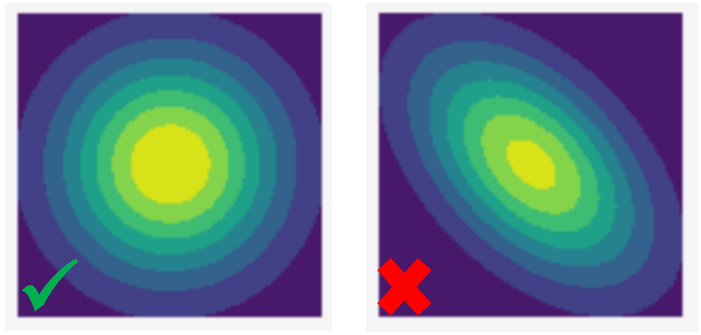
Characterization of Fully Coupled Linear Optics with Turn-by-Turn Data

Yongjun Li, Victor Smaluk, Robert Rainer | Brookhaven National Lab

NAPAC2022-WEPA74

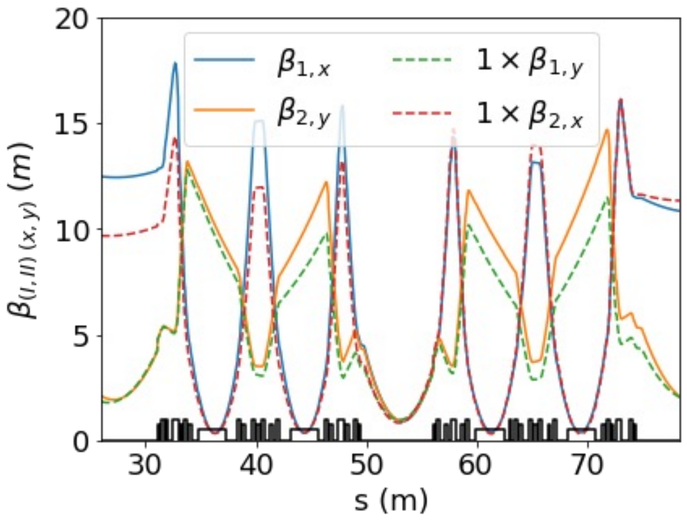
Round beam



Random error sources

- roll errors of normal quads
- closed orbit distortion

Randomness in linear optics



Asymmetric, random

Ripken Twiss

- Two modes, four sets of Twiss function
- Turn-by-turn data seen by BPMs

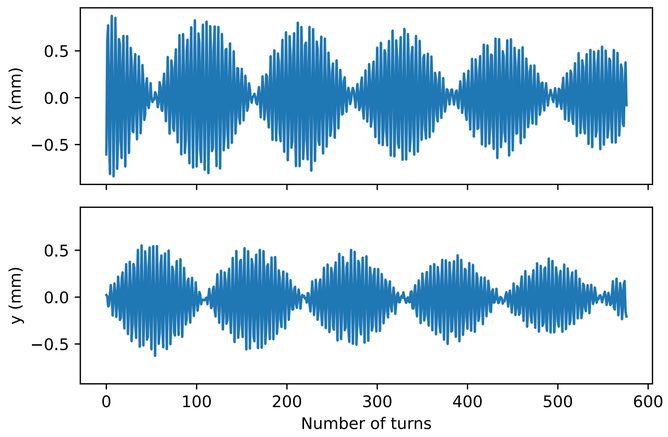
$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \sqrt{2J_1\beta_{1,x}}\cos(i\cdot 2\pi\nu_1+\phi_{1,x})+ \\ \sqrt{2J_2\beta_{2,x}}\cos(i\cdot 2\pi\nu_2+\phi_{2,x}) \\ \sqrt{2J_1\beta_{1,y}}\cos(i\cdot 2\pi\nu_1+\phi_{1,y})+ \\ \sqrt{2J_2\beta_{2,y}}\cos(i\cdot 2\pi\nu_2+\phi_{2,y}) \end{bmatrix} + \begin{bmatrix} x_{co,i} \\ \\ y_{co,i} \end{bmatrix}$$

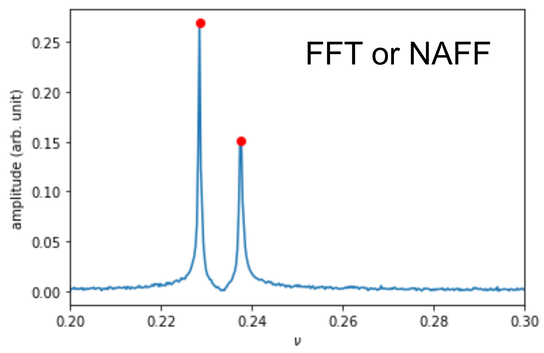
Harmonic analysis

- Extracting tunes, Twiss functions, and phase advances at BPMs

This research supported by DoE office of science under Contract No. DE-SC0012704.

Formulae





- Complex amplitude: cosine and sine

$$\begin{aligned} C_{(1,2),x} &= \sum_{i=1}^{N} x_i \cdot \cos(2\pi\nu_{1,2}i) \\ S_{(1,2),x} &= \sum_{i=1}^{N} x_i \cdot \sin(2\pi\nu_{1,2}i) \\ C_{(1,2),y} &= \sum_{i=1}^{N} y_i \cdot \cos(2\pi\nu_{1,2}i) \\ S_{(1,2),y} &= \sum_{i=1}^{N} y_i \cdot \sin(2\pi\nu_{1,2}i) \end{aligned}$$

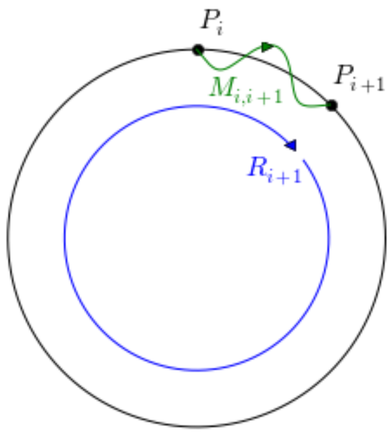
$$\begin{aligned} C_{(1,2),x} &\approx \frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),x}}\cos\phi_{1,2} \\ S_{(1,2),x} &\approx -\frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),x}}\sin\phi_{1,2} \\ C_{(1,2),y} &\approx \frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),y}}\cos\phi_{1,2} \\ S_{(1,2),y} &\approx -\frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),y}}\sin\phi_{1,2} \end{aligned}$$

- Twiss from complex amplitudes

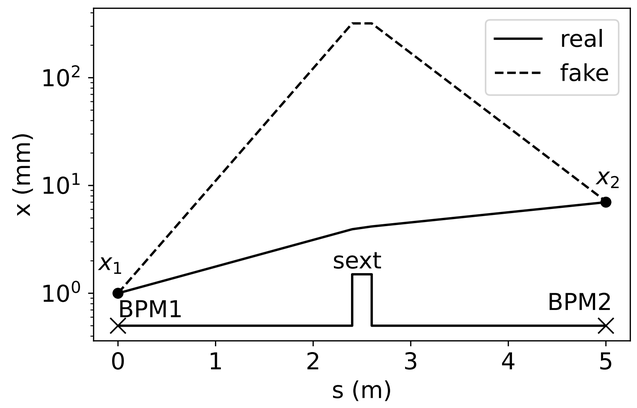
$$A_x = \sqrt{2J_{1,2}\beta_{(1,2),x}} = \frac{2}{N}\sqrt{C_{(1,2),x}^2 + S_{(1,2),x}^2}$$
$$A_y = \sqrt{2J_{1,2}\beta_{(1,2),y}} = \frac{2}{N}\sqrt{C_{(1,2),y}^2 + S_{(1,2),y}^2}$$

$$\phi_{(1,2),x} = -\tan^{-1}\frac{S_{(1,2),x}}{C_{(1,2),x}},\ \phi_{(1,2),y} = -\tan^{-1}\frac{S_{(1,2),y}}{C_{(1,2),y}}$$

- Absolute calibration at one location



One-turn matrix

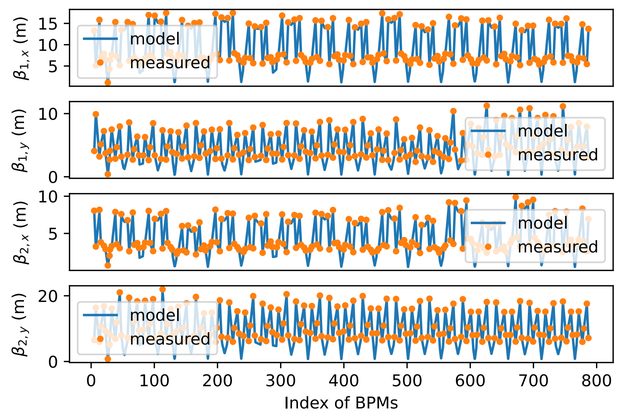


$$M = \begin{bmatrix} 0.11431 & 1.73718 & -0.07327 & -0.18993 \\ -0.55839 & 0.07913 & 0.01842 & 0.11278 \\ -0.03139 & 0.00295 & 0.09589 & 1.13780 \\ -0.08562 & 0.18194 & -0.86040 & -0.02086 \end{bmatrix}$$

- Ripken parameterization

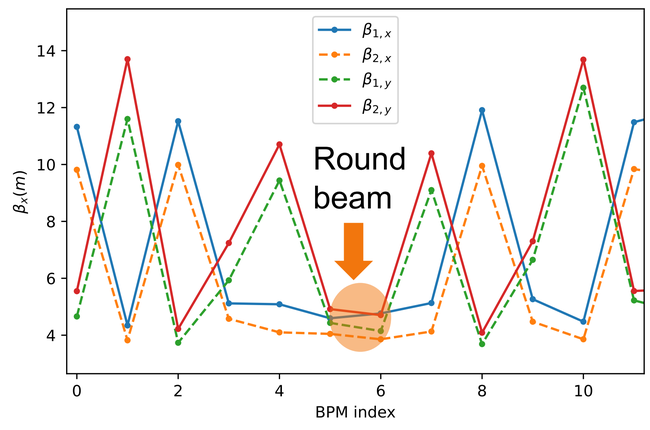
$$\nu_1 = 0.22455,\ \beta_{1,x} = 1.10m,\ \beta_{1,y} = 0.39m$$
$$\nu_2 = 0.25391,\ \beta_{2,x} = 0.67m,\ \beta_{2,y} = 0.75m$$

Simulation



RMS Errors 2~3 cm with BPM error

Beam experiment



Measured coupled Twiss for one supercell

Plan

Re-constructing lattice model, fitting quadrupole roll errors