Dynamics in Beam Driven Structures

S.S. Baturin
University of Chicago
NAPAC 2019
09/02/2019

Outline

- Introduction to dynamics problem
- Adaptive focusing
- Generalized BNS damping and stability analysis

Introduction

- Initial study

A.W. Chao, B. Richter and C.Y. Yao, Nucl. Instr. and Meth. A, 178 (1980)

- Extensive analysis and full BBU theory for smooth focusing

J.R. Delayen

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **6**, 084402 (2003)

Cumulative beam breakup in linear accelerators with arbitrary beam current profile

J. R. Delayen*

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **7**, 074402 (2004)

Cumulative beam breakup in linear accelerators with random displacement of cavities and focusing elements

J. R. Delayen*

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **8**, 024402 (2005)

Cumulative beam breakup in linear accelerators with time-dependent parameters

J. R. Delayen*

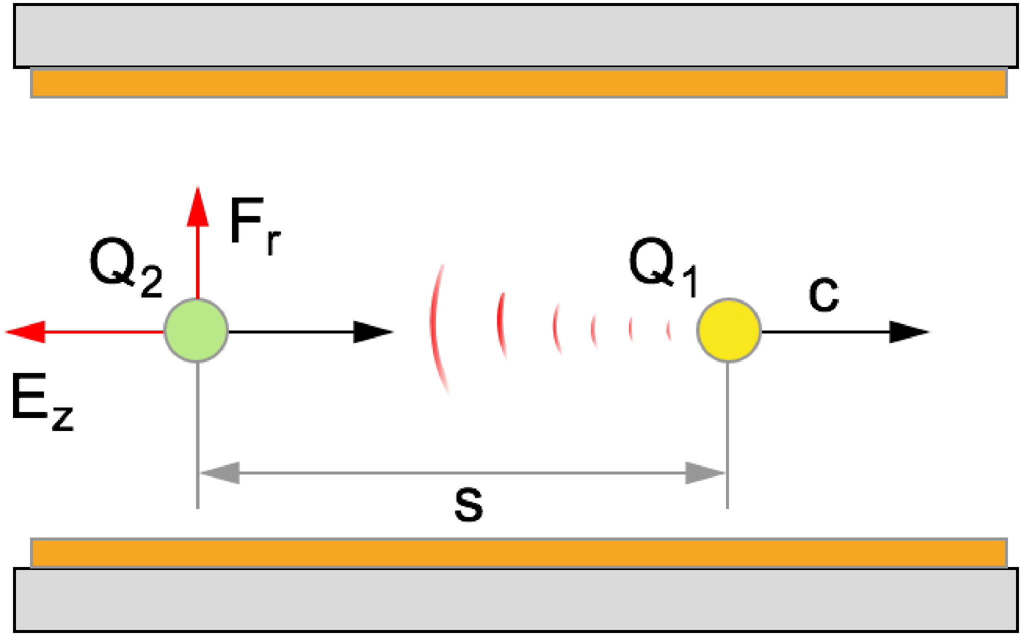
Introduction

Longitudinal wakefield potential

$$W_z = -\frac{1}{Q_1}\int_{-\infty}^{\infty} E_z\left(z, t = \frac{z+s}{c}\right) dz$$

Transverse wakefield potential

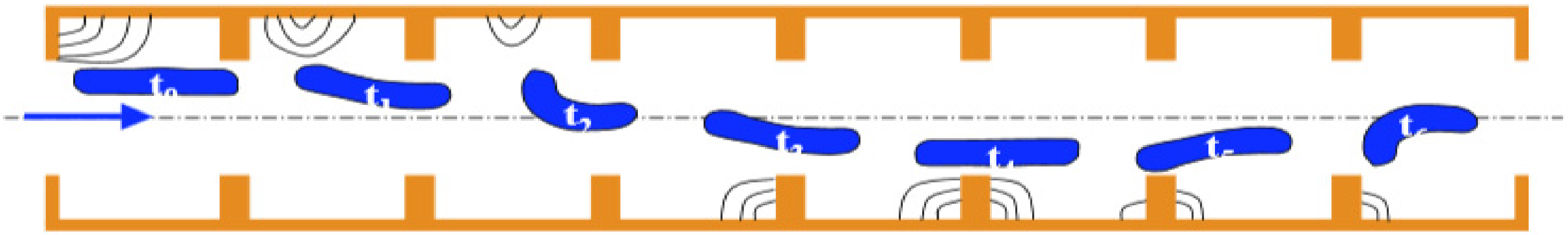
$$\vec{W}_\perp = \frac{1}{Q_1}\int_{-\infty}^{\infty} \vec{F}\left(z, t = \frac{z+s}{c}\right) dz$$



Energy change of the test particle

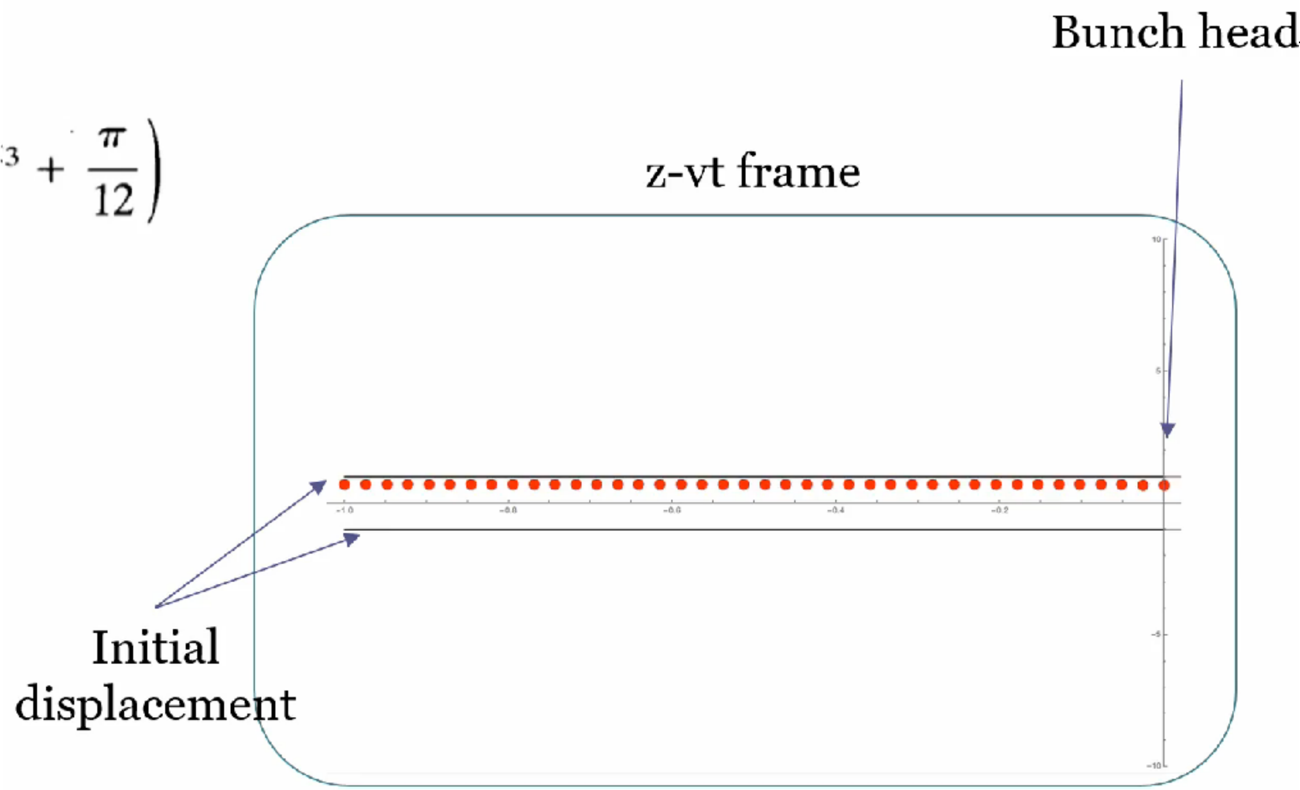
$$\Delta E = -Q_1 Q_2 W_z(s)$$

Beam breakup instability (BBU)



$$y(L_0, z) \approx \frac{\hat{y}}{\sqrt{6\pi}} \Upsilon^{-1/6} \exp\left(\frac{3^{3/2}}{4}\Upsilon^{1/3}\right) \cos\left(k_\beta L_0 - \frac{3}{4}\Upsilon^{1/3} + \frac{\pi}{12}\right)$$

$$\Upsilon = \frac{N r_0 L_0 W_0}{k_\beta \gamma L}\left(\frac{1}{2} - \frac{z}{l}\right)^2$$

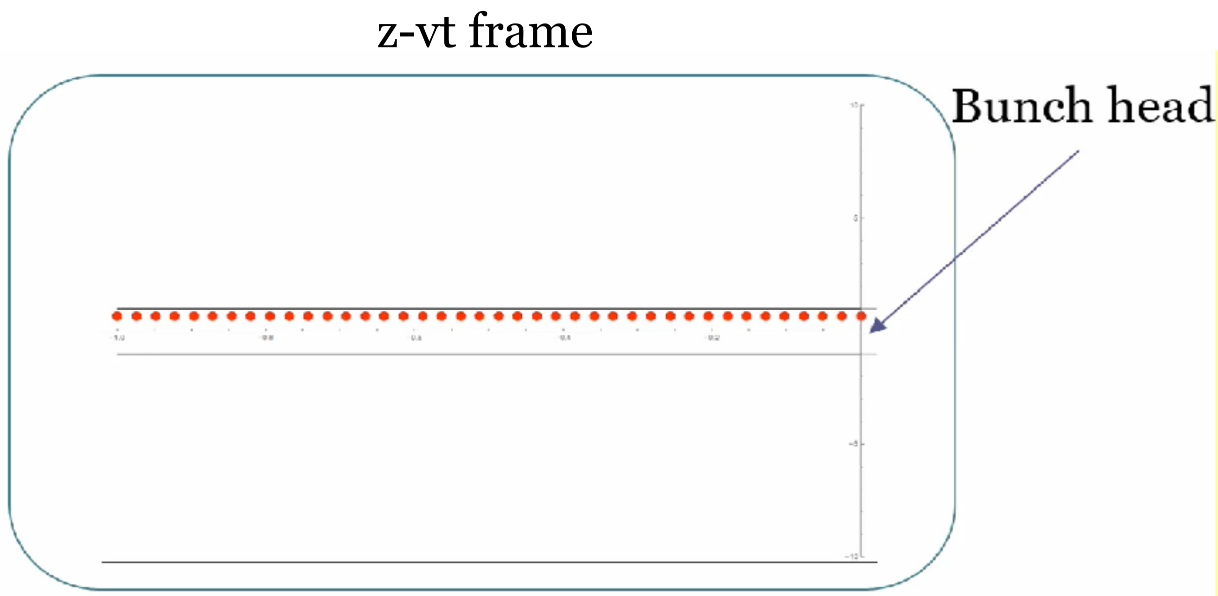


*A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (Wiley and Sons, New York, 1993).

Balakin, Novohatsky, Smirnov method (BNS)*

Wake field effectively produce tune spread and causes a resonant interaction.

Proposition is to introduce a tune inside the beam in such a way that it will compensate tune spread induced by the wake.



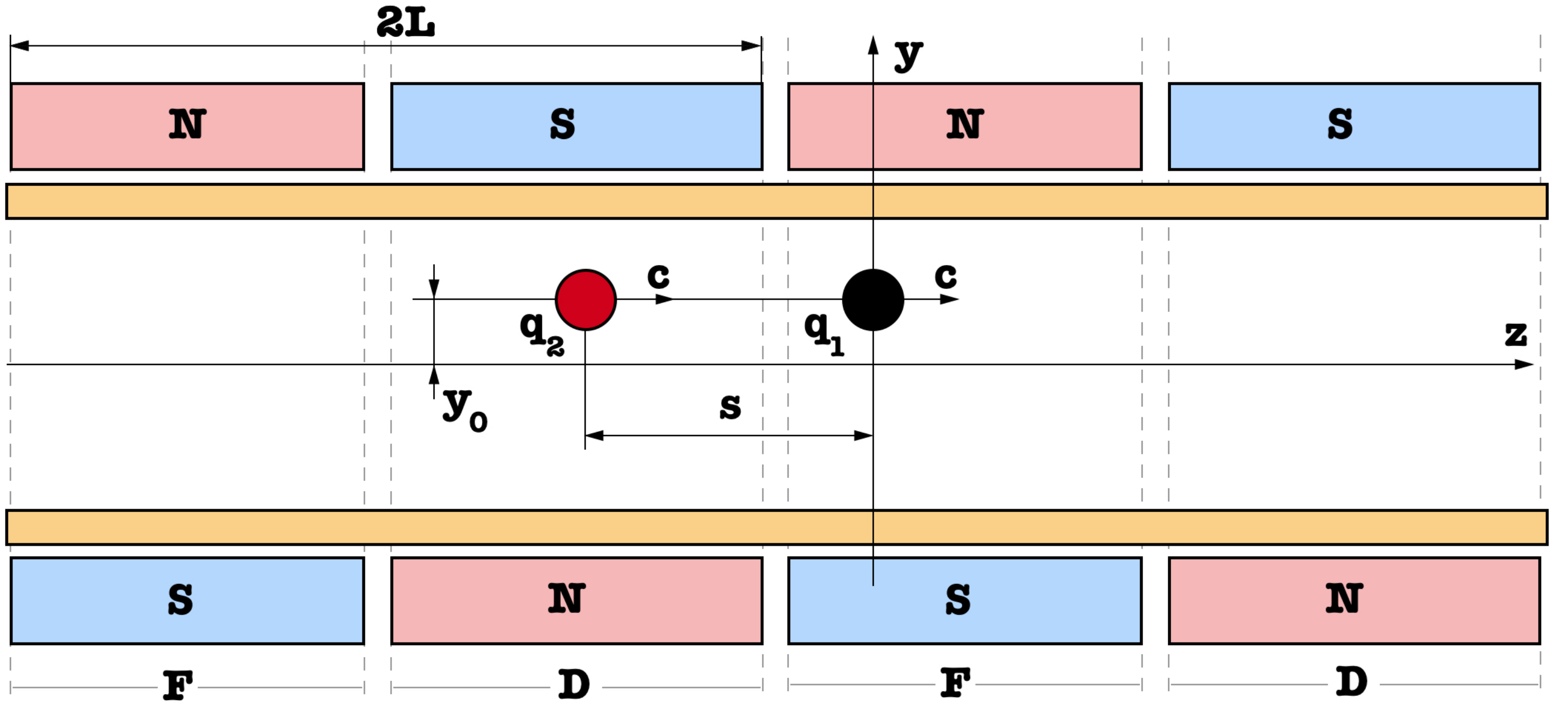
$$\frac{d^2y(z,s)}{dz^2} + \left(k_\beta^2 + 2k_\beta \Delta k_\beta + \Delta k_\beta^2\right) y(z,s) = \frac{1}{\gamma m c^2} \int_0^s G_y(s-s_0) y(s_0) q(s_0) ds_0$$

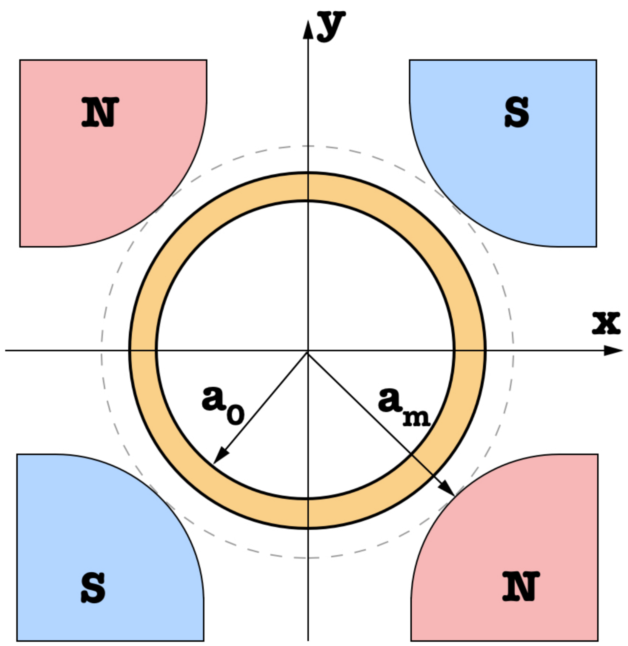
$$\frac{\Delta k_\beta}{k_\beta} = \frac{1}{2k_\beta^2 \gamma m_e c^2} \int_0^s G_y(s-s_0) q(s_0) ds_0.$$

*V. Balakin, S. Novokhatsky, and V. Smirnov, in Proceedings of 12th International Conference on High Energy Accelerators, Batavia, Illinois (Fermilab, Batavia, 1983) p. 119.

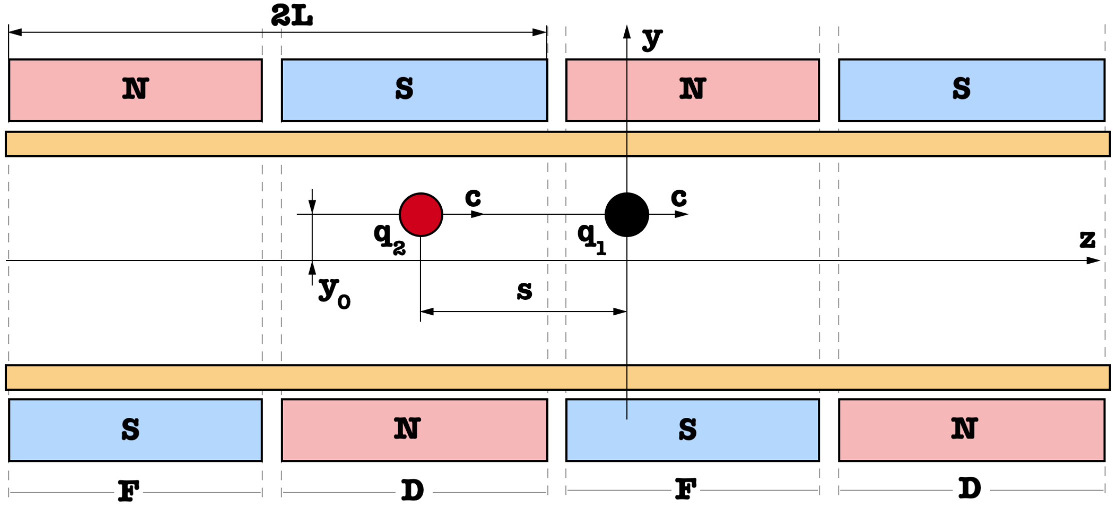
Can we apply existing theory directly to analyze CWA?

The model





The model



Equations of motion

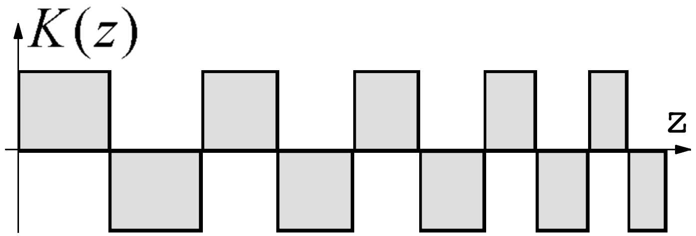
$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\gamma_1(z)}{\gamma_0}\dfrac{dy_1}{dz}\right)+K(z)y_1=0 \\ \dfrac{d}{dz}\left(\dfrac{\gamma_2(z)}{\gamma_0}\dfrac{dy_2}{dz}\right)+K(z)y_2=w(s)y_1 \end{cases}$$

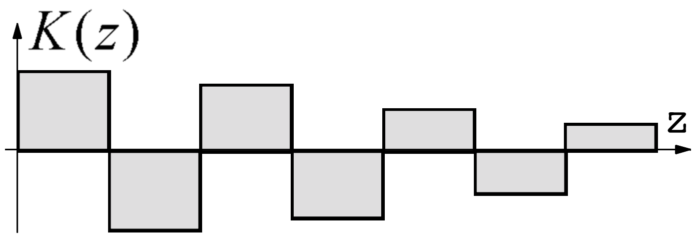
Adaptive focusing

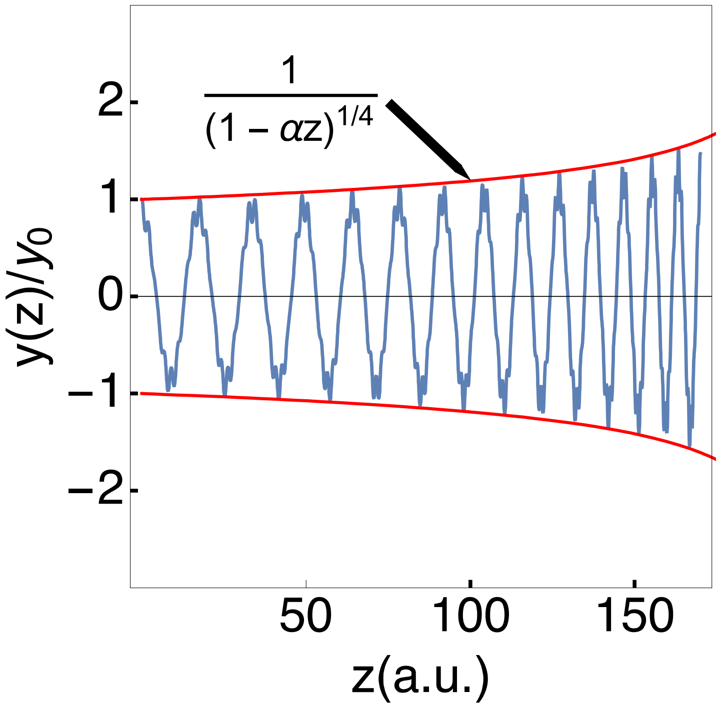
$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\gamma_1(z)}{\gamma_0}\dfrac{dy_1}{dz}\right) + K(z)y_1 = 0 \\ \dfrac{d}{dz}\left(\dfrac{\gamma_2(z)}{\gamma_0}\dfrac{dy_2}{dz}\right) + K(z)y_2 = w(s)y_1 \end{cases}$$

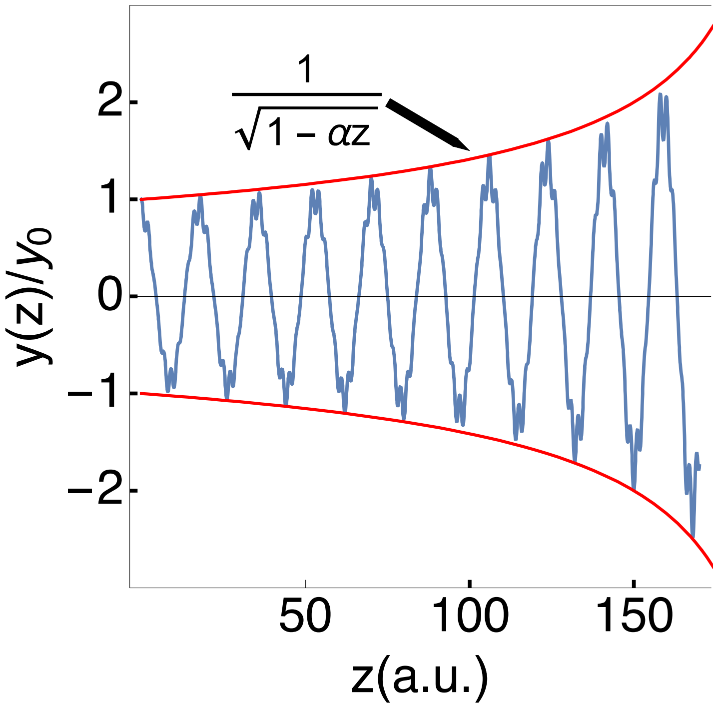
- How we keep constant phase advance?
- How to tweak magnetic system to match the energy of the first particle?
- We know that to mitigate the wakefield we need energy spread.

Adaptive lens length L







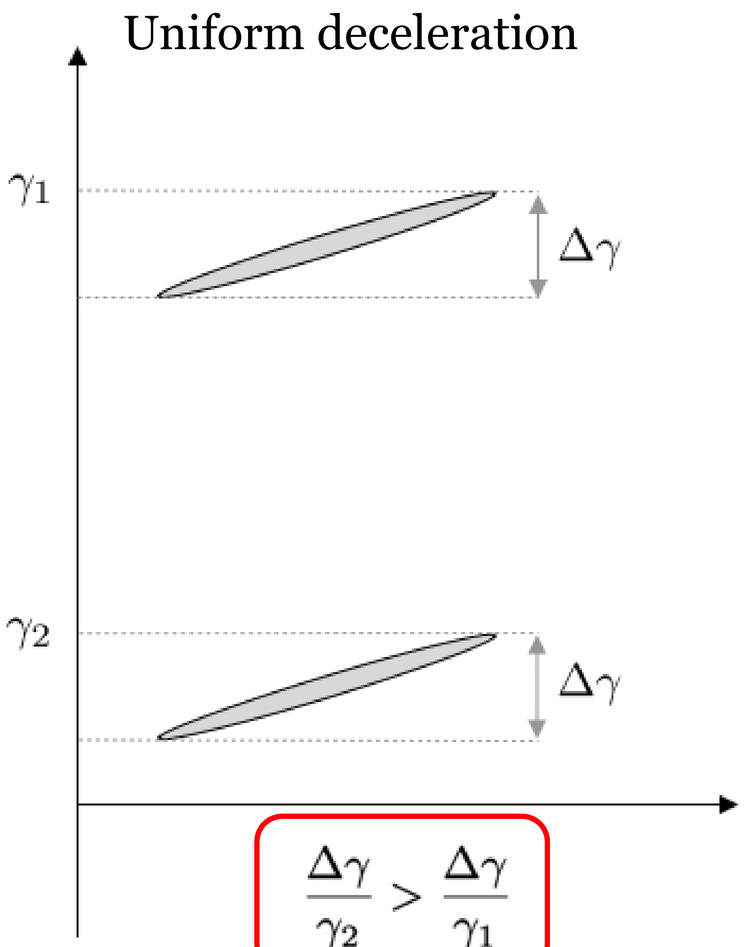


$$K(z(u))$$

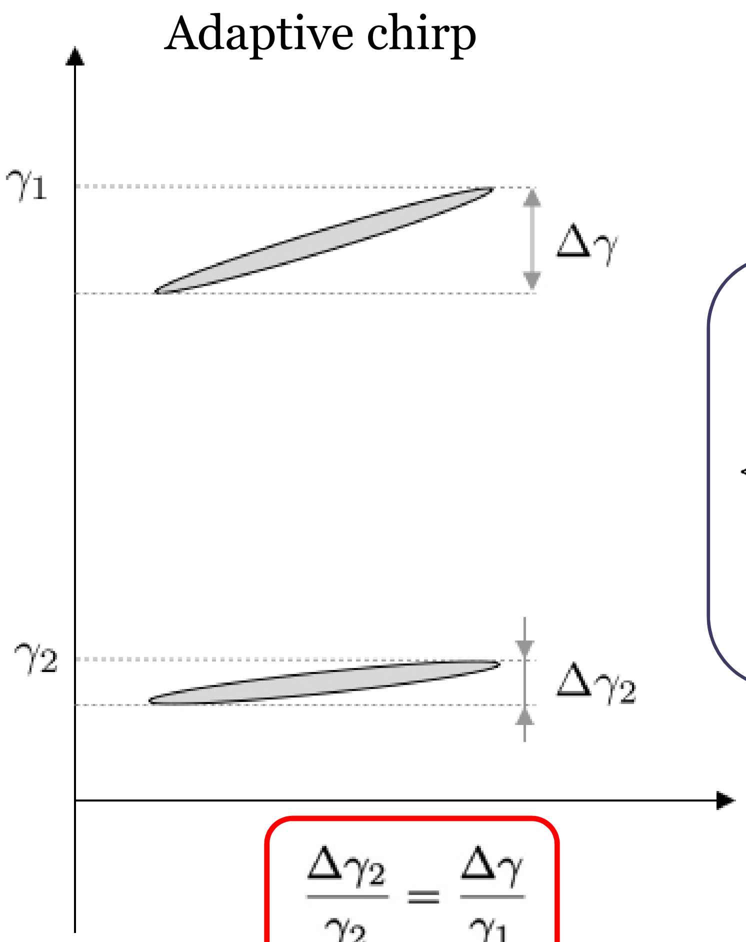
Periodic function of u

$$L = L_0\sqrt{1-\alpha z}$$

Adaptive energy chirp



$$\frac{\Delta\gamma}{\gamma_2} > \frac{\Delta\gamma}{\gamma_1}$$



$$\frac{\Delta\gamma_2}{\gamma_2} = \frac{\Delta\gamma}{\gamma_1}$$

$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\gamma_1(z)}{\gamma_0}\dfrac{dy_1}{dz}\right) + K(z)y_1 = 0 \\ \dfrac{d}{dz}\left(\dfrac{\gamma_2(z)}{\gamma_0}\dfrac{dy_2}{dz}\right) + K(z)y_2 = w(s)y_1 \end{cases}$$

Adaptive energy chirp

$$\begin{cases} (1-\alpha z)\dfrac{d^2 y_1}{dz^2} - \alpha\dfrac{dy_1}{dz} + K(z)y_1 = 0 \\ (1-\alpha z)\dfrac{d^2 y_2}{dz^2} - \alpha\dfrac{dy_2}{dz} + \dfrac{K(z)}{1-f(s)}y_2 = \dfrac{w(s)}{1-f(s)}y_1 \end{cases}$$

we introduce new variables

$$y_{1,2}(z) = \frac{g_{1,2}(u)}{\sqrt{u}}$$

$$u = \sqrt{1-\alpha z}$$

$$u > 0.1$$

$$\begin{cases} \dfrac{d^2 g_1}{du^2} + K(z(u))g_1 = 0 \\ \dfrac{d^2 g_2}{du^2} + \dfrac{K(z(u))}{1-f(s)}g_2 = \dfrac{w(s)}{1-f(s)}g_1 \end{cases}$$

Adaptive focusing summary

- Adaptive chip

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

- Adaptive lens length variation

$$L = L_0\sqrt{1-\alpha z}$$

Wakefield cancelation

$$\begin{cases} \dfrac{d^2 x_1}{dt^2} + K(t)x_1 = 0 \\ \dfrac{d^2 x_2}{dt^2} + \dfrac{K(t)}{1-f(s)}x_2 = \dfrac{w(s)}{1-f(s)}x_1 \end{cases}$$

Wakefield cancelation

$$\begin{pmatrix} x_1(t) \\ p_1(t) \end{pmatrix} = \mathbf{X}_1^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

$$\begin{pmatrix} x_2(t) \\ p_2(t) \end{pmatrix} = \mathbf{X}_2^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix} + \frac{w(s)}{1 - f(s)} \mathbf{X}_2^t \int_0^t \left(\mathbf{X}_2^\tau\right)^{-1} \begin{pmatrix} 0 \\ p_1(\tau) \end{pmatrix} d\tau$$

Vladimir I. Arnol'd, Ordinary Differential Equations,
Springer-Verlag, Berlin (1992)

Wakefield cancelation

$$\begin{pmatrix} x_1(t) \\ p_1(t) \end{pmatrix} = \mathbf{X}_1^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

$$\begin{pmatrix} x_2(t) \\ p_2(t) \end{pmatrix} = \mathbf{T}^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

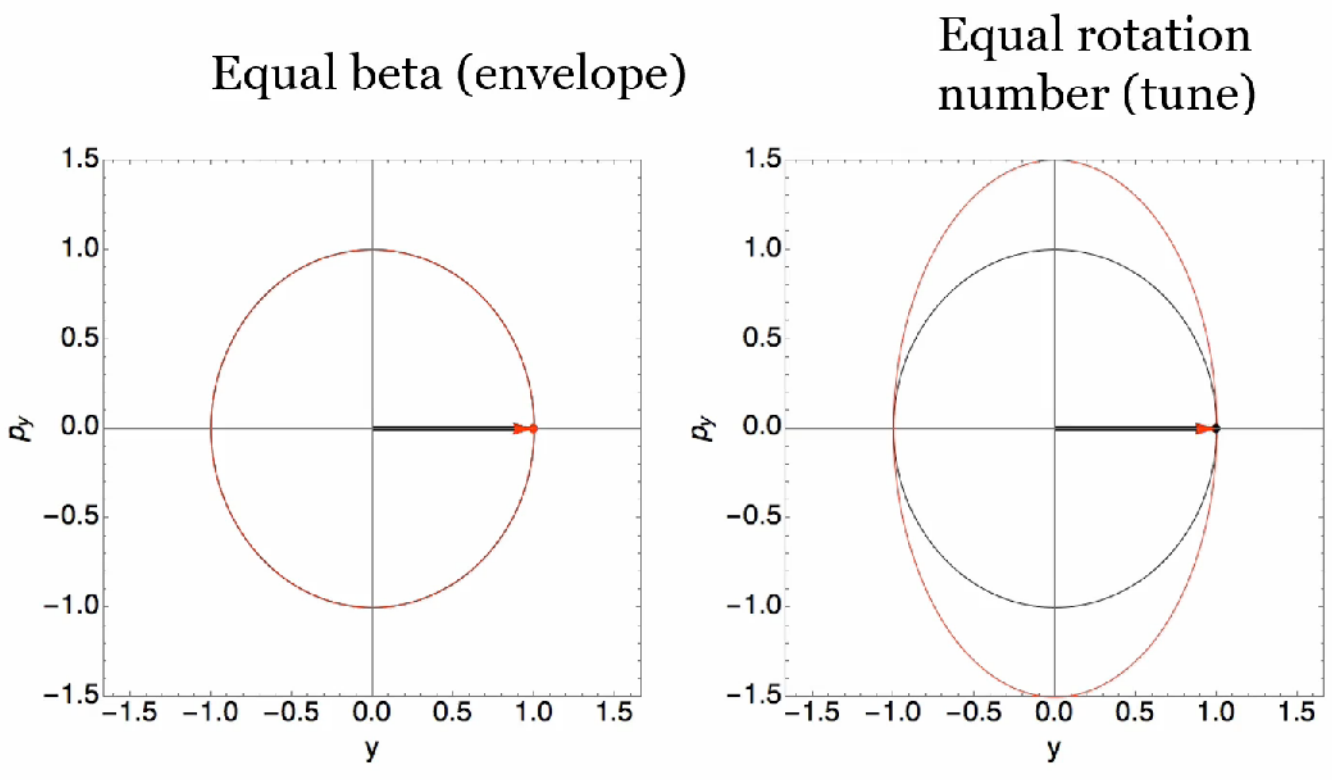
$$\mathbf{T}^t = \mathbf{X}_2^t + \mathbf{X}_2^t \int_0^t \left(\mathbf{X}_2^\tau\right)^{-1} \mathbf{W} \mathbf{X}_1^\tau d\tau$$

$$\mathbf{W} = \begin{pmatrix} 0 & 0 \\ \dfrac{w(s)}{1 - f(s)} & 0 \end{pmatrix}$$

Wakefield cancelation

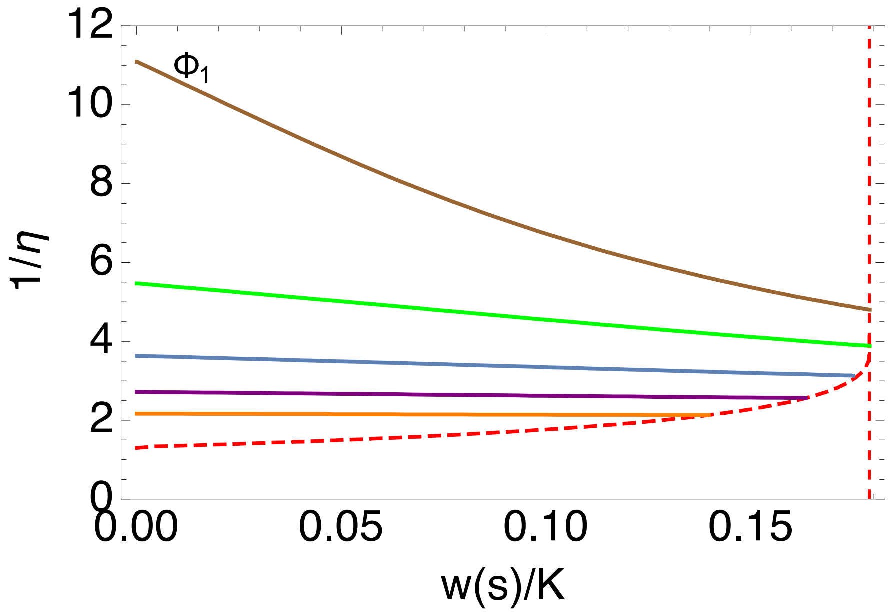
General idea of BNS damping for determining f(s)

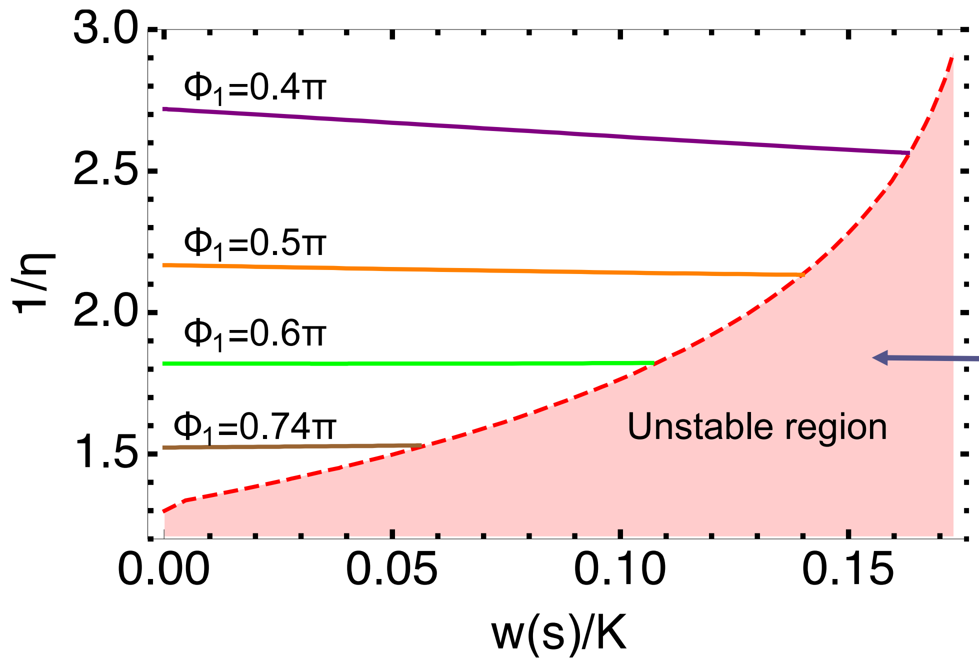
$$\mathrm{Tr}\left[\mathbf{X}_1^{2L}\right] = \mathrm{Tr}\left[\mathbf{T}^{2L}\right]$$



Wakefield cancelation. Stability.

$$f(s) = \frac{1}{\eta}\frac{w(s)}{\max|K|}$$





$$\left|\mathrm{Tr}\left[\mathbf{X}_2^{2L}\right]\right| > 2$$

Stability condition

$$f(s) \approx \frac{1}{\eta} \frac{\int_0^s w(s-s_0)q(s_0)ds_0}{\max|K|}$$

$$E_z(s) = 2\kappa_{\|} \int_0^s w_{\|}(s-s_0)q(s_0)ds_0$$

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

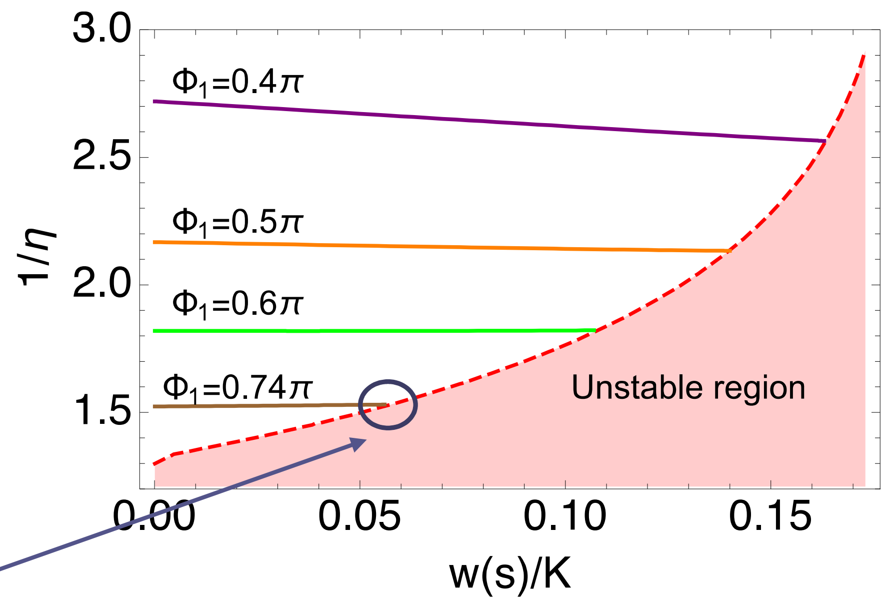
Stability condition

$$\left|\frac{\Delta\gamma}{\gamma}\right| > \frac{1}{\eta}\frac{a_m}{cB_0}\frac{\kappa_\perp}{2\kappa_\parallel}\frac{\max\left|E_+\right|}{k_0}\frac{\sqrt{R^2-1}}{R}$$

Stability condition

$$\Phi_1 = 0.74\pi$$

$$\frac{1}{\eta} = 1.53$$



$$0.086 > \left|\frac{\Delta\gamma}{\gamma}\right| > 2\frac{1.53}{k_0 a_0}\frac{\max\left|E_+\right|}{cB_0}$$

Summary

- Adaptive chip

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

- Adaptive lens length variation

$$L = L_0\sqrt{1-\alpha z}$$

- Restrictions on chirp

$$0.086 > \left|\frac{\Delta\gamma}{\gamma}\right| > 2\frac{1.53}{k_0 a_0}\frac{\max|E_+|}{cB_0}$$

Perspectives

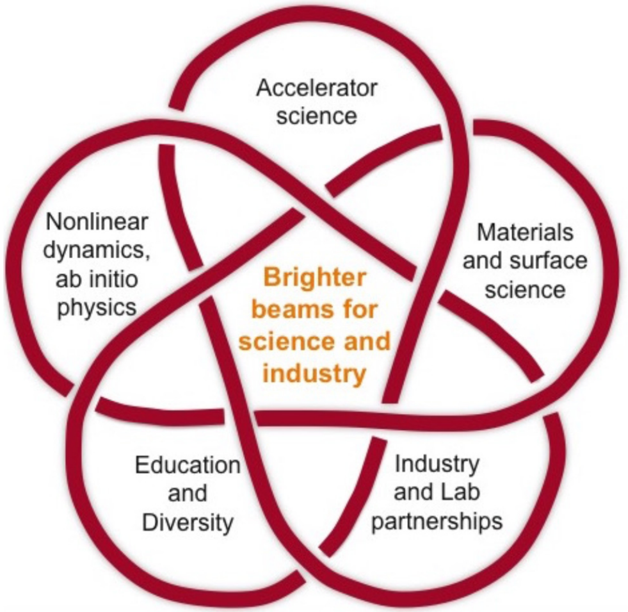
- Nonlinear focusing. Use octupoles to relax conditions on energy chirp.

- Account for nonlinear (in displacement) wakefield.

Thank you!

Acknowledgments

Center for Bright Beams and NSF



Awards No. PHY-1549132 and No. PHY-1535639

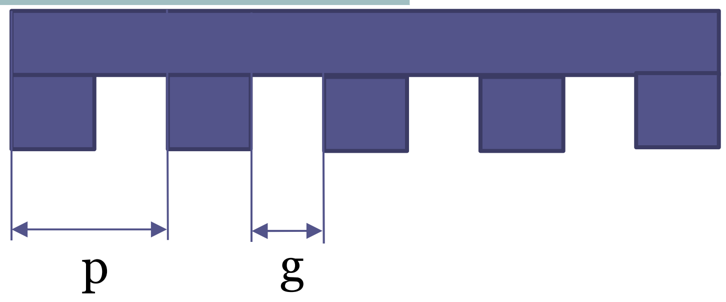
Numeric example

$$0.086 > \left|\frac{\Delta\gamma}{\gamma}\right| > 2\frac{1.53}{k_0 a_0}\frac{\max\left|E_+\right|}{cB_0}$$

$$B = 1T$$
$$cB \approx 300MV/m$$

$$\max|E_+| = 8.43k_0a_0\,[MV/m]$$

Numeric example



$$\max|E_+| = 8.43 k_0 a_0 \, [MV/m]$$

100 MV/m → $k_0 a_0 \approx 12$

$$k_0 \approx \sqrt{\frac{p}{g}}\sqrt{\frac{2}{a_0 \Delta}} \qquad \sqrt{\frac{p}{g}} = \sqrt{2}$$

$$\Delta \approx \frac{a_0}{36}$$

$$a_0 = 1mm$$

$$\Delta = 28\mu m$$