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EMITTANCE GROWTH DUE TO RF PHASE NOISE IN CRAB CAVITIES*

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Abstract

The Electron-Ion Collider (EIC) incorporates beam crabbing to recover geometric luminosity loss from the nonzero crossing angle at the interaction point (IP). It is well-known that crab cavity imperfections can cause growth of colliding beam emittances, thus degrading collider performance. Here we report a particle tracking study to quantify these effects. Presently the study is focused on crab cavity RF phase noise. Simulations were carried out using Bmad. Dependence of emittance growth on phase noise level was obtained which could be used for developing crab cavity phase control specifications. We also benchmarked these simulations with theory.

INTRODUCTION

Crab crossing provides a head-on beam-beam collision for beams with a nonzero crossing angle. When a bunch passes through a crab cavity, as in an RF deflector, the phase is set so there is no kick on the longitudinal bunch centroid; only the head and tail of a bunch receive transverse kicks in opposite directions. Previous work [1] has shown that some imperfections of RF crab cavities could cause degradation of stored beams, leading to collider performance reduction.

One relevant imperfection is crab cavity RF phase noise, which can induce additional momentum kicks on passing particles and cause colliding beam emittance growth in the crabbing plane [2, 3]. Some key crabbing system parameters are the crab cavity synchronization and noise tolerances. Constraints on these tolerances come from the maximum acceptable emittance growth rate and maximum acceptable luminosity reduction. Previous studies found the transverse emittance growth is dominated by the crab cavity phase noise [2, 3], so it is critical to evaluate and quantify this growth to provide an important input to design specifications of the crab cavity controls.

In this paper, we focused our investigations and studies on the emittance growth due to the RF phase noise in crab cavities. In the first section, the simplified formulas are applied to estimate the value of emittance growth rate for a bunch passed through the crab cavities. The second section addresses what we observed in the numerical calculations and demonstrates the benchmarking work.

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ANALYTIC THEORY AND PREDICTIONS

P. Baudrenghien and T. Mastoridis developed a theoretical model for the growth of beam transverse emittance induced by crab cavity noises [2, 3]. They argued the growth is dominated by phase noises, which can be written as,

$$\frac{d\varepsilon_x}{dt} = \beta_{cc}\left(\frac{eV_0 f_c}{2E_b}\right)^2 C_{\Delta\phi}(\sigma_\phi) \cdot \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} S_{\Delta\phi}[(k \pm \nu_b)f_c]\rho(\nu)d\nu \tag{1}$$

where β_{cc} is the horizontal β function at the crab cavity location, V_0 is the crab cavity voltage amplitude, f_c is the beam circulation frequency, E_b is the beam energy, $\rho(\nu)$ is the betatron tune distribution, $S_{\Delta\phi}$ is the phase noise power spectral density (PSD), and ν_b is the horizontal betatron tune. σ_ϕ is the rms longitudinal bunch line density (in radians at the crab cavity frequency) and $C_{\Delta\phi}(\sigma_\phi)$ is a function describing the growth rate dependence on the bunch length:

$$C_{\Delta\phi}(\sigma_\phi) = e^{-\sigma_\phi^2}\left[I_0(\sigma_\phi^2) + 2\sum_{l=1}^{\infty} I_{2l}(\sigma_\phi^2)\right]. \tag{2}$$

$I_{2n}(x)$ is the modified Bessel function of the first kind. As the bunch length increases, the effect of phase noise on transverse emittance growth is reduced.

To simplify evaluation, we assumed that the betatron tune spread σ_{ν_b} is sufficiently narrow, since the power spectral density (PSD) is even symmetric, the effect of noise is independent of the actual tune distribution, then the following analytic formula was used for evaluating the emittance growth rate in our numerical predictions,

$$\frac{d\varepsilon_x}{dt} \approx 2\beta_{cc}\left(\frac{eV_0 f_c}{2E_b}\right)^2 S_{\Delta\phi}\left[\min_k |k \pm \nu_b| f_c\right] \tag{3}$$

For general practice, the filtered noise centered around the betatron frequency is also used on PSD. In our calculation, suppose that Ergodic theory works here for the introduced phase noise, considering a stationary Gaussian random process with zero mean, a white Gaussian noise was included in crab cavities, and $S_{\Delta\phi}$ is simply as

$$S_{\Delta\phi} = \frac{\sigma_{\Delta\phi}^2}{f_c}, \tag{4}$$

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here $\sigma_{\Delta\phi}$ is the phase noise, it needs to be controlled in order to avoid degradation of the beam emittance, particularly of the cooled ion beam, then the tolerance of crab RF phase noise for a ring can be evaluated as,

$$\sigma_{\Delta\phi} < \frac{2E_b}{eV_0 f_c}\sqrt{\frac{f_c}{2\beta_{cc}}\frac{d\varepsilon_x}{dt}} \quad . \qquad (5)$$

Then the numerical prediction for each turn is:

$$\frac{d\varepsilon_x^N}{dN} \approx \frac{p/mc}{2N_{cavities}\,\beta_x^*}\left(\frac{c\theta_{cc}}{\omega_{RF}}\right)^2 \sigma_{\Delta\phi}^2 \, . \qquad (6)$$

SIMULATION RESULTS

Simulations will provide tracking methodology and benchmarking with the analytical model. Because EIC ion collider ring design has not been completed yet, simulations up to this point were carried out for the ion collider ring based on an earlier version of Jefferson Lab Electron-Ion Collider (JLEIC) [4, 5] which features a crossing angle of 50 mrad leading to a Piwinski angle of 16.5 rad. Table 1 lists some JLEIC parameters those had been used in our simulations. Without compensation of the crossing angle at the physics program requirements, the well-developed software, Bmad, had been implemented for tracking particles, investigating the dynamic stability of the lattice and calculating all emittance in our simulations.

Table 1: JLEIC Parameters in Simulations

Beam		Proton
Beam Energy	(GeV)	100.0
Beam Circulation Frequency	(MHz)	0.139
RF Cavity Frequency	(MHz)	950.9
RF Cavity Voltage	(MV)	42.6
Crab Cavities Frequency	(MHz)	950.9
Crab Cavities Voltage	(MV)	20.8
Normalized Emittance ε_x^N	(µm rad)	0.35
Normalized Emittance ε_x^N	(µm rad)	0.07
$\beta_{\text{Horizontal}}$ at Crab Cavitiy β_{cc}	(m)	450.0
RMS Bunch Length σ_z	(cm)	1.0

For a particle with offset of centroid phase position $(\Delta x, \Delta z, \Delta(dp/p)) = (0.\ 38\text{mm}, 0.44\text{mm}, 0.0001)$, in two cases, we tracked a particle movement in the simulations: one simulation had crab cavities in the lattice, the other didn't. Figure 1 demonstrated the status of dynamic stability of the lattice design. As parts of our studies, we also investigated the fractional parts of frequency content of the betatron tune spectrum by tracking a particle with offset of centroid phase position at every turn. Figure 2 shows that when the crab cavities are on, additional harmonics appear in the spectrum The addition shift has been found as well in the tune distribution in detail. Physically, the emittance growth depends on the frequency domain overlap between the noise spectrum and the betatron tune distribution. For a bunch, because of particle's motion is coupled by the synchro-motion and betatron-motion, statistically, this tiny drift of the tune could finally affect the geometric emittances in both longitudinal part and transverse part.

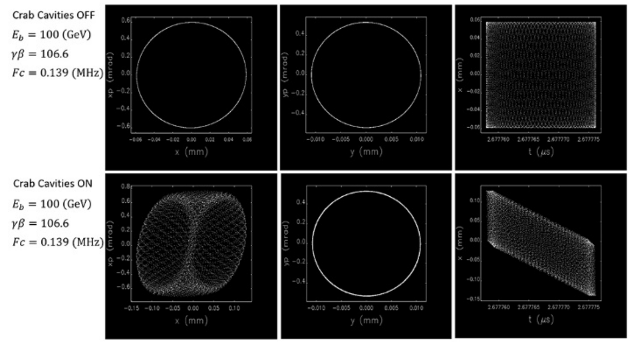

Figure 1: Particle's tracking: a particle with the offset of $(\Delta x, \Delta z, \Delta(dp/p)) = (0.\ 38\text{mm}, 0.44\text{mm}, 0.0001)$.

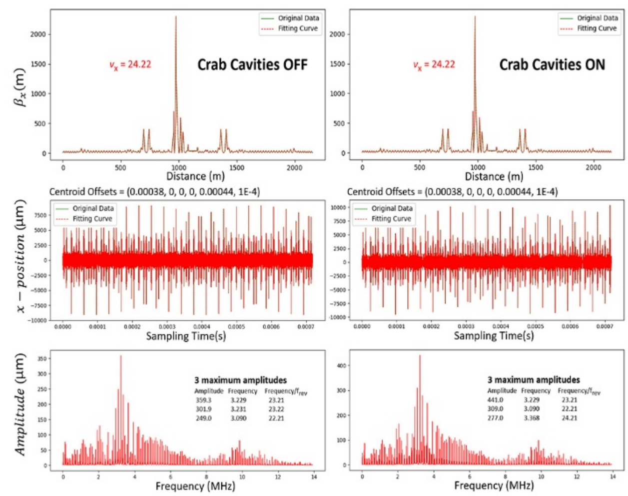

Figure 2: Betatron tune spectrum due to the offset of the centroid phase-coordinate.

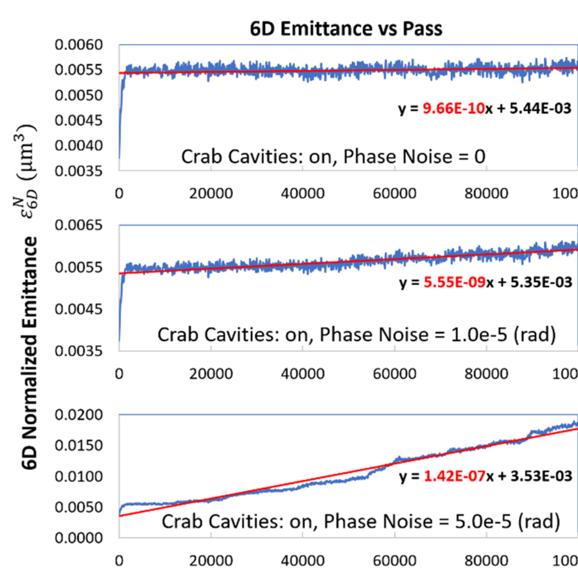

Figure 3: 6D normalized emittance curves.

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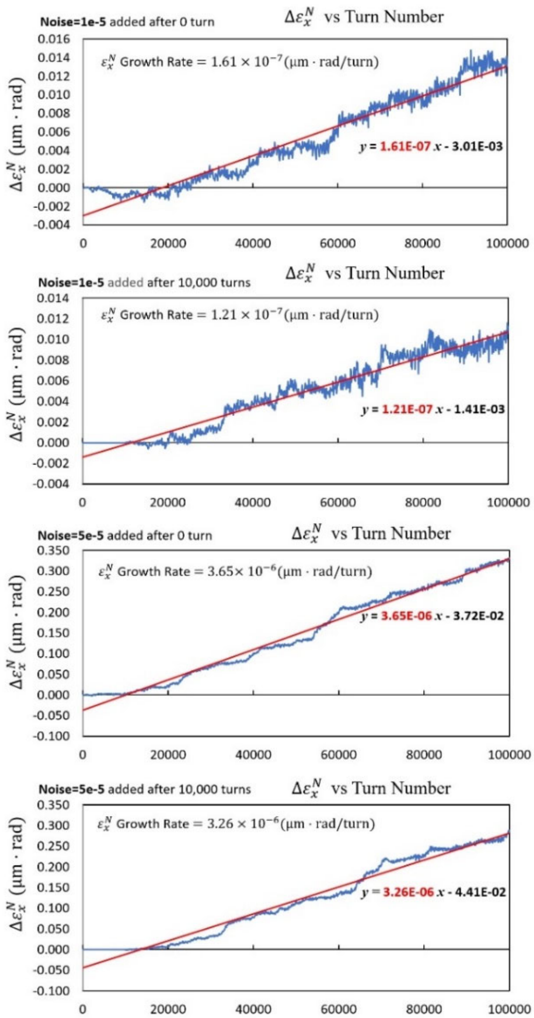

Figure 4: ε_x^N caused by the crab cavities phase noise.

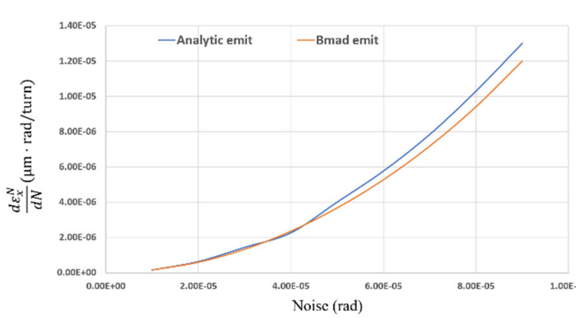

Figure 5: Normalized horizontal emittance growth rate as a function of crab cavity RF phase noise.

When we induced RF phase noise into crab cavities, the 6D emittance curves show an order of magnitude growth trend (see Fig. 3). For monitoring the emittance's variations, we demonstrated the effect of delaying the entry of RF phase noise. In Fig. 4, the vertical axis represents the pure effect of the phase noise which is the difference between the emittance without noise and the emittance with noise. The results show the significant different in the emittance growth. We then performed a series of simulations to verify the values predicted by the theory. Same as the analytic calculations, white noise of varying PSD was injected in simulations. Table 2 summarizes growth rates for the noises equivalent to 10 to 90 μrad crossing angles. The same data set is also shown in Fig. 5. It is clear that simulation results agree very well with the theoretical predictions. The data provides an induction of how serious such growth could affect collider operation. For example, at 30 μrad noise level, the growth rate is 1.3×10^{-6} μm·rad/turn, which means over every 7.5×10^{5} turns (~5.7 sec) in the ring, the proton beam normalized emittance grows 1.0 μm·rad. It is clear a feedback mechanism is required.

Table 2: Normalized Transverse Emittance Growth Rates

PSD (rad²/Hz)	Noise (dBc)	Noise (μrad)	Analytic $\frac{d\varepsilon_x^N}{dN}$ (μm rad /turn)	Simulation $\frac{d\varepsilon_x^N}{dN}$ (μm rad /turn)
$7.2\text{x}10^{-16}$	−103.0	10.0	$1.6\text{x}10^{-7}$	$1.6\text{x}10^{-7}$
$2.9\text{x}10^{-15}$	−97.0	20.0	$6.4\text{x}10^{-7}$	$6.1\text{x}10^{-7}$
$6.5\text{x}10^{-15}$	−93.5	30.0	$1.4\text{x}10^{-6}$	$1.3\text{x}10^{-6}$
$1.2\text{x}10^{-14}$	−91.0	40.0	$2.6\text{x}10^{-6}$	$2.4\text{x}10^{-6}$
$1.8\text{x}10^{-14}$	−89.0	50.0	$4.0\text{x}10^{-6}$	$3.7\text{x}10^{-6}$
$2.6\text{x}10^{-14}$	−87.5	60.0	$5.8\text{x}10^{-6}$	$5.3\text{x}10^{-6}$
$3.5\text{x}10^{-14}$	−86.1	70.0	$7.8\text{x}10^{-6}$	$7.2\text{x}10^{-6}$
$4.6\text{x}10^{-14}$	−85.0	80.0	$1.0\text{x}10^{-5}$	$9.4\text{x}10^{-6}$
$5.8\text{x}10^{-14}$	−83.9	90.0	$1.3\text{x}10^{-5}$	$1.2\text{x}10^{-5}$

CONCLUSION

We have performed a tracking simulation study to evaluate JLEIC proton beam emittance growth induced by crab cavity RF phase noise, and benchmarked simulations with a theoretical model. It is found they agree very well. This study will be extended to EIC as soon as its ion ring lattice design is available.

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