Longitudinal Feedback Dynamics in Storage Rings With Small Synchrotron Tunes

Ryan Lindberg

Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60559

Argonne National Laboratory is a U.S. Department of Energy laboratory managed by U Chicago Argonne, LLC.

TUPA27

Abstract

We analyze the dynamics of multibunch longitudinal instabilities including bunch-by-bunch feedback under the assumption that the synchrotron tune is small. We find that increasing the feedback response does not always guarantee stability, even in the ideal case with no noise. As an example, we show that if the growth rate of a cavity-driven mode is of the order of the synchrotron frequency, then there are parameter regions for which the instability cannot be controlled by feedback irrespective of its gain. We verify these calculations with tracking simulations relevant to the APS-U, and find that the dynamics do not depend upon whether the longitudinal feedback relies on phase-sensing or energy-sensing technology. Hence, this choice should be dictated by measurement accuracy and noise considerations.

1. INTRODUCTION

To better understand the feedback performance for the planned APS-U, we simplify the analysis using the fact that synchrotron tune is very small $\lesssim 0.002$).

2. FEEDBACK MODELING

- We assume that the longitudinal feedback acts as follws:
  1. The pickup measures either the average rf phase $\langle\varphi\rangle$ or the mean energy energy deviation $\langle\delta\rangle$.
  2. The previous N turns of the pickup record is converted into an energy correction Δ_δ using a finite impulse response (FIR) filter.
  3. The longitudinal feedback cavity applies energy kick Δ_δ.
- The FIR filter coefficients give Δ_δ and the damping performance characterized by the transfer function $\mathcal{T}(\omega)$:

	Kick Δ_δ	Transfer function $\mathcal{T}(\omega)$
Phase detection based feedback:	$-\frac{3G}{\pi h\alpha_c}\sum_{p=0}^{N-1}\mathcal{C}_p\langle\varphi\rangle_{-p}$	$\sum_{p=0}^{N-1}\mathcal{C}_p e^{ip\omega T_0}$
Energy detection based feedback:	$-G\sum_{p=0}^{N-1}\mathcal{K}_p\langle\delta\rangle_{-p}$	$\sum_{p=0}^{N-1}\frac{i\omega T_0}{6}\mathcal{K}_p e^{ip\omega T_0}$

- Phase detection feedbacks should have zero DC component, $\sum\mathcal{C}_p = 0$, and act as a derivative, $\mathcal{T}(\omega)\propto i\omega T_0 + O(\omega^2 T_0^2)$.
- Two examples of phase detection FIR filters are

"Usual" differentiator FIR coefficients:[1]
$$\mathcal{C}_p = -\frac{\tan(\pi/N)}{3N}\sin\left[\frac{2\pi}{N}(p+1)\right]$$
Linear regression-based FIR coefficiants[2]
$$\mathcal{C}_p = -\frac{(N-1)-2p}{N(N^2-1)}$$

- We consider and energy detection-based scheme that simply takes the average energy deviation, so that $\mathcal{K}_p = 1/N$.

3. COMPARISON OF PHASE AND ENERGY DETECTION

The dispersion relation for multi-bunch instability when the wakefield varies slowly over the bunch length is

$$1 = \frac{4\pi\sigma_t^2}{\alpha_c T_0^2}\int d\mathcal{I}\,\bar{F}(\mathcal{I})\sum_{m=1}^{\infty}\frac{m^2\,|z_m(\mathcal{I})/\sigma_z|^2}{[\Omega/\omega(\mathcal{I})]^2 - m^2} \times\left[\frac{iI_{\text{tot}}T_0}{(\gamma mc^2/e)}\sum_{p=-\infty}^{\infty}\omega_{p,\mu}Z_\parallel(\omega_{p,\mu}) - \frac{6G}{\alpha_c}\mathcal{T}(\Omega)\right]$$

Factor involving the single particle equilibrium: $\frac{4\pi\sigma_t^2}{\alpha_c T_0^2}\int d\mathcal{I}\,\bar{F}(\mathcal{I})\sum_{m=1}^{\infty}\frac{m^2\,|z_m(\mathcal{I})/\sigma_z|^2}{[\Omega/\omega(\mathcal{I})]^2 - m^2}$

Impedance of long-range instability: $\frac{iI_{\text{tot}}T_0}{(\gamma mc^2/e)}\sum_{p=-\infty}^{\infty}\omega_{p,\mu}Z_\parallel(\omega_{p,\mu})$

$\bar{F}(\mathcal{I})$: Equilibrium distribution function of particle action $\mathcal{I}$.
$z_m(\mathcal{I})$: m^{th} Fourier component of the longitudinal coordinate z.
$\omega(\mathcal{I})$: Oscillation frequency in longitudinal potential.
$T_0 = 2\pi/\omega_0$: Revolution period.
α_c : Momentum compaction.
$\sigma_z = c\sigma_t$: RMS bunch length.
σ_δ : RMS energy spread.
I_{tot} : Total current.
γmc^2 : Beam energy.
$Z_\parallel(\omega_{p,\mu})$: Longitudinal impedance at $\omega_{p,\mu} = \omega_0(pM+\mu)+\Omega$.

- If we assume that the synchtron frequency and multibunch growth rate are $\ll$ than the inverse time over which the FIR record is kept:

$$\begin{array}{l}|\Omega|\,NT_0 \ll 1\\ \langle\omega\rangle NT_0 \ll 1\end{array} \Rightarrow \mathcal{T}(\Omega)\approx\frac{i\omega T_0}{6}$$ for all the FIR filters considered.

- The two phase detection schemes should have similar damping performance as the energy detection scheme.
- We verified this using tracking for the APS-U.

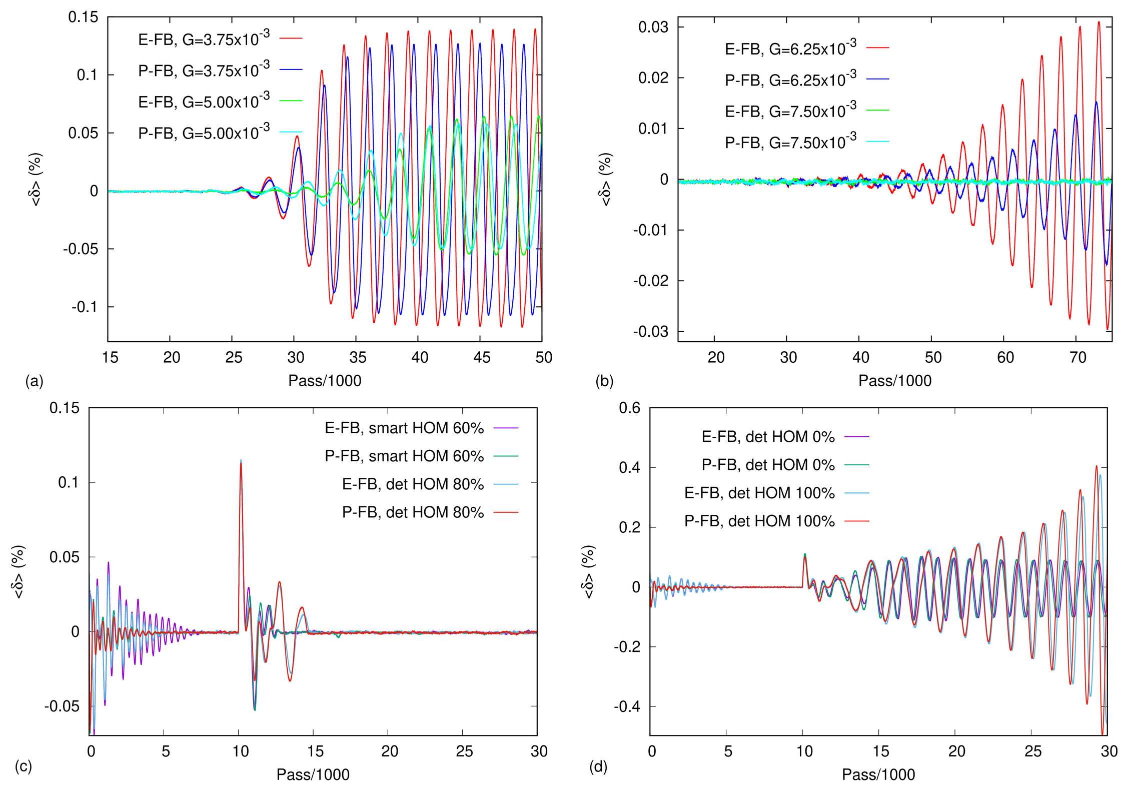

Figure 1: *Damping performance for longitudinal feedbacks using a pickup with either energy detection (E-FB) and phase detection (P-FB) obtained with* `elegant`*[3] tracking. (a) and (b) show the damping of a single cavity HOM for various levels of gain. (c)-(d) show stable and unstable dynamics when a single bunch is lost for various HOM configurations.*

4. FEEDBACK DYNAMICS

- We investigate the dynamics further by restricting out attention to an instability driven by a single cavity higher-order mode (HOM).
- The dispersion relation for a single HOM reduces to

$$1 = \int d\mathcal{I}\,4\pi\bar{F}(\mathcal{I})\sum_{m=1}^{\infty}\frac{m^2\,|z_m(\mathcal{I})/\sigma_z|^2}{[\Omega/\omega(\mathcal{I})]^2-m^2} \times\frac{\sigma_t}{\alpha_c\sigma_\delta}\left[2\Lambda\frac{i+\varpi}{1+\varpi^2}+\frac{6\sigma_t G}{\alpha_c\sigma_\delta T_0^2}\mathcal{T}(\Omega)\right],$$

where, for the HOM shunt impedance R_s quality factor Q, the maximum growth rate Λ and normalized detuning ϖ are

$$\Lambda = \frac{\sigma_t\omega_{\text{HOM}}I_{\text{tot}}R_s}{2\sigma_\delta(\gamma mc^2/e)T_0}, \qquad \varpi = \frac{2Q}{\omega_{\text{HOM}}}\left(\omega_{\text{HOM}} - p\omega_0 - \Omega\right).$$

- Now, we simplify the theory further by assuming
  1. Longitudinal rf potential is quadratic in z, so that
  $$2\pi\sigma_\delta\sigma_z\bar{F}(\mathcal{I})\propto e^{-\mathcal{I}/\sigma_z\sigma_\delta} \qquad z_{\pm1} = \sigma_z\sqrt{\mathcal{I}/2\sigma_z\sigma_\delta}$$
  2. HOM resonance is much broader than the complex mode frequency and synchrotron frequency, $\omega_{\text{HOM}}/2Q \gg |\Omega|,\ \omega_s$
  3. FIR feedback filter retains a small number of turns such that both $\omega_s T_0$ and $|\Omega T_0|$ are $\ll 1/N$
- Under these assumptions the complex frequency satisfies

$$\Omega^2 - \omega_s^2 = 2\omega_s\Lambda\frac{i+\varpi}{1+\varpi^2} - \frac{iG}{T_0}\Omega. \qquad (1)$$

- We have two simple limits:
  1. Synchrotron frequency much larger than $|\Omega|$ and G/T_0:
  $$\Omega_\pm \approx \pm\omega_s\left(1+\frac{\Lambda}{\omega_s}\frac{\varpi}{1+\varpi^2}\right) - i\left(\frac{G}{2T_0}-\frac{\Lambda}{1+\varpi^2}\right). \Rightarrow$$ Stable if $G > 2\Lambda T_0$.
  2. Gain is large such that $G \gg \omega_s T_0,\ |\Omega|\,T_0$
  $$\Omega_+ \approx \frac{2\omega_s\Lambda T_0}{G(1+\varpi^2)} - \frac{i\omega_s}{G}\left(\omega_s T_0 + \frac{2\Lambda T_0\varpi}{1+\varpi^2}\right) \Rightarrow$$ Beam is unstable if $-2\Lambda\varpi > \omega_s(1+\varpi^2)$ regardless of the feedback gain G

HOMs detuned by $-\omega_{\text{HOM}}/2Q$ are unstable when $\Lambda > \omega_s$.

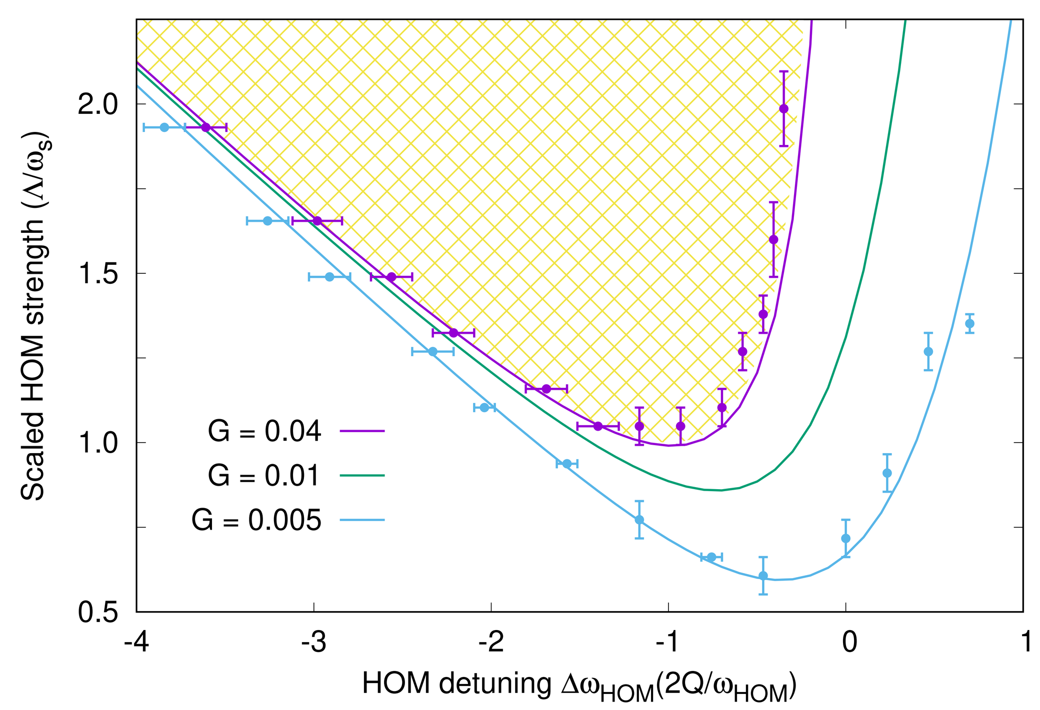

Figure 2: *Regions of multibunch stability as a function of the HOM detuning ϖ for a single rf system. Theory predicts that the regions below the purple, green and blue lines are stable for feedback gains labelled. The points plot results from* `elegant` *simulations that assume $G = 0.04$ and $G = 0.005$; the top/right or bottom/left of the "error bars" indicate parameters where tracking displays unstable or stable motion, respectively. The yellow cross-hatched region is predicted to be unstable for any feedback gain $G < 1$.*

5. DYNAMICS IN QUARTIC POTENTIAL

- The APS-U lengthens the bunch with a passive harmonic cavity.
- Our `elegant` tracking simulations have
  1. 48 bunches tracked for 100K turns through the APS-U lattice.
  2. Linear and lowest-order nonlinearities of the lattice simulated using the `ILMATRIX` element.
  3. Synchrotron radiation applied once per turn using the `SREFFECTS` element.
  4. An `RFMODE` element to simulate one cavity HOM with frequency near 921 MHz chosen to exite the $m = 29$ multibunch mode. Uses fixed $Q = 10.4\times10^4$ and variable R_s and detuning ϖ.
  5. Longitudinal feedback applied using paired `TFBPICKUP` and `TFBDRIVER` elements with $N = 10$ FIR filter.
  6. RF cavity parameters tuned such that $\sigma_t \approx 52$ ps.
- In the previous part we satisfied item 6 above by introducing fictious rf cavities at 39.1 MHz ($\omega_s/2\pi \approx 160$ Hz).
- Here, we include a passive rf cavity operating at the 4^{th} harmonic of the fundamental 352 MHz cavities.
- Quantitatively, the stability region depends upon the specifics of the longitudinal potential including the short-range impedance $Z_\parallel$.

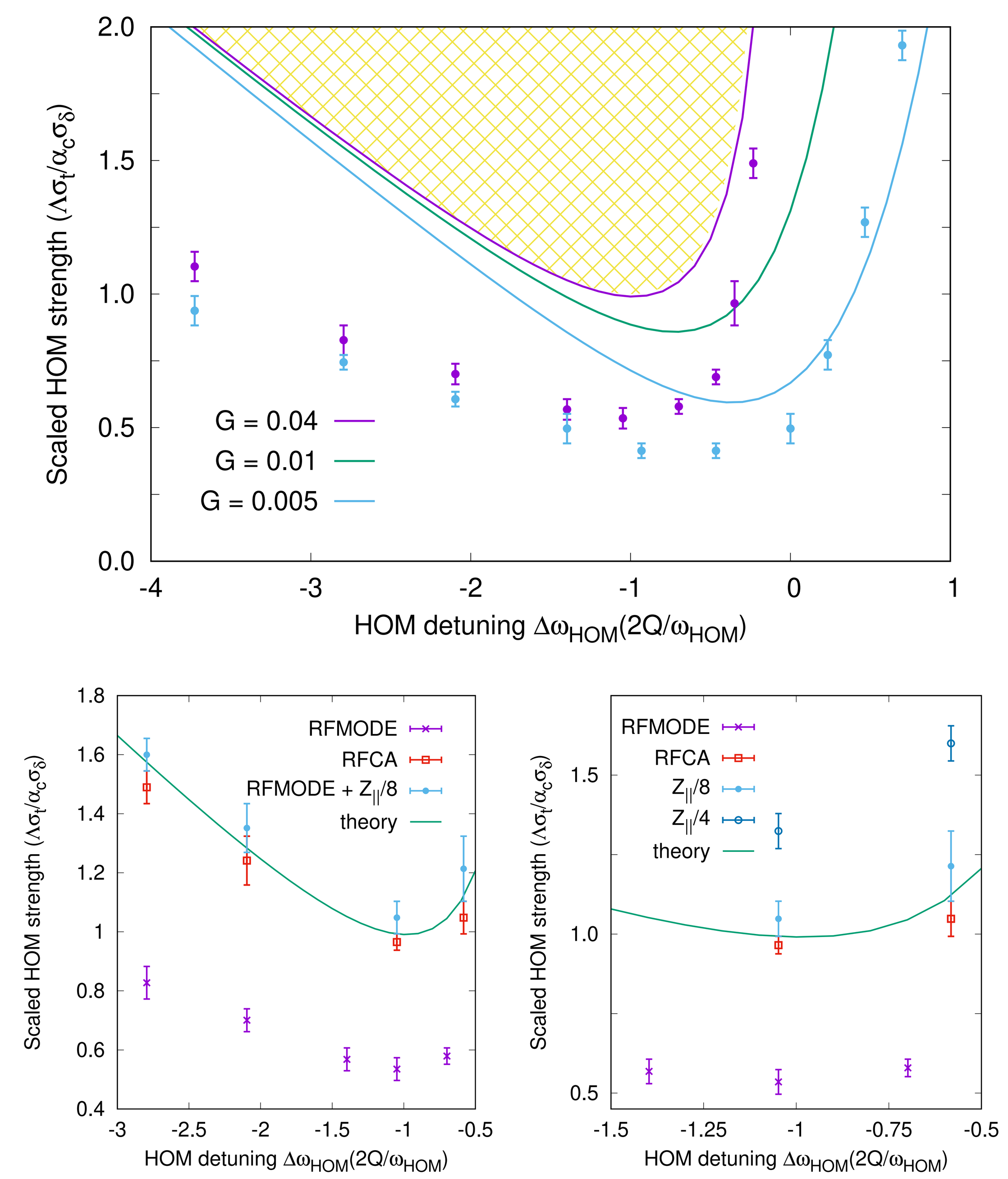

Figure 3: *Regions of multibunch stability for a flattened rf potential. Top: The theory for a flattened potential is indistiguishable from that in Fig. 2, while the simulation points include the APS-U's self-consistent double rf system with two* `RFMODE` *elements. The bottom panels plot results for a self-consistent double rf system with no impedance (purple), with a prescribed harmonic potential using* `RFCA` *elements (red), and with the ring $Z_\parallel/8$ (blue) or $Z_\parallel/4$ (green).*

Acknowledgments

We acknowledge useful discussions with Uli Wienands and Michael Borland from the APS/APS-U, and Marco Venturing from ALS/ALS-U.
Work supported by the U.S. Deptartment of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357.

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