THE FREE-ELECTRON LASER COLLECTIVE INSTABILITY AND THE DEVELOPMENT OF X-RAY FELS*

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Abstract

We discuss the physics of collective instabilities in particles beams, and consider one particular case: the free-electron laser (FEL) collective instability. We present a review of the main characteristics of this instability, and its application to the development of Self Amplified Spontaneous Emission (SASE) FELs from the infrared to the X-ray region.

INTRODUCTION

In this paper we discuss some aspects of the physics of particle beam collective instabilities and consider in more detail one example, the free-electron laser (FEL) collective instability. We review the properties of the FEL instability, and present some recent data obtained at wavelengths from the infrared to the ultraviolet. These results provide the theoretical and experimental support to design and build an FEL at about 1Å, with peak power of tens of GWatts and subpicosecond pulse length, as proposed recently by the LCLS and TESLA projects [1]. This paper is not a complete review of the subject, and it only mentions some of the initial contributions. A more complete list of references is found in the Proceedings on the International FEL Conference, published by Nuclear Instruments and Methods.

1. PARTICLE BEAMS AND COLLECTIVE INSTABILITIES

A particle beam is an ensemble of particles occupying a small volume in a 6-D position-momentum phase-space, $\vec{r}, \vec{p}$. To describe the beam we can introduce a central trajectory and a central momentum $\vec{r_0}(t), \vec{p_0}(t)$, and a phase-space distribution function $f(\vec{r}, \vec{p}, t)$. Since the beam occupies a small phase-space volume we can write the distribution function as

$$f(\vec{r}, \vec{p}, t) = F(\vec{r} - \vec{r}_0, \vec{p} - \vec{p_0}, t), \qquad (1)$$

where the F is different from zero only when $\vec{p} - \vec{p}_0 << \vec{p}_0, \vec{r}_T - \vec{r}_{T,0} << \sigma$, where a subscript T means perpendicular to the central momentum and σ is the transverse beam size.

If we neglect collisions and dissipative effects, the 6-D phase-space distribution F is constant (Liouville theorem):

Consider now a system consisting of two parts: a linear focusing transport channel, described by an Hamiltonian H_0, and another element described by H_1, which introduces an interaction between the particles in the beam.

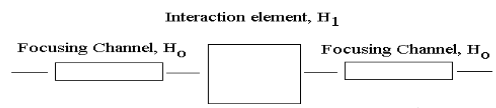

Figure 1: Transport channel and interaction element

Assume that when in the transport channel the beam is in an equilibrium state, described by a distribution function $F(H_0)$. When the beam enters the interaction element it is no more in equilibrium if $H_0 \neq H_1$, and the beam distribution function will evolve in time toward a new function of H_1. This evolution is a beam instability. How the beam evolves depends on the type and the strength of the interaction. The beam can evolve toward a state that can be useful to us, or that can lead to its destruction.

2. FEL COLLECTIVE INSTABILITY

Let us consider the situation when the element H_1 is an undulator magnet. A relativistic electron propagating along the undulator oscillates around the undulator axis and emits a nearly monochromatic radiation field. The radiation produced by one electron acts on other electrons changing their energy, introducing an interaction which depends on the undulator characteristics, the distance between the electrons, and the electron density. As a result the beam evolves toward a distribution with the electrons regularly spaced within the beam, with a separation about equal to the undulator radiation wavelength. The transition to the new state is characterized by the undulator length needed for this process to occur.

2.1 Radiation from one electron

The emission of undulator radiation from one electron is the fundamental process of a free-electron laser. We summarize here its main characteristics, considering the case of a helical undulator:

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a. Central wavelength:

$$\lambda = \frac{\lambda_W}{2\gamma^2}(1 + K^2 + \gamma^2\theta^2), \qquad (2)$$

where θ is the emission angle respect to the undulator axis, $K=eB_w \lambda_w/2\pi mc^2$ is the undulator parameter, λ_W and B_W are the undulator period and magnetic field, and $\gamma=E/mc^2$ is the electron beam central energy in rest mass units.

b. Line-width on axis (θ=0): $\Delta\lambda/\lambda=1/N_W$, where N_w is the number of undulator periods.

c. Coherent angle, corresponding to the emission of radiation within the line-width on axis: $\theta_C=(\lambda\lambda_W N_W)^{1/2}$.

d. Effective source radius corresponding to the emission of diffraction limited radiation: $a_C=(1/4\pi)(\lambda/\lambda_W N_W)^{1/2}$. Notice that $a_C\ \theta_C=\lambda/4\pi$.

e. Number of coherent photons, within the coherent solid angle $\Delta_C=\pi\theta_C^2$, and within the line width on axis:

$$N_C = \pi\alpha K^2/(1+K^2) \qquad (3)$$

where α is the fine structure constant.

Since the undulator parameter is typically of the order of 1, we have from (3) that the number of coherent photons per electron is of the order of 10^{-2}, a rather low efficiency.

2.2 Radiation from many electrons

In the case of an ensemble of electrons, the total radiation field is the sum of the fields generated by all electrons. These fields differ only by a phase factor depending on the initial position of the electron within the bunch, z_{n0}. The field can thus be written as

$$E = E_0 B_0 \equiv E_0 \sum_{n=1}^{N_e} \exp(2\pi i z_{n0}/\lambda), \qquad (4)$$

where E_0 is a common factor and the sum B_0 of the phase factors represents the superposition of the fields from all the electrons. The quantity B_0, is the "bunching factor" or beam order parameter. The intensity of the field is proportional to E^2, and thus to B_0^2.

Let us consider the simple case when the distribution of the electron longitudinal position does not change during the undulator crossing. We call this the "short undulator case", and we will clarify in the next section when this assumption is justified. For a short undulator we consider three cases:

I. Uniform beam current: $B_0=0$, I=0;Bunch length <λ: $B_0\sim N_e$, $I\sim N_e^2$, coherent radiation;

III. electrons from a thermoionic cathode or photo-cathode and bunch length >>λ; B_0 is a random number, with an average value $<B_0>=0$, and a mean square average $<|B_0|^2>=N_e$, $I\sim N_e$.

In the last case the radiation is produced by the beam "noise", the initial value of the bunching factor, and is what is called "spontaneous radiation". The total number of spontaneous radiation coherent photons is given, using (3), by $N_C = \pi\alpha N_e K^2/(1+K^2)$, or about 1% of the number of electrons.

2.3 The FEL collective instability

We can change the bunching factor and increase the number of coherent photons for case III using the interaction between electrons and the radiation. The mechanism is as follows.

I. Electrons, propagating through the undulator, interact with the electromagnetic field generated by other electrons. Since the electrons have a component of the velocity transverse to the undulator axis and parallel to the radiation electric field, there is an energy exchange and the electron energy is modulated on the scale of the radiation wavelength λ.

II. In the undulator magnetic field the trajectory of electrons with larger (smaller) energy is bent less (more); as a result the electrons are bunched on the scale of the wavelength, and the bunching factor increases;

III. A larger bunching means a larger electromagnetic field, and in turn a larger energy exchange and bunching. The bunching and the field amplitude grow exponentially. The exponential growth saturates when the bunching factor becomes of the order of N_e.

A mathematical analysis of this process gives a dispersion relation for the field and bunching factor, with real and complex roots. The imaginary part of the complex root gives the exponential growth rate along the undulator, the gain length L_G. The theoretical derivation of the existence of imaginary solutions of the FEL dispersion relation goes back to the late 70, early 80s [2]

For the exponential process to take place the undulator length, L_W, must be larger than the gain length, $L_W>>L_G$. The instabilty can start from an external electromagnetic field, in which case we call the system a High Gain FEL Amplifier, or it can start from B_0, the random bunching of the beam at the undulator entrance. In this case we call the system a Self Amplified Spontaneous Emission FEL (SASE-FEL). A 1D theory of a SASE-FEL including saturation was given in 1984 by R. Bonifacio, C. Pellegrini and L. Narducci [3]. This paper introduced the universal FEL parameter **ρ**, which gives all the basic of properties of a SASE-FEL.

The instability growth rate, or gain length, is given, in a 1-D model by[3]

$$L_G = \frac{\lambda_u}{4\sqrt{3}\pi\rho}, \qquad (6)$$

where ρ is given by

$$\rho = \{\frac{K}{4\gamma}\frac{\Omega_p}{\omega_W}\}^{2/3}, \qquad (7)$$

$\omega_W=2\pi\ c/\lambda_W$ is the undulator frequency, $\Omega_p=(4\pi c^2 r_e n_e/\gamma)^{1/2}$ is the beam plasma frequency, n_e is the electron density, and r_e is the classical electron radius.

For a long undulator, $L_W \gg L_G$, the intensity grows along the undulator as

$$I \sim \frac{I_0}{9}\exp(z/L_G), \qquad (8)$$

where I_0 is the spontaneous coherent undulator radiation intensity for an undulator with a length L_G, and is proportional to the square of the initial value of the bunching factor, $|B_0|^2$.

In a SASE-FEL saturation occurs after about 20 gain lengths, and the radiated intensity at saturation is $I_{sat}=\rho N_e E_{beam}$ [3] It follows from (6) that the saturation length is $L_S \sim 20L_G \sim \lambda_U/\rho$, and .that the number of periods needed in to reach saturation is $N_{US} \sim 1/\rho$. To keep this number at reasonable value we need ρ to be of the order of 10^{-3} or larger. This requirement puts restrictions on the beam density and energy, for any given wavelength.

The number of photons per electron at saturation is $N_{sat}=\rho E_{beam}/E_{ph}$. Let us consider as an example an X-ray FEL with $E_{ph} \sim 10^4$ eV, E~15GeV, $\rho \sim 10^{-3}$. We have $N_{sat} \sim 10^3$, i.e. an increase of almost 5 orders of magnitude in the number of photons per electron.

Another important characteristic of a SASE-FEL is related to causality, as shown in Fig.2. The radiation propagates faster than the electron (it moves forward, "slips" by λ per undulator period); thus electrons communicate only with the ones in front of them, at a distance not larger than the "slippage" distance $S=N_W\lambda$.

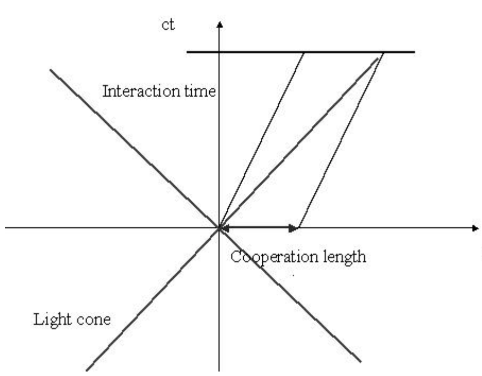

Figure 2 The difference in axial velocity between the electrons and the radiation defines the distance over which electrons in the bunch can interact, the cooperation length.

In a SASE-FEL one amplifies the initial noise, described by B_0, the bunching factor, which changes along the bunch on the scale of λ. The radiation from one group of electrons, interacting with the electrons in front of them, establishes a correlation, and smoothes out the random changes in the radiation intensity due to the random nature of B_0. This effect takes place over a distance of the order of the cooperation length [4], the slippage in one gain length, $L_C=\lambda/4\pi\rho$. . As a result the temporal distribution of the radiation intensity changes along the undulator as shown in Fig. 3, from an initial variation on the scale of λ, to a final variation on the scale of L_C. In the final state, near saturation, the temporal distribution of the radiation is in the form of spikes, and the number of "spikes" is given by N_S=bunch length/$2\pi L_C$. The intensity in each spike is a random quantity with a negative exponential distribution. For a long electron bunch, with many spikes, the rms relative intensity fluctuation is 1/(number of spikes $)^{1/2}$

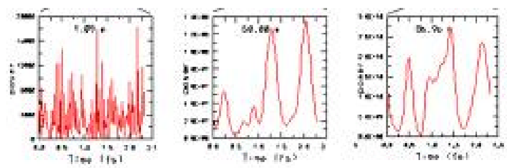

Figure 3 The temporal structure of the LCLS SASE-FEL near the beginning of the undulator, during the exponential growth and at saturation. (Courtesy H.-D. Nuhn)

This analysis applies when the bunch is much longer than the cooperation length, and $N_S \gg 1$. In the opposite limit of a short bunch, the dynamics of the process changes, as discussed in ref. [4]

The FEL instability can develop only if the undulator length is much larger than the gain length, and some other conditions are satisfied:

beam emittance smaller than the wavelength: $\varepsilon<\lambda/4\pi$; beam energy spread smaller than the FEL parameter: $\sigma_E<\rho$; gain length shorter than the radiation Raileigh range: $L_G<L_R$, where the Raileigh range is defined as $L_R=\pi w_0^2/\lambda$, and w_0 is the radiation rms beam radius. The first is a phase-space matching condition between the electron beam and the radiation field. The second condition requires that the wavelength of the emitted radiation be within the FEL gain bandwidth. The last condition describes the 3D effects of diffraction and optical guiding in an FEL first discussed by G. T. Moore [5], and by E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele [6].

The first and last conditions depend on the beam radius and the radiation wavelength, and are not independent. If they are satisfied we can use with good approximation the 1-D model. If they are not satisfied the gain length is larger than the 1-D value.

These conditions, together with the requirement that $\rho \sim 10^{-3}$, define the characteristics of the electron beam needed to drive the instability, and can be used to obtain a scaling law for SASE-FELs with wavelength [7]. The scaling law shows that as the wavelength is reduced the beam phase-space density must increase. Hence the electron accelerator driving the FEL must satisfy more stringent requirements. In particular collective effects in the accelerator, due to space-charge and wakefields, must be strictly controlled to avoid an increase of the beam phase density. Operating a SASE-FEL at short wavelength is a balancing act between controlling the unwanted wakefield induced collective effects in the accelerator, and letting the FEL collective instability develop in the undulator.

4. THE DEVELOPMENT OF SASE-FELS

The first proposal to use the FEL collective instability to produce IR radiation starting from noise was published by A.M. Kondratenko and E.L. Saldin [8], in 1980. The first proposal to use the instability for a soft X-ray FEL starting from noise was published by J.B. Murphy and C. Pellegrini [9] in 1985, using a bypass in a storage ring to provide the electron beam. At that time an electron storage ring was the accelerator delivering the electron beam, with the highest phase-space density. However the limitations on emittance, peak current and energy spread due storage ring collective effects, like the microwave instability or the Touschek effect, limited the shortest FEL wavelength to about a few hundred Å.

The first proposal to use the instability for a 1 Å X-ray FEL starting from noise and using the SLAC linac was published by C. Pellegrini [10] in 1992. Reaching 1 Å was made possible by the development of a novel electron source, the photoinjector, by J.S. Fraser, R.L. Sheffield, and E.R. Gray [11], which increased the beam phase-space density by two or more order of magnitudes..

While the theory of a SASE-FEL was mainly developed in 80s and 90s, an experimental verification had to wait the development of high quality electron beams in the late 90s. A demonstration [12] of exponential gain at 16μm, over about 4 gain lengths, in a 60cm long undulator was done in 1998.

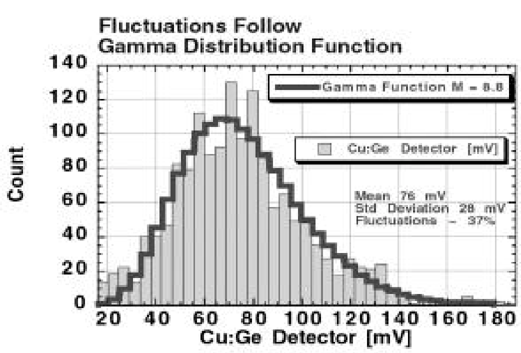

Figure 4 Intensity fluctuations in the UCLA-LANL-RRCKI-SSRL 12 μm SASE-FEL, showing the effect of the start-up from noise.

A much larger gain, $3x10^5$, and a demonstration of fluctuations and spikes, at 12μm, as shown in figure 4, was obtained using a 2m long undulator by a UCLA, Kurchatov, LANL, SSRL group [13]. A direct measurement of microbunching using coherent transition radiation was also done in the same experiment [14].

More recently a group at Argonne [15] has reached saturation in the LEUTL SASE-FEL at 530 and 320 nm. A DESY group [16], operating the TESLA SASE-FEL, has obtained large exponential gain down to of 80nm, the shortest wavelength obtained in an FEL. The VISA experiment [17], a BNL, SLAC, LLNL, UCLA collaboration, designed to obtain high gain using a beam with characteristics similar to those of LCLS has also reached saturation, as shown in figure 5.

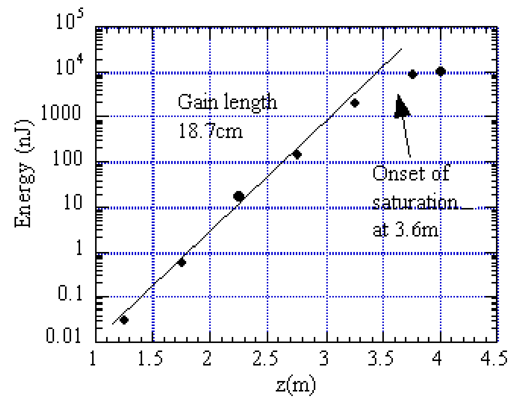

Figure 5 Exponential growth and saturation in the VISA experiment at 830nm, in a 4 m long undulator.

All these results give support to the LCLS and TESLA proposals [1] to build a 1 Å X-ray SASE-FEL, with peak power of tens of GWatts, subpicosecond pulse length, transversely coherent, diffraction limited radiation, with a line width < 0.001, a powerful tool to explore matter and fundamental physics.

The development of SASE-FELs shows that the collective instabilities of particle beams can be used to our advantage. The instability also offers an interesting example of self-organization of an ensemble of particles, leading to novel collective properties.

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6. REFERENCES

[1] LCLS Design Study Report, SLAC-R-521 (1998); TESLA Technical Design Report, DESY (2001).
[2] N.M. Kroll and McMullin, Phys.Rev.A17,300 (1977); A.M. Kondratenko and E.L. Saldin, Dokl. Aka. Nauk SSSR 249, 843 (1979); P. Sprangle and R.A Smith, Phys. Rev. A21, 293 (1980).
[3] R. Bonifacio, C. Pellegrini and L. Narducci, Optics Comm.50, 313 (1984)0
[4] R. Bonifacio et al., Phys. Rev. Lett. 73, 70 (1994)
[5] G. T. Moore, Nucl. Instr. and Meth. A239, 19(1985).
[6] E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele,Phys. Rev. Lett. 54, 1925 (1985)
[7] C. Pellegrini, Nucl. Instr. and Meth A272, 364 (1988).
[8] A.M. Kondratenko and E.L. Saldin, Part. Acc.10, 207 (1980).
[9] J.B. Murphy and C. Pellegrini, J. Opt. Soc. Am. B2, 259 (1985)

[10] C. Pellegrini, "A 4 to 0.1 nm FEL based on the SLAC Linac", in Proc. of the Workshop on IV Generation Light Sources, M.Cornacchia and H. Winick eds., SSRL-SLAC Report 92/02, p. 364 (1992)
[11] J.S. Fraser, R.L. Sheffield, and E.R. Gray, Nucl. Instr. and Meth. in Phys. Res., 250, 71, (1986).
[12] M.Hogan et al., Phys. Rev. Lett. 80, 289 (1998).
[13] M. Hogan et al. Phys. Rev. Lett. 81, 4897 (1998).
[14] A. Tremaine et al., Phys. Rev. Lett. 81, 5816 (1998).
[15] S. Milton and al., Science*xpress,* May 17, (2001).
[16] J. Rossbach et al., Phys. Rev. Lett. 85, 3825 (2000).
[17] A. Tremaine and al. Proc. of this Conference.