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TRACKING WITH SPACE HARMONICS IN ELEGANT CODE*

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Abstract

The ELEGANT code has the capability of simulating particle motion in accelerating or deflecting RF cavities, with a simplified (or ideal) model of the electromagnetic fields. To improve the accuracy of RF cavity simulations, the ability to track with space harmonics has been added to the elegant code. The sum of all the space harmonics will mimic the real electromagnetic fields in the RF cavity. These space harmonics will be derived from electromagnetic fields simulation of the RF cavity. This method should be general, which can be applied to any traveling wave multi-cell RF cavity structure, including accelerating and deflecting cavities. In this paper this approach is illustrated with the deflecting cavity example.

INTRODUCTION

There are several ways to perform particle tracking simulations through an electromagnetic field which changes with time. The most complicated and possibly most accurate method is to directly tracking through a three dimensional field map with fine mesh size, and integrate the momentum/coordinates change due to the electromagnetic force $\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$.

The simplest method is to model the electromagnetic field as a thin element with zero length. The momentum change (or angular kick) is applied with an effective impulse of the dominant fields component which is usually integrated near-axis with some approximations. This method is efficient in simulation computing time, but may be less accurate.

An intermediate approach would be to expand the thin-lens single kick method to a combination of multiple components. These multiple components may be derived from measured or simulated three dimensional electromagnetic field map. One possibility is to use the space harmonics that are determined by the geometry of the structure, which provides boundary conditions of the electromagnetic field. This approach could provide a more accurate model, and at the same time are still efficient in simulation computing time. In the following sections the work is discussed on these three approaches.

THIN-LENS DEFLECTING CAVITY

The ELEGANT code [1] is capable of simulating particle's motion in the accelerating or deflecting RF cavities, with a simplified model of the electromagnetic fields. Take the horizontal deflecting cavity as an example, the Hamiltonian to describe the thin deflecting cavity is

$$H_0 = q\bar{V} \cdot \sin(kz + \phi_0) \cdot x, \tag{1}$$

where H denotes the Hamiltonian, q the particle charge, $\bar{V}$ the effective voltage, ϕ_0 the synchronous phase of the thin deflecting cavity rf wave, $k = \omega/c$ the wave number, ω the angular frequency of the fundamental deflecting mode, c the velocity of light, z the longitudinal coordinate relative to the bunch center, x the horizontal coordinate.

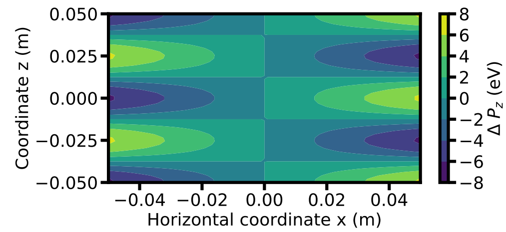

Figure 1: Longitudinal kick ΔP_z on x–z space. 6GHz deflecting cavity with $\bar{V} = 1$.

Using the Hamiltonian, it is possible to derive the deflection in the three dimensions from an ideal cavity, as listed in Eqs. (2)–(4). These are used for an ideal deflecting cavity in ELEGANT [1]:

$$\Delta P_x = -\frac{\partial H_0}{\partial x} = -q\bar{V} \cdot \sin(kz + \phi_0), \tag{2}$$

$$\Delta P_z = -\frac{\partial H_0}{\partial z} = -q\bar{V} \cdot k \cdot x \cdot \cos(kz + \phi_0), \tag{3}$$

$$\Delta P_y = 0, \tag{4}$$

where ΔP_x denotes the change of momentum in horizontal plane, ΔP_z denotes the change of momentum in longitudinal plane.

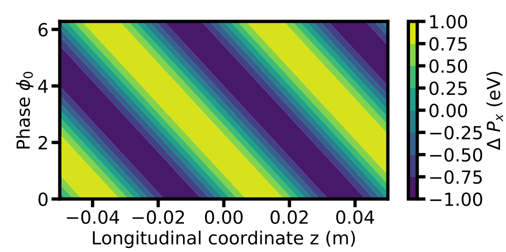

Figure 2: Horizontal kick ΔP_x on z–ϕ space. 6GHz deflecting cavity with $\bar{V} = 1$.

Figure 1 shows the longitudinal kick from a 6 GHz thin deflecting cavity with voltage of 1 volt. The horizontal kick on the z–ϕ space is shown in Fig. 2 for same deflecting cavity.

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DIRECT TRACKING WITH FIELD MAP

As mentioned above, the most complicated and possibly most accurate method is to directly tracking through a three dimensional field map with fine mesh size. An integration of the electromagnetic force through the field map will provide update for the momentum/coordinates.

The simulated 3D EM fields would have a total of 12 columns, which are the real and imaginary parts of **E** and **B**. These EM fields contain all the space harmonic components in the deflecting cavity, which can be considered to be static EM fields at time $t = 0$:

$$\tilde{\mathbf{E}}(x, y, z) = \mathbf{E_x}(x, y, z)\hat{\mathbf{x}} + \mathbf{E_y}(x, y, z)\hat{\mathbf{y}} + \mathbf{E_z}(x, y, z)\hat{\mathbf{z}}, \quad (5)$$

$$\tilde{\mathbf{B}}(x, y, z) = \mathbf{B_x}(x, y, z)\hat{\mathbf{x}} + \mathbf{B_y}(x, y, z)\hat{\mathbf{y}} + \mathbf{B_z}(x, y, z)\hat{\mathbf{z}}. \quad (6)$$

The time varying electromagnetic fields are then expressed as

$$\tilde{\mathbf{E}}(x, y, z, t) = \tilde{\mathbf{E}}(x, y, z) \cdot \mathrm{e}^{-i\omega t}, \quad (7)$$

$$\tilde{\mathbf{B}}(x, y, z, t) = \tilde{\mathbf{B}}(x, y, z) \cdot \mathrm{e}^{-i\omega t}. \quad (8)$$

Work is in progress to include this capability in direct tracking of time varying electromagnetic field map in ELEGANT [1]. This will be reported later, and here we focus on the space harmonic approach.

SPACE HARMONIC APPROACH

As discussed in the above sections, the space harmonic are determined by the periodic structures/geometry of the cavity. The amplitude and phase of the space harmonics can be obtained by performing singular value decomposition on the electromagnetic field data from a cylindrical surface centered on z axis [2]. The electric field (deflecting mode) in the inside region of the iris of the deflecting cavity is expressed as [2, 3]

$$E_z(r, \Phi, z, t) = \sum_{n=-\infty}^{\infty} A_n \frac{I_m(\alpha_n r)}{I_m(\alpha_n a)} \cdot \mathrm{e}^{-ik_n z} \cdot \mathrm{e}^{-im\Phi} \cdot \mathrm{e}^{-i\omega t}, \quad (9)$$

$$k_n = \frac{\varphi_0 + 2\pi n}{d}, \quad (10)$$

$$\alpha_n^2 + k_n^2 = k_0^2, \quad (11)$$

where I_m is first kind modified Bessel function, k_n wave number of n^{th} space harmonic, n an integer number, φ_0 the phase advance per cavity period, d the cavity period length, α_n the wave number in the radial direction, m wave number (per 2π) in the angular direction. The other components of the electromagnetic field TM mode can be derived from E_z. It is observed that the EM fields of lowest harmonic ($n = 0$) depends linearly on radial coordinates. The cavity period length d and inner radius a are shown in Fig. 3. In the following part, the space harmonics with negative n are neglected (no backward traveling waves).

For $2\pi/3$ mode deflecting cavity ($2\pi/3$ phase advance per cavity period), the wave number k_n of space harmonic 1–10 is shown in Fig. 4.

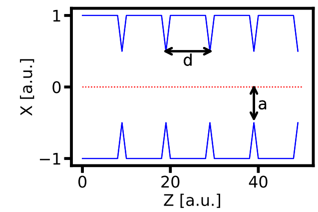

Figure 3: Notation of d and a on a five cell rf cavity.

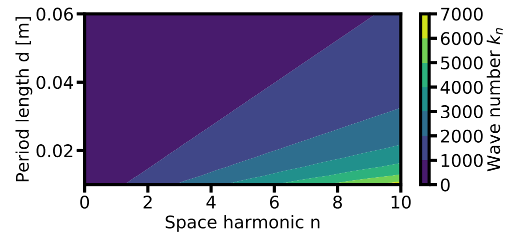

Figure 4: Space harmonic wave number k_n on the n–d space, for $2\pi/3$ phase advance per cavity period.

The first kind modified Bessel function I_m can be defined by a series expansion around $x = 0$, as shown below. For deflecting (dipole) mode, one finds that $m = 1$:

$$I_m(x) = \sum_{j=0}^{\infty} \frac{1}{j! \cdot \Gamma(j + m + 1)} \cdot \left(\frac{x}{2}\right)^{2j+m}. \quad (12)$$

As the electromagnetic fields are linearly decomposed into the space harmonics, from the principle of superposition, the Hamiltonian to describe the thin deflecting cavity with space harmonics also follows superposition principle. It can be shown that for the zero-th space harmonic with Hamiltonian H_0, the Hamiltonian has no dependency on the angular coordinate Φ with first order approximation. Keep the first two terms in I_1 ($m = 1$), neglect the vertical plane, after some derivations, one finds (with $r = \sqrt{x^2 + y^2}$ and $x = r\cos\Phi$)

$$\mathbf{H} = H_0 + \sum_{n=1}^{\infty} q\bar{V}_n \cdot \sin(k_n z + \phi_n) \cdot (\frac{1}{2}\alpha_n \cdot x + \frac{1}{16}\alpha_n^3 \cdot (x^2 + y^2) \cdot x). \quad (13)$$

The thin kicks from the space harmonics ($n \geq 1$) are

$$\Delta P_x = -\frac{\partial(\mathbf{H} - H_0)}{\partial x} = \sum_{n=1}^{\infty} -q\bar{V}_n \cdot \sin(k_n z + \phi_n) \cdot (\frac{1}{2}\alpha_n + \frac{1}{16}\alpha_n^3 \cdot (3x^2 + y^2)), \quad (14)$$

$$\Delta P_z = -\frac{\partial(\mathbf{H} - H_0)}{\partial z} = \sum_{n=1}^{\infty} -q\bar{V}_n \cdot k_n \cdot \cos(k_n z + \phi_n) \cdot (\frac{1}{2}\alpha_n \cdot x + \frac{1}{16}\alpha_n^3 \cdot (x^2 + y^2) \cdot x). \quad (15)$$

Assume that there is a 6 GHz deflecting cavity with up to three space harmonics, it is possible to perform particle

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phase space tracking and illustrate the impacts from space harmonics. The cavity period length is $d = 0.03$ m. The effective voltage and the synchronous phase of these three space harmonics are

- $n = 0$: $\bar{V}_0 = 1$, $\phi_0 = 0$,
- $n = 1$: $\bar{V}_1 = 0.2$, $\phi_1 = \pi/3$,
- $n = 2$: $\bar{V}_2 = 0.15$, $\phi_2 = -\pi/3$.

An electron beam with Gaussian distributions is employed in the tracking simulation where the longitudinal coordinates follow a uniform distribution just for illustration purpose. With these relatively strong space harmonic components, the phase space is shown in Fig. 5 and Fig. 6.

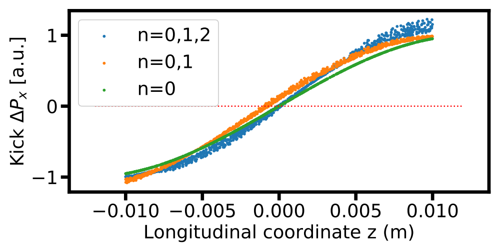

Figure 5: Phase space of z–ΔP_x, after passing by the deflecting cavity with up to three space harmonics.

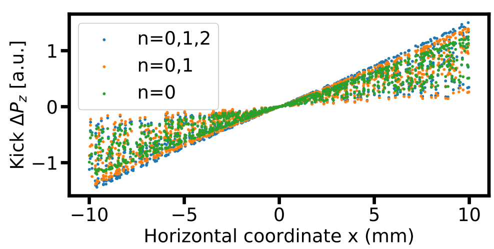

Figure 6: Phase space of x–ΔP_z, after passing by the deflecting cavity with up to three space harmonics.

IMPLEMENTATION IN ELEGANT CODE

The ELEGANT code [1] has a simplified model of the electromagnetic fields in deflecting cavities, which is called "RFDF". This element's major function is discussed in a previous section "Thin-lens deflecting cavity".

To include space harmonic components in the tracking simulations, a new element named "SHRFDF" (space harmonic RF deflector) is created in ELEGANT code [1]. The cavity period length d is a scalar input, and the cavity cell phase advance is another scalar input. The effective voltage and the synchronous phase of the space harmonics will be input as two arrays. Using some different parameters as

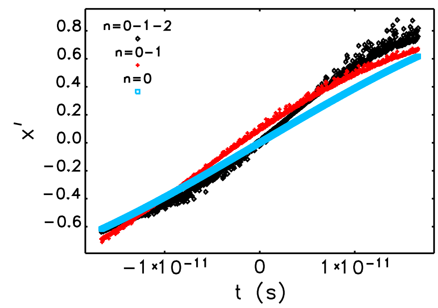

Figure 7: ELEGANT simulations [1], phase space of $t-\Delta x'$, after passing by the deflecting cavity with up to three space harmonics.

compared to Fig. 5 and Fig. 6, a demonstration of the new ELEGANT simulation capability is shown in Fig. 7.

CONCLUSIONS

Several possible ways are discussed to perform particle tracking simulations through electromagnetic fields in the RF cavity. The space harmonic approach could provide an accurate model, and at the meantime are still efficient in simulation computing time. The details and the implementations of the space harmonic approach are presented. It is a reasonable tradeoff between the simplest model (with zeroth harmonic) and the direct tracking through 3D electromagnetic fields map. Work is in progress to directly tracking through a three dimensional field map and integrate the momentum/coordinates change due to the electromagnetic force.

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