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CHARACTERIZATION OF FULLY COUPLED LINEAR OPTICS WITH TURN-BY-TURN DATA*

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Abstract

In future diffraction-limited light source rings, generation of round beam through fully coupled linear optics may be preferable. When the machine tunes approach linear difference resonances, small random errors, such as quadrupole rolls, can result in the desired fully coupled optics. Consequently, some uncertainty exists in such a configuration due to random error distributions. With turn-by-turn data from beam position monitors, the harmonic analysis method is demonstrated for the coupled Ripken Twiss parameters characterization.

INTRODUCTION

For some beamline applications in light source communities, round beam is preferred in place of traditional flat beam. The increased vertical beam size also has the added benefit of increasing the lifetime which is particularly desirable for low emittance storage rings. Considering such benefits, future diffraction-limited light source facilities such as ALS-U [1] and APS-U [2], have plans to operate with a round beam mode. Most light source rings have only horizontal bending magnets, which leads to an intrinsically flat beam. The beam motion can be coupled transversely through either dedicated devices such as skew quadrupoles, or through intrinsic magnet imperfections such as quadrupole roll errors. Conventionally in electron machines, geometrically round beam is obtained by: (1) equally distributing the natural horizontal emittance into the horizontal and vertical planes $\epsilon_x = \epsilon_y$ by shifting the machine's tune close to a linear difference resonance $\nu_x - \nu_y - n = 0$, with n as an integer, (2) adjusting the envelope Twiss functions so that $\beta_x = \beta_y$ at the location of radiators. As achromat lattices are often used for light source rings, it is assumed that the radiators are located at non-dispersive sections.

In the presence of linear coupling, the uncoupled 2-dimensional Courant-Snyder parameterization [3] can be generalized to the 4-dimensional coupled case. One such parameterization was proposed by Ripken et. al [4, 5] and further developed by Lebedev and Bogacz [6]. For our application, we used the harmonic analysis method [7] to characterize the coupled Twiss parameters with turn by turn (TbT) data from Beam Position Monitors (BPM). Some other exact parameterizations were also available such as [8–11], which are equivalent to Ripken's parameters, but not used in our application.

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RIPKEN PARAMETERIZATION AND HARMONIC ANALYSIS

For a linearly coupled optics, after being kicked, beam performs a free betatron oscillation. The turn-by-turn data observed at the location a Beam Position Monitor (BPM) reads as,

$$\begin{bmatrix} x_i \\ \\ y_i \end{bmatrix} = \begin{bmatrix} \sqrt{2J_1\beta_{1,x}}\cos(i\cdot 2\pi\nu_1+\phi_{1,x})+ \\ \sqrt{2J_2\beta_{2,x}}\cos(i\cdot 2\pi\nu_2+\phi_{2,x}) \\ \sqrt{2J_1\beta_{1,y}}\cos(i\cdot 2\pi\nu_1+\phi_{1,y})+ \\ \sqrt{2J_2\beta_{2,y}}\cos(i\cdot 2\pi\nu_2+\phi_{2,y}) \end{bmatrix} + \begin{bmatrix} x_{co,i} \\ \\ y_{co,i} \end{bmatrix}, \quad (1)$$

here $J_{1,2}$ are the action variables associated with the initial amplitude, $\beta_{(1,2),(x,y)}$ are the envelope functions for the mode 1 or 2 at the horizontal x or vertical y planes respectively, i is the index of turns, $\phi_{(1,2),(x,y)}$ are the initial phases at the location of the BPM, $x_{co,i}$ and $y_{co,i}$ are the static closed orbit, and $\nu_{1,2}$ are the mode tunes. Depending on the stop-band width (SBW), they are separate with $|\nu_1 - \nu_2| \geq \Delta\nu_{SBW}$.

The analysis of two harmonics of $\nu_{1,2}$ can be implemented by computing their cosine and sine parts,

$$\begin{array}{rcl} C_{(1,2),x} &=& \sum_{i=1}^N x_i\cdot\cos(2\pi\nu_{1,2}i) \\ S_{(1,2),x} &=& \sum_{i=1}^N x_i\cdot\sin(2\pi\nu_{1,2}i) \\ C_{(1,2),y} &=& \sum_{i=1}^N y_i\cdot\cos(2\pi\nu_{1,2}i) \\ S_{(1,2),y} &=& \sum_{i=1}^N y_i\cdot\sin(2\pi\nu_{1,2}i) \end{array}. \quad (2)$$

Substituting Eq. (1) into Eq. (2), and considering a sufficient number of samples N and the orthogonality of trigonometric functions, Eq. (2) can be approximated as,

$$\begin{array}{rcl} C_{(1,2),x} &\approx& \frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),x}}\cos\phi_{1,2} \\ S_{(1,2),x} &\approx& -\frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),x}}\sin\phi_{1,2} \\ C_{(1,2),y} &\approx& \frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),y}}\cos\phi_{1,2} \\ S_{(1,2),y} &\approx& -\frac{N}{2}\sqrt{2J_{1,2}\beta_{(1,2),y}}\sin\phi_{1,2} \end{array}. \quad (3)$$

The amplitudes $A_{(1,2),(x,y)}$ of the betatron oscillation observed at each BPM can be obtained as,

$$\begin{array}{l} A_x = \sqrt{2J_{1,2}\beta_{(1,2),x}} = \frac{2}{N}\sqrt{C^2_{(1,2),x} + S^2_{(1,2),x}} \\ A_y = \sqrt{2J_{1,2}\beta_{(1,2),y}} = \frac{2}{N}\sqrt{C^2_{(1,2),y} + S^2_{(1,2),y}} \end{array}, \quad (4)$$

and the phases as well,

$$\phi_{(1,2),x} = -\tan^{-1}\frac{S_{(1,2),x}}{C_{(1,2),x}},\ \phi_{(1,2),y} = -\tan^{-1}\frac{S_{(1,2),y}}{C_{(1,2),y}}, \quad (5)$$

here the quadrant of the phase ϕ depends on the signs of C and S.

The amplitudes in Eq. (4) are mixed with the global actions $J_{1,2}$, and s-dependent $\beta_{(1,2),(x,y)}$. In order to extract

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the β, we need to calibrate J first. For a weakly-coupled optics, calibration can be done by scaling the measured amplitudes $A^2_{j,(x,y)}$ (with j the index of BPMs) to the design model $\beta_{j,(x,y)}$ level. However, when the coupling raises from random quadrupole roll errors, no accurate optics model is available. To solve this problem, we need to calibrate β_s at a specific location s and then extract the global action $J = \frac{A_s^2}{2\beta_s}$. Our strategy is to construct the one-turn matrix at s. Therefore, a pair of neighboring BPMs are chosen, and the linear transfer matrix in-between is assumed to be known in advance. Ideally with no magnetic elements in-between, the angle (momentum) coordinates are

$$x'_i = (x_{i,2} - x_{i,1})/L, y'_i = (y_{i,2} - y_{i,1})/L, \quad (6)$$

where $x_{i,(1,2)}$ and $y_{i,1,2}$ are the i^{th} turn horizontal and vertical positions measured at BPMs 1 (upstream) and 2 (downstream) respectively, and L is the distance in-between. With the turn-by-turn phase space coordinates $(x_i, x'_i, y_i, y'_i)^T$, their one-turn matrix M can be fitted out with the linear regression. From the coupled 4×4 matrix, the Ripken parameterization at these two BPMs can be implemented. Meanwhile, from the harmonic analysis Eq. (4), the amplitude $A_{(1,2),(x,y)}$ are already known, and therefore $J_{1,2}$ can be calibrated. The β functions can be obtained at the rest of BPMs.

If BPM pairs are not separated with drift spaces purely, but instead with nonlinear magnets, we can reconstruct the trajectories with the BPM reading by taking the magnets actual settings into account as illustrated in Fig. 1. However, some assumptions are needed in this case. It turns out that when the number of magnets is limited, and the excitation amplitude is relatively small (within the linear region), the errors introduced by the assumptions are controllable. For electron machines, the primary errors are the radiation damping and beam decoherence. The determinate of the one-turn matrix is observed off unity. Therefore it is no longer symplectic. Re-symplectification might be considered necessary before implementing the Ripken parameterization.

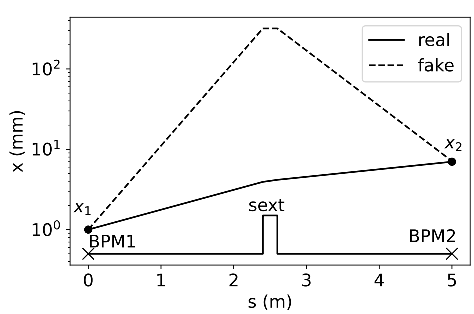

Figure 1: Reconstruction of trajectory with a sextupole located between two BPMs. Given only BPM readings, there may be more than one possible trajectory, but the paraxial trajectory is real.

SIMULATION

A set of simulated TbT data obtained with the ELEGANT code [12] was used to test this method in conjunction with the NSLS-II double bend achromat lattice. All 300 normal quadrupoles are randomly tilted with 1 mrad (RMS) angles, and the tune was shifted close to the difference resonance. The TbT data was collected at 182 BPMs, among them two BPMs are separated by a drift space $L = 2.798$ m. The TbT data observed at the upstream BPM is illustrated in Fig. 2, in which a random 12.5 µm reading error has been included. The one-turn-matrix at the upstream BPM was fitted,

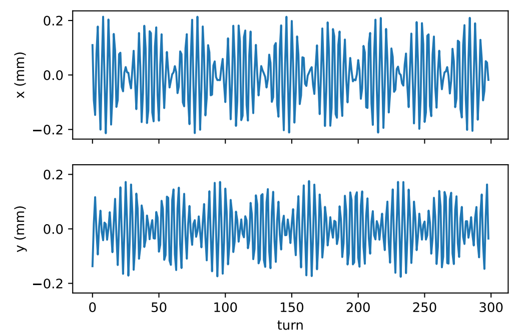

Figure 2: Simulated TbT data at one BPM.

$$M = \begin{bmatrix} 0.11431 & 1.73718 & -0.07327 & -0.18993 \\ -0.55839 & 0.07913 & 0.01842 & 0.11278 \\ -0.03139 & 0.00295 & 0.09589 & 1.13780 \\ -0.08562 & 0.18194 & -0.86040 & -0.02086 \end{bmatrix} \quad (7)$$

Note that the $det(M) = 0.95$, which means that the matrix is not exactly symplectic due to the added BPM reading errors. A symplectification was applied, however, and the difference of parameterization result was small. The self-consistency (see below) between the TbT spectrum and the tune extracted from the matrix was checked to determine if the symplectification was needed.

From Eq. (7), the tune fractional parts and coupled Twiss functions can obtained with the Ripken parameterization

$$\begin{matrix} \nu_1 = 0.22455, \beta_{1,x} = 1.10\,\text{m}, \beta_{1,y} = 0.39\,\text{m} \\ \nu_2 = 0.25391, \beta_{2,x} = 0.67\,\text{m}, \beta_{2,y} = 0.75\,\text{m} \end{matrix} . \quad (8)$$

At the location of upstream BPM, the betatron oscillation amplitude is $\sqrt{2J_1\beta_{1,x}} = 1.16 \times 10^{-4}$ m from the harmonic analysis. With $\beta_{1,x} = 1.10$ m from Eq. (8), the action J_1 is calibrated as 6.13×10^{-9} m. Another action $J_2 = 7.08 \times 10^{-9}$ is obtained in the same way. Then β at the rest 180 BPMs can be scaled with the actions as illustrated in Fig. 3. By comparing the Twiss parameters extracted from the simulated TbT data against the lattice model, the RMS errors are at a level of $2 \sim 3$ cm.

ONLINE MEASUREMENTS

Experimentation and testing of our method was carried out using the NSLS-II ring. First, an on-resonance round

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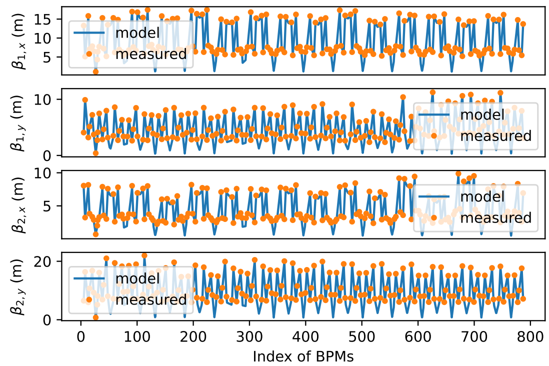

Figure 3: Comparison of the Ripken Twiss functions between the lattice model and the extraction from simulated TbT data.

beam lattice [13] was set up. The TbT data (Fig. 4) decayed due to the radiation damping and beam decoherence. Therefore, only 475 turns data (after being kicked) was used to characterize the Twiss parameters. The measured Ripken Twiss function for one supercell is shown in Fig. 5.

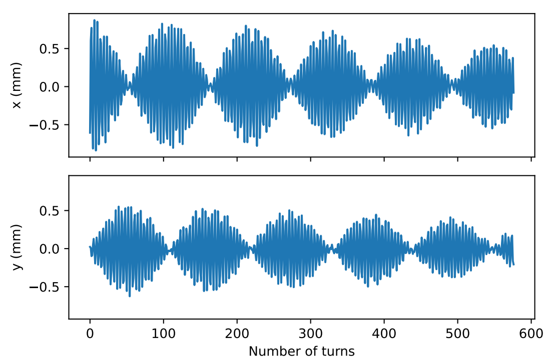

Figure 4: Measured TbT data at one BPM.

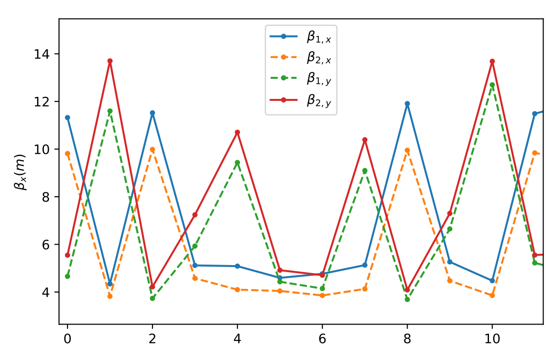

Figure 5: Measured Ripken Twiss parameters for one supercell (cell 30 and 1). Here the round beam was obtained at the short straight section between BPM 5 and 6.

We also controlled the stop-band width (SBW) using dedicated skew quadrupoles as was done at ALBA [14]. Before shifting the tune close to the resonance, the linear coupling can be increased (having wide SBWs) or decreased (having narrow SBWs). However, the narrow SBW resonance optics is more sensitive to errors, and two entangled modes are difficult to distinguish. The Twiss characterization is also not as accurate as the wide SBW case. From an operations point of view, maintaining a round beamwith a narrow SBW is difficult. Changes in undulator gaps or slow drifts of equipment such as power supplies can move the optics off the coupling and consequently change the beam sizes.

Besides the β-functions, with Eq. (5), the phase advance between neighboring BPMs was also measured since the TbT data collected from all BPMs are synchronized. The result is not shown here due to limited space.

SELF-CONSISTENCY CHECK

Some parameters, such as fractional tune, actions, can be used to check the self-consistency of the optics characterization. Beside computing the eigenvalues of a one-turn matrix, the fractional tune can also be obtained in several ways with TbT data, e.g, using direct FFT spectral analysis or Numerical Analysis of Fundamental Frequencies (NAFF) [15]. The results should be self-consistent no matter how we compute them. Actually NAFF provides not only frequencies, but their complex amplitudes, which can be used to measure β and phase as well.

The two actions $J_{1,2}$ are entangled with four $\beta_{(1,2),(x,y)}$ in TbT data. Although in each plane, we can calibrate them independently, they should be approximately the same. For example, J_1 can be extracted as

$$J_{1,x} = J_{1,y} = \frac{A_x^2}{2\beta_{1,x}} = \frac{A_y^2}{2\beta_{1,y}}, \tag{9}$$

so is J_2. In our online experiment, they are measured as $J_{1,y} = 7.136 \times 10^{-9}$m , $J_{1,x} = 7.201 \times 10^{-9}$ m, and $J_{2,y} = 5.971 \times 10^{-9}$m , $J_{2,x} = 5.814 \times 10^{-9}$ m. The discrepancies between them are about 1-2%. If multiple BPM pairs are used to construct one-turn matrix, the self-consistency can be cross-validated.

CONCLUSION

The harmonic analysis of TbT data has been applied to characterize strong coupled linear lattices. When the SBW is sufficiently wide ($\Delta\nu \geq 0.005$), its performance is reliable. When the SBW is narrow, however, the optics become sensitive to errors and the performance of the characterization needs improvement. The accuracy of measurement can be checked with the self-consistency between/among certain parameters. The harmonic analysis can also be used for some other parameterizations. For example, in Ref. [11], $\beta_{a,b}, \gamma, \bar{C}$-matrix elements and phase in Eqs. (54) and (55) can be analyzed similar to obtaining $\beta_{(1,2),(x,y)}$ etc.

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