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EFFECTS OF TRANSVERSE DEPENDENCE OF KICKS IN SIMULATIONS OF MICROBUNCHED ELECTRON COOLING*

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Abstract

Microbunched electron cooling (MBEC) is a cooling scheme in which a beam of hadrons to be cooled induces energy perturbations in a beam of electrons. These electron energy perturbations are amplified and turned into density modulations, which in turn provide energy kicks to the hadrons, tending to cool them. For simplification, previous work has modelled the electron-hadron interactions using a disc-disc model, assuming that the inter-particle kicks depend only on the longitudinal distances between individual hadrons and electrons. In reality, these kicks will also have a transverse dependence, which will impact the cooling process. We incorporate this transverse kick dependence into our simulations of the cooling process, allowing us to better understand the physics and provide improved design goals for the MBEC cooler for the Electron-Ion Collider.

INTRODUCTION

Microbunched electron cooling (MBEC) is a promising technique for cooling dense hadron beams in future colliders, and, in particular, is planned to be used in the Electron-Ion Collider (EIC) in order to counteract the effects of intrabeam scattering (IBS) [1]. This technique was first described in [2] and expanded upon in [3–6]. In short, the hadrons to be cooled co-propogate with a beam of electrons through a straight modulator section. This imprints an energy modulation on the electron beam which is correlated with the density of the hadron beam. One then separates the electrons and hadrons. The electrons are sent through a series of chicanes and straight sections which amplify their initial energy perturbations through the microbunching instability and turn them into density perturbations. Meanwhile, the hadrons pass through their own chicane, with their path length dependent on their energy and transverse offsets and angles in the modulator. The electrons and hadrons meet up again in a "kicker" straight. Through a proper choice of hadron optics, we can arrange for each hadron to receive an energy kick from the electron perturbation which it had induced which tends to reduce its initial energy offset and transverse actions.

Previous simulations [7–9] have focused on the simple 1D model of [3]. The electron/electron and electron/hadron interactions in the modulator and kicker are treated as those of 2D discs, with the force between the particles averaged over the transverse sizes of the beams and independent of their real transverse coordinates. In order to incorporate the effect of the transverse position of each hadron in the modulator and kicker on its received wake, we model the hadrons as point charges, but treat the electrons as discs. We first consider a simplified theory where the transverse kick to the hadron falls off as a Gaussian in its transverse coordinates in the kicker. We then extend the theory in [5] to compute the real energy kick between a point-like hadron and disc-like electron and incorporate this new wake into our multi-turn simulations of MBEC at the EIC.

SIMPLIFIED THEORY

As a first attempt to understand the effect of transverse offsets of the hadrons on the cooling dynamics, we consider the simplified model where the kick which an electron disc imparts to a hadron in the kicker is given by

$$\Delta\eta(x,z) = \sqrt{1+\Sigma_{h,x}^2/\Sigma_{e,x}^2}\, e^{-x^2/2\Sigma_{e,x}^2} A\Delta z \tag{1}$$

where x and z are respectively the horizontal and longitudinal offsets of the hadron relative to the electron disc, $A\Delta z$ is the linearized wake function between two discs, and the Σs are the hadron and electron horizontal beam sizes. A similar prefactor can be included to account for vertical position, but we ignore it here for simplicity.

The longitudinal cooling rate (defined so a negative sign implies cooling) is given by

$$\lambda_z = \frac{\langle \Delta(\eta^2)\rangle}{2T_{rev}\sigma_\eta^2} \approx \frac{\langle 2\eta\Delta\eta\rangle}{2T_{rev}\sigma_\eta^2} \tag{2}$$

where the factor of 2 in the denominator represents the fact that the longitudinal action is split evenly between the energy offset and longitudinal position offset, but we only change the former. We have also made the approximation that the energy kick is small relative to the original energy offset, so that we only keep the linear term.

For a Gaussian beam with horizontal emittance ϵ and fractional energy spread σ_η, we can do the relevant integrals analytically, arriving at

$$\begin{aligned}\lambda_z =& \frac{A}{T_{rev}(\Sigma_{x,h}^2+\Sigma_{x,e}^2)}[R_{56}(\Sigma_{x,e}^2+\epsilon\beta) \\ &+ R_{16}(D'\Sigma_{x,e}^2 + D'\epsilon\beta + D\epsilon\alpha) - R_{26}D\Sigma_{x,e}^2]\end{aligned} \tag{3}$$

where all the coordinates and optics parameters are evaluated at the *kicker* and we have made use of the expression for $\Delta\eta$ in Eq. (1) and the expression for longitudinal delay in terms of the kicker coordinates [5]

$$\Delta z = R_{16}x' - R_{26}x + R_{56}\eta \tag{4}$$

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The transverse cooling rate is given by

$$\lambda_x = \frac{\langle \Delta J \rangle}{T_{rev}\epsilon} \approx \frac{-\langle(\beta x'_\beta D' + \alpha x_\beta D' + \alpha x'_\beta D + \gamma x_\beta D)\Delta\eta\rangle}{T_{rev}\epsilon} \tag{5}$$

where x_β and x'_β are betatron coordinates at the kicker with dispersion subtracted out and again we keep this linear in $\Delta\eta$. We may once again use Eq. (1) and perform the average over a Gaussian bunch, arriving at

$$\lambda_x = \frac{A}{T_{rev}(\Sigma_{x,h}^2 + \Sigma_{x,e}^2)}[R_{56}D^2\sigma_\eta^2 - R_{16}(D'\Sigma_{x,e}^2 + D'\epsilon\beta + D\epsilon\alpha) + R_{26}D\Sigma_{x,e}^2] \tag{6}$$

In contrast to the case without transverse wake dependence [5], it is possible here to have transverse cooling with $D = D' = 0$ in the modulator. Although the introduction of a transverse dependence of the kick to the hadrons changes the redistribution of the cooling rate between the horizontal and longitudinal emittance from the case without such a dependence (recovered by taking $\Sigma_{e,x} \to \infty$), the summed cooling rate $\lambda_x + \lambda_z = \frac{AR_{56}}{T_{rev}}$ is unchanged.

The diffusion rate in both planes is proportional to the mean squared kick from all the other hadrons in the beam,

$$\lambda_D \propto n \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x)\Delta\eta^2(x,z)dzdx \tag{7}$$

where n is the longitudinal hadron density and $\rho(x)$ is the transverse distribution of hadrons (normalized to 1). Taking the case where $\Delta\eta(x,z) = \Delta\eta(z)\sqrt{1+\Sigma_{h,x}^2/\Sigma_{e,x}^2}e^{-x^2/2\Sigma_{e,x}^2}$ and $\rho(x) = \frac{1}{\sqrt{2\pi}\Sigma_{h,x}}e^{-x^2/2\Sigma_{h,x}^2}$, we find

$$\lambda_D \propto \frac{1+\Sigma_{h,x}^2/\Sigma_{e,x}^2}{\sqrt{1+2\Sigma_{h,x}^2/\Sigma_{e,x}^2}} n\int_{-\infty}^{\infty}\Delta\eta^2(z)dz \tag{8}$$

A similar prefactor will be added if one also considers averages over a Gaussian vertical beam distribution. We see then that diffusion will increase with the addition of a transverse dependence to the kick to the hadrons.

SIMULATION

Exact Theory

Appendix C of [5] details the procedure of computing the force in the rest frame of a disc of electrons on a disc of hadrons with arbitrary Gaussian distributions in the two transverse coordinates for the two beams:

$$F_z(z) = \frac{4Q_hQ_ez}{4\pi\epsilon_0\sqrt{\pi}}\int_0^\infty d\lambda\lambda^2 \times \frac{e^{-\lambda^2z^2}}{\sqrt{1+2\lambda^2\Sigma_{h,x}^2+2\lambda^2\Sigma_{e,x}^2}\sqrt{1+2\lambda^2\Sigma_{h,y}^2+2\lambda^2\Sigma_{e,y}^2}} \tag{9}$$

where the Σs are the RMS beam sizes in x and y for the hadron and electron beams, the Qs are the charges of the two discs, and z is the longitudinal position of the hadrons relative to the electrons.

If we follow the same procedure, but leave the hadron as a point charge and only average over the electron beam, we arrive at:

$$F_z(x,y,z) = \frac{4Q_hQ_ez}{4\pi\epsilon_0\sqrt{\pi}}\int_0^\infty d\lambda\lambda^2 \times \frac{e^{-\lambda^2z^2}e^{-\lambda^2x^2/(1+2\lambda^2\Sigma_{e,x}^2)}e^{-\lambda^2y^2/(1+2\lambda^2\Sigma_{e,y}^2)}}{\sqrt{1+2\lambda^2\Sigma_{e,x}^2}\sqrt{1+2\lambda^2\Sigma_{e,y}^2}} \tag{10}$$

where x, y, and z are the hadron coordinates relative to the center of the electron disc. This change in definition is then carried forward into the wake function using the same techniques as in [5]. Note that this affects the electron-hadron interaction in both the modulator and kicker.

Fast Interpolation

In order to run our multi-turn simulation with 100,000 macroparticles in a reasonable time, we need a fast way to obtain the wake function for a hadron with arbitrary offsets in the modulator and kicker. Rather than interpolating the full 5D generalized wake function (depending on the hadron's longitudinal shift as well as its transverse coordinates in both modulator and kicker), we perform scans of the longitudinal wake as a function of the hadron's 4 transverse offsets individually. We parameterize each wake using a modified version of the fit function in [10]:

$$w(z) = A\sin(\kappa z)e^{-z^2/2\lambda^2} \tag{11}$$

and interpolate A, κ, and λ as functions of the transverse hadron offset. We then assume that these are separable functions, so that, for example, $A(x_m, y_m, x_k, y_k) = A_0A_{xm}(x_m)A_{ym}(y_m)A_{xk}(x_k)A_{yk}(y_k)$, where A_0 is the value A takes for an on-axis hadron, the coordinates refer to the x and y offsets in modulator and kicker, and A_{xm}, etc, refer to fractional changes in the wake from the on-axis case. To account for the change in the position of the hadron as it moves through the modulator and kicker, which act like drifts for the hadrons, we average the fit parameters discussed above over the lengths of the modulator and kicker for each hadron at each step of our simulation, eg, $A(x_m, y_m, x_k, y_k) = A_0\langle A_{xm}(x_m)A_{ym}(y_m)\rangle_m\langle A_{xk}(x_k)A_{yk}(y_k)\rangle_k$, where the averages are over the hadron position in the modulator and kicker, respectively. The diffusive kick is fit in a similar way, but we only consider offsets in the kicker, since this is a whole-beam effect and so we already average over the whole hadron beam in the modulator.

As a check that the above gives reasonable results, we compare the output of these interpolated fit wakes to that of the "true" wake for hadrons of random transverse offsets and angles. Figure 1 shows good agreement for a typical case.

Results

Using the current meters presented in Table 1, we compute the cooling rates for a variety of cases, as shown in

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Table 1: Parameters for Longitudinal and Transverse Cooling

Parameter	Value
Lengths of the Modulator and Kicker (m)	55
Lengths of the Two Amplifier Straights (m)	27
Proton / Electron Energy (GeV)	275 / 0.15
Protons per Bunch	6.9×10^{10}
Proton Bunch Length (cm)	6
Proton Fractional Energy Spread	6.8×10^{-4}
Proton Emittance (x/y) (nm)	11.3 / 1
Horizontal / Vertical Proton Betas in Modulator (m)	26 / 20
Horizontal / Vertical Proton Dispersion in Modulator (m)	0.5 / 0
Horizontal / Vertical Proton Dispersion Derivative in Modulator	-0.040 / 0
Proton Horizontal/Vertical Phase Advance (rad)	4.0 / 4.8
Proton R56 between Centers of Modulator and Kicker (mm)	−1.25
Electron Bunch Charge (nC)	1
Electron Bunch Length (mm)	8
Electron Fractional Slice Energy Spread	1×10^{-4}
Electron Normalized Emittance (x/y) (mm-mrad)	2.8 / 2.8
Horizontal / Vertical Electron Betas in Modulator (m)	64 / 11
Horizontal / Vertical Electron Betas in Kicker (m)	16.5 / 2
Horizontal / Vertical Electron Betas in Amplifiers (m)	0.5 / 0.5
R56 in First Two Electron Chicanes (mm)	2.5
R56 in Third Electron Chicane (mm)	−6.3

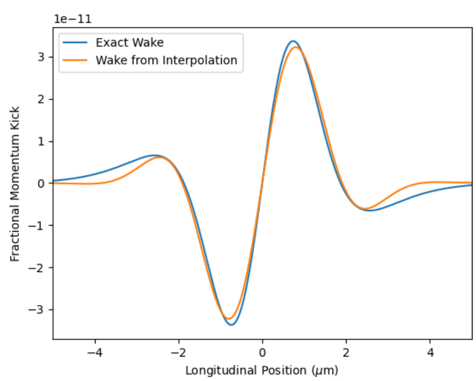

Figure 1: Comparison of the real wake for a hadron with (x, x', y, y') coordinates of (-76µm, -33µrad, -224µm, -3µrad) in the modulator and (715µm, 18µrad, 52µm, -11µrad) in the kicker with that obtained by interpolation of 1D fit functions. Good agreement is observed.

Table 2. (The hadron optics are symmetric between modulator and kicker.) The hadron (x, y) beam sizes are (0.64, 0.14)mm, while the electrons are (0.40, 0.14)mm for the (16.5, 2)m betas and (0.20, 0.20)mm for the (4, 4)m betas. We see that without diffusion, the addition of a transverse dependence to the wake has a minimal effect, in accordance with what we had found in the simple theory earlier. However, adding diffusion notably worsens the cooling rates, and this deterioration is more pronounced when the ratio of electron to hadron beam sizes is reduced. In order to keep the diffusion rate reasonable, we must not let the hadron beam size become too much larger than that of the electrons in the kicker. We also note in Eq. (10) that the electron-hadron interparticle force is large if the electron beam size is small. We therefore will need to seek a happy medium between these two constraints.

Table 2: Comparison of cooling times when modelling the hadrons as point charges and discs with and without diffusion.

Electron x / y Betas in Kicker (m)	x / z Cooling Times for Disc-Hadron Wakes (hr)	x / z Cooling Times for Point-Hadron Wakes (hr)
16.5 / 2	0.7 / 1.1	0.9 / 1.6
16.5 / 2 No Diffusion	0.5 / 0.8	0.5 / 0.9
4 / 4	0.7 / 1.1	1.1 / 2.0
4 / 4 No Diffusion	0.5 / 0.8	0.5 / 1.0

CONCLUSIONS

We have examined the effect of adding a transverse dependence to the electron-hadron interaction strength in MBEC in both theory and simulation. The raw cooling rate is not much affected by this addition, except to redistribute cooling between the longitudinal and transverse emittances. However, the diffusion is increased for hadron beam sizes large relative to those of the electrons. Future optimizations must keep this in mind.

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