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Comparison of Numerical Methods for the Calculation of Synchrotron Radiation from Electrons*

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Introduction

- Modeling near-field SR *accurately* & *efficiently* is critical to understanding *self-consistent* beam dynamics

- Many methods and simulation tools have been developed, mostly based on 1D models

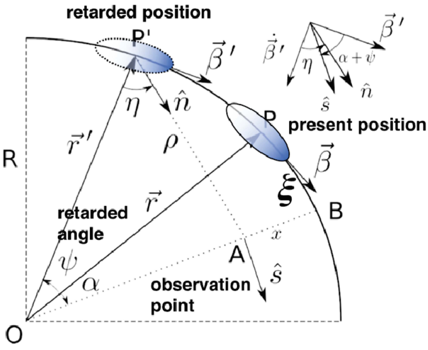
- We compare a few popular methods in order to develop a new simulation framework:
  - 2D analytic model based on Liénard-Wiechert (LW) equation
  - Particle-in-Cell (PIC) based on Finite Difference Time Domain
  - Jefimenko equation
  - A Lagrangian method for real-time calculation

2D analytic model based on LW equation

$$\vec{E} = \frac{e(\hat{n}-\vec{\beta}')}{\gamma^2\rho^2(1-\hat{n}\cdot\vec{\beta}')^3} + \frac{e\hat{n}\times[(\hat{n}-\vec{\beta}')\times\dot{\vec{\beta}}']}{c\rho(1-\hat{n}\cdot\vec{\beta}')^3}$$

$$E_s^{vel} = \frac{e\beta^2[\sin(\eta+\alpha+\psi)-\beta\cos(\alpha+\psi)]}{\gamma^2R^2\psi^2(1-\beta\sin\eta)^3}$$

$$E_s^{rad} = \frac{e\beta^3(\sin\eta-\beta)\cos(\eta+\alpha+\psi)}{R^2\psi(1-\beta\sin\eta)^3}$$



- Extension from 1D line charge model
- Applies to exact circular trajectory
- The key is to solve retarded angle $\psi = \psi(\alpha, x)$

$$1+(1+x)^2 - 2(1+x)\cos(\alpha+\psi) = \psi^2/\beta^2$$

Huang *et al.*, Phys. Rev. Accel. Beams **16**, 010701 (2013)

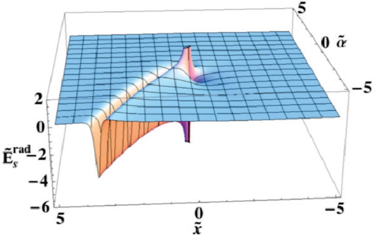
Self-similarities of single-particle kernel

Self-similarity in radiation field

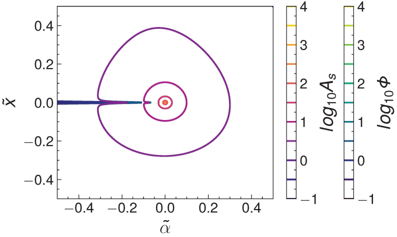
$$\tilde{x}^2 + 2\tilde{\alpha}\tilde{\psi} + (\tilde{x}-1)\tilde{\psi}^2 - \tilde{\psi}^4/12 = 0$$

$$\tilde{x} \equiv x\gamma^2, \tilde{\alpha} \equiv \alpha\gamma^3, \tilde{\psi} \equiv \psi\gamma$$

$$\tilde{E}_s^{rad} \equiv E_s^{rad}/(e\gamma^4/R^2)$$



Self-similarity in potentials



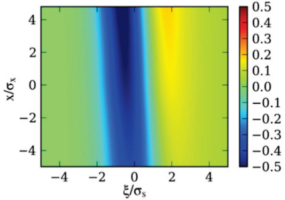
$$\Phi = \frac{e}{R\tilde{\psi}(1+\tilde{z}^2)}[\gamma^3(\beta + \sin\eta) - \gamma\sin\eta]$$

$$A_s = \frac{e}{R\tilde{\psi}(1+\tilde{z}^2)}[\gamma^3(\beta+\sin\eta)-\gamma(\beta^{-1}\tilde{z}^2+\beta+\sin\eta]$$

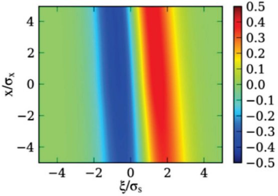
Convolution to obtain steady-state/transient fields for an electron beam

$$\varepsilon_s^i(\xi, x) = \iint E_s^i(\xi - \xi', x - x')\rho(\xi', x')d\xi' dx', i = \mathrm{vel, rad}$$

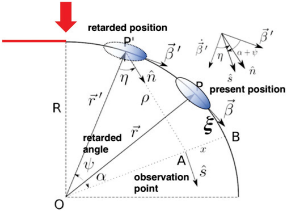
Steady-state field



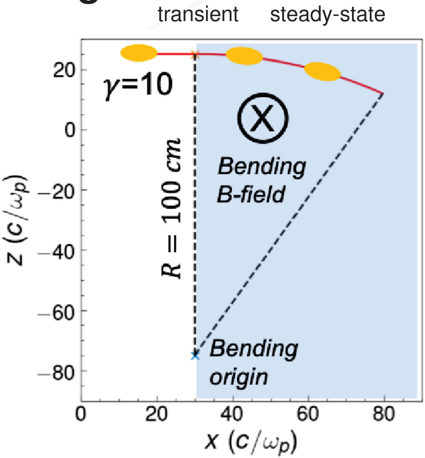
Transient field



Transition into bends



3D PIC simulation: High resolution needed to mitigate numerical errors

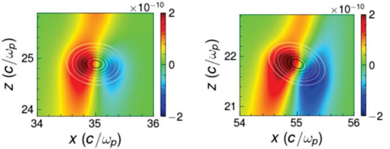


2D bi-Gaussian beam

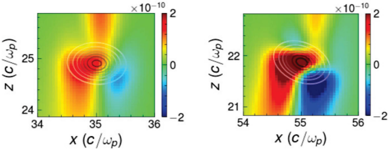
Yee solver

~3000 cpu-hrs

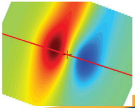
High resolution: $\sigma_x/dx = 25$



Low resolution: $\sigma_x/dx = 5$



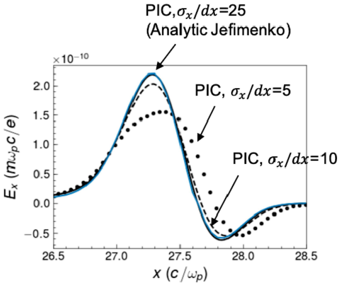
2D analytical model

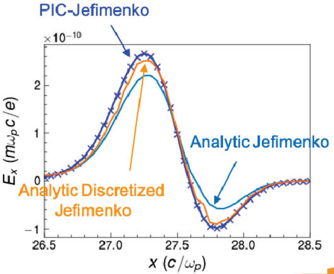


Jefimenko calculation & Comparison with PIC

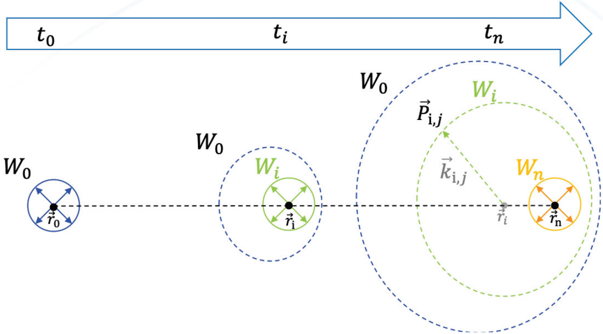
$$\vec{E}(\vec{r},t) = \frac{1}{4\pi}\int\left(\frac{\rho(\vec{r}',t')\hat{n}}{R^2} + \frac{\dot{\rho}(\vec{r}',t')\hat{n}}{R} - \frac{\dot{\vec{J}}(\vec{r}',t')}{R}\right)d^3\vec{r}'$$

	Discretization	Deposition	Dispersion	Integration
PIC-Jefi.	Yes	Yes	-	-*
Analytic-Jefi.	-	-	-	-*
Analytic-Discretized-Jefi.	Yes	-	-	-*
PIC	Yes	Yes	Yes	-





A Lagrangian method: Real-time animation of SR by tracing emission wavefronts



Construct wavefronts from emission points:

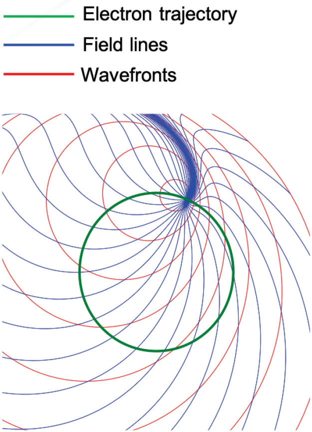
$$\vec{P}_{i,j}(N\Delta t) = \vec{r}_i + c(N-1)\Delta t\vec{k}_{i,j}, (N > i)$$

Emission angle determined by β:

$$k_x = (\cos\theta' + \beta)/(1 + \beta\cos\theta')$$
$$k_y = \sin\theta'/\gamma(1 + \beta\cos\theta')$$

T. Shintake, Nucl. Instr. Methods Phys. Res. A **507**, 89-92 (2003)

Adaptive mesh & Radiation field due to electron acceleration



Electric fields in electron frame:

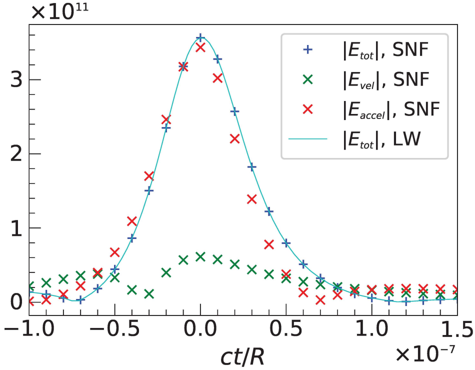
$$\vec{E}'_{vel} = -e\frac{\hat{n}}{\rho^2}\,;\ \vec{E}'_{acc} = e\frac{\hat{n}\times(\hat{n}\times\dot{\vec{\beta}}')}{\rho}$$

Total electric field in lab frame:

$$\vec{E} = \frac{e(\hat{n}-\vec{\beta}')}{\gamma^2\rho^2(1-\hat{n}\cdot\vec{\beta}')^3} + \frac{e\hat{n}\times[(\hat{n}-\vec{\beta}')\times\dot{\vec{\beta}}']}{c\rho(1-\hat{n}\cdot\vec{\beta}')^3}$$

KirK T. McDonald, Notes (2017)

Double Lorentz transform & Equivalence to LW equation



- Transform to electron frame
- Obtain velocity field due to charge at rest
- Obtain radiation field due to charge acceleration
- Transform fields back to lab frame

Li, Huang, *et al.*, Proc. 10th Int. Particle Accelerator Conf. 397-399 (2019)

Summary

	Pros.	Cons.
Analytic models	Accurate Scaling	Case limited Non self-consistent (History searching)
PIC (FDTD)	Self-consistent	Numerical issues Expensive (small dx, dt)
Larangian (Shintake)	Accurate Free of searching Adaptive mesh Real-time Self-consistent	Can be expensive but inherently parallel