The positive sign of the two-beam Green function wake field (11) indicates that the wake force points towards the center of the chamber, i.e. the two-beam wake tends to counteract the direct long-range beam-beam force. The effect of the two forces cancels for

$$\frac{cL}{\sigma} \approx \left(\frac{b^6}{a^4}\right) \frac{\pi^2}{16} \ . \tag{15}$$

Assuming parameters typical of the LHC inner triplet — $b \approx 30$ mm, $L \approx 50$ m, $b/a \approx 5$ —, we obtain $\sigma \approx 4 \times 10^{10}$ s^{-1}, about seven orders of magnitude smaller than the conductivity of copper or aluminum. Considering instead approximate parameters for an LHC two-beam tertiary tungsten collimator, namely $b \approx 1.5$ mm, $b/a \approx 5$, and $L \approx 1$ m, the force cancellation occurs for $\sigma \approx 4 \times 10^{11}$ s^{-1}, which is still six orders of magnitude below the conductivity of tungsten.

CONCLUSIONS

We have derived the transverse two-beam resistive-wall wake function describing the coupling of two beams moving in opposite direction via the impedance of a round resistive vacuum chamber. The Green function wake field (11) equals that of a single beam but it has the opposite sign. The modification of the long-range beam-beam deflection by the resistive-wall wake field is small for typical LHC parameters. Larger effects would arise either for smaller beam pipes or for chamber walls of higher resistivity.

ACKNOWLEDGEMENTS

REFERENCES

[1] A. Renieri and C. Pellegrini, Report Adone T-64, Laboratori Nazionali di Frascati (1974).

[2] C. Pellegrini, CERN-LEP/TH/86-17 (1986)

[3] J.M. Wang, CERN-LEP/TH/87-65 (1987)

[4] A.W. Chao, Physics of Collective Instabilites in High Energy Accelerators, John Wiley, 1993.

[5] P.L. Morton, V.K. Neil, A.M. Sessler, J. Appl. Phys. 37, 10, 3875–3883, 1966.

[6] K.L.F. Bane, M. Sands, AIP Conf. Proc. 367: 131-149, 1996.

[7] W.K.H. Panofsky, W.A. Wenzel, Rev. Sci. Instrum. 27, 967 (1956).

[8] K. Oide, private communication (2007).

05 Beam Dynamics and Electromagnetic Fields

D04 Instabilities - Processes, Impedances, Countermeasures