

# ALGORITHMS USED IN ACTION AND PHASE JUMP ANALYSIS TO ESTIMATE CORRECTIONS TO QUADRUPOLE ERRORS IN THE INTERACTION REGIONS OF THE LHC

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## Abstract

Action and phase jump analysis has been used to estimate corrector strengths in the high luminosity interaction regions of the LHC. It has been proven that these corrections are effective to eliminate the beta-beating that is generated in those important regions and that propagates around the ring. More recently, it was also shown that the beta-beating at the interaction point can also be suppressed by combining  $k$ -modulation measurements with action and phase jump analysis. Applying this technique to the re-commissioning of the LHC in 2021 requires a good knowledge of the software developed for action and phase jump analysis over the years. In this paper a detailed description is made of all the modules that are part of this software and the corresponding algorithms.

## INTRODUCTION

A one-turn particle trajectory in a linear lattice is conventionally described by

$$z(s) = \sqrt{2J_c\beta_r(s)}\sin(\psi_r(s) - \delta_c), \quad (1)$$

where  $z$  represents the distance between the transverse position of the particle respect to the design trajectory,  $s$  is the longitudinal position of the particle around the ring,  $J_c$  and  $\delta_c$  are the action constants, and finally  $\beta_r$  and  $\psi_r$  correspond to the lattice functions with errors. These lattice functions can also be expressed as

$$\beta_r = \beta_n + \Delta\beta, \quad (2)$$

$$\psi_r = \psi_n + \Delta\psi, \quad (3)$$

where  $\beta_n$  and  $\psi_n$  are the nominal lattice functions while  $\Delta\beta$  and  $\Delta\psi$  correspond to the beating produced by the magnetic errors in the accelerator. A one-turn particle trajectory can also be described by

$$z(s) = \sqrt{2J(s)\beta_n(s)}\sin(\psi_n(s) - \delta(s)) \quad (4)$$

In this case,  $J$  and  $\delta$  change along the axial coordinate in accordance with the magnetic errors present around the ring, resulting in what is called Action and Phase Jump (APJ) Analysis [1]. This approach has proven to be effective in estimating magnetic errors in the Interaction Regions (IRs) of high energy accelerators [1–3]. Under APJ analysis, actions and phases are inferred from one-turn beam trajectory and the nominal lattice functions. Plots of these action and phases as function of the longitudinal coordinate  $s$  reveals significant jumps at the IRs. These jumps can be used to

estimate how the strength of some of the quadrupole of the Interaction Region should be changed to suppress the jump. This is done by first estimating the equivalent magnetic deflection or kick that all the quadrupole errors of the IR would make in the beam trajectory. Then, analytical expressions are used to find the corresponding corrections. The IRs are mainly formed by two sets of three quadrupoles called triplets, one located to the left of the IP and the other to the right. The first scheme of correction estimated with APJ analysis used only two quadrupoles of the IR, one for each triplet [3]. The effectiveness of this correction was tested verifying that the jumps in action and phase were significantly reduced after applying the correction. The effectiveness of this correction was also tested through the beta-beating. This test revealed that the 2-corrector scheme is effective in suppressing the beta-beating around the ring but not at the Interaction Point (IP). This problem was solved with a new scheme of correction that uses the variables associated to  $k$ -modulation measurements as shown in [4]. Different to the previous scheme of correction, this new scheme of correction uses four quadrupoles per IR. Description of  $k$ -modulation measurements and their associated variables can be found in [5].

## BEAM TRAJECTORIES FROM TURN-BY-TURN DATA

Beam used for APJ analysis consist of only one bunch, which has been excited to large amplitude oscillations with a special device called the AC dipole. The bunch centroid is measured with the 500 dual Beam Position Monitors (BPMs) of the LHC for 6600 turns. All this data is stored in an ASCII file called a Turn-By-Turn (TBT) data set, which allows to draw a trajectory for every turn (simple trajectory) as seen in Fig. 1. In this paper, average trajectories are used rather than simple trajectories. The average trajectories are obtained by averaging simple trajectories of certain selected turns. The most common average trajectory is the closed orbit, which is obtained after averaging all the 6600 simple trajectories stored in a TBT data set. Other average trajectories are built so that they have maximum values of at specific points of the accelerator where the errors want to be estimated. Those average trajectories are called average max trajectories.

## MODULES OF THE APJ SOFTWARE

The software is divided in several modules, which are executed according to the flowchart shown in Fig. 2. Each module performs a specific task as explained below.

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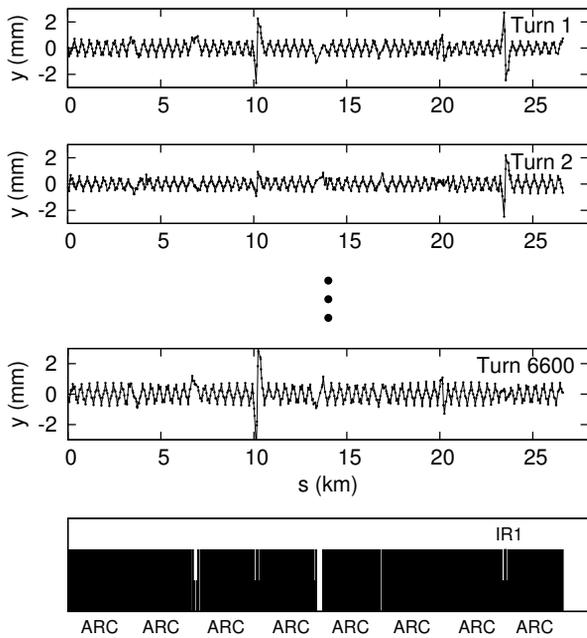


Figure 1: Contents of a TBT data set of 6600 turns. The small dots represent BPM measurements. The collection of small dots in each graph represent what is called a simple trajectory. Then, a simple trajectory is any one-turn trajectory extracted from a particular TBT data set.

### Module A (Nominal Files)

In this module the LHC magnet lattice, which is an ordered list of the magnets and other elements of the accelerator, is used to generate the nominal lattice functions (file lattice.asc) through a program called MADX [6]. Two additional files are also created in this module. The first is a rudimentary sketch of the principal magnets of the accelerator as a function of the longitudinal coordinate, examples of which can be seen in the lower part of Fig. 1. The second is a list of integrals of the beta functions of the triplet quadrupoles (file triplet.dat), which are used to estimate the corrector strengths through modules E and F. To calculate these integrals, it is necessary to subdivide the triplet quadrupoles into smaller quadrupoles since the MADX program only provides one value of the beta function for each element. The integrals are estimated by the Romberg integration method. Other integration methods like the trapezoidal method and the Simpson's method were used (in [7] and [8] respectively), but they are slower than the Romberg integration method since they need a finer subdivision of the quadrupoles to obtain the same accuracy. The outputs of this module are used for simulations and experimental data analysis.

### Module B (Experimental Lattice)

Experimental quantities related to the lattice functions are obtained by a software developed at CERN called GetLLM [9]. The output of this software is taken by module B and converted into the experimental lattice\_err.asc file.

Additionally, since the AC dipole produces an additional jump in the action and phase graphs, the axial coordinate of lattice\_err.asc must be reassigned so that the jump moves to the edge of the graphs. In this way, calculations associated with the action and phase graphs are not affected by these jumps.

### Module C (Reassigning s in TBT data)

This module takes all the BPMs of a TBT data set and reassigns their longitudinal coordinate to avoid the effect of the AC dipole in the action and phase graphs, as explained in the previous module.

### Module D (Average Trajectories)

As indicated above, average trajectories are created by averaging simple trajectories of certain turns of a particular TBT data set, so that the excursion of the average trajectory is a maximum at the axial location of the equivalent kick. Selection of turns is made according to the average phase of the corresponding simple trajectory. Therefore, the first task performed by this module is to estimate the phase of each simple trajectory of a TBT data set, which is done by applying the algorithms developed in [2] to estimate action and phase in each BPM and averaging the phase measurements for each turn of the TBT data set. The second task of this module is to choose the turns that will be part of the average trajectory. This is done by choosing turns according to their average phases with respect to the phase that provides the maximum excursion of the trajectory

### Module E (Equivalent Kicks and Their Quad Components)

Once average trajectories are created, the equivalent kicks and their components can be estimated from the action and phases extracted from this kind of trajectories and the mathematical relations of [3]. The equivalent kicks should be estimated for at least two kinds of average max trajectories in both planes and for the two triplets of the IR under study. The inputs of this module are the four kinds of average trajectories created in the previous module, the axial location of the equivalent kick, the quadrupoles that will be used as correctors and the extension of the arc segments to left and to the right of the IR under study. The results of this module are printed in 4 ASCII files (one for each plane and each triplet) with information about the quadrupole components, the beam position at the location of the equivalent kick and related lattice parameters for subsequent calculations.

### Module F (Corrector Strengths)

Corrector strengths can be estimated from the output files of the previous module. There are two ways of making these estimates: the first uses equations (39) of [3] with two average max trajectories and the second finds the corrector strengths through an adjustment procedure involving all four average max trajectories and equation (38) of [3]. Both give satisfactory results.

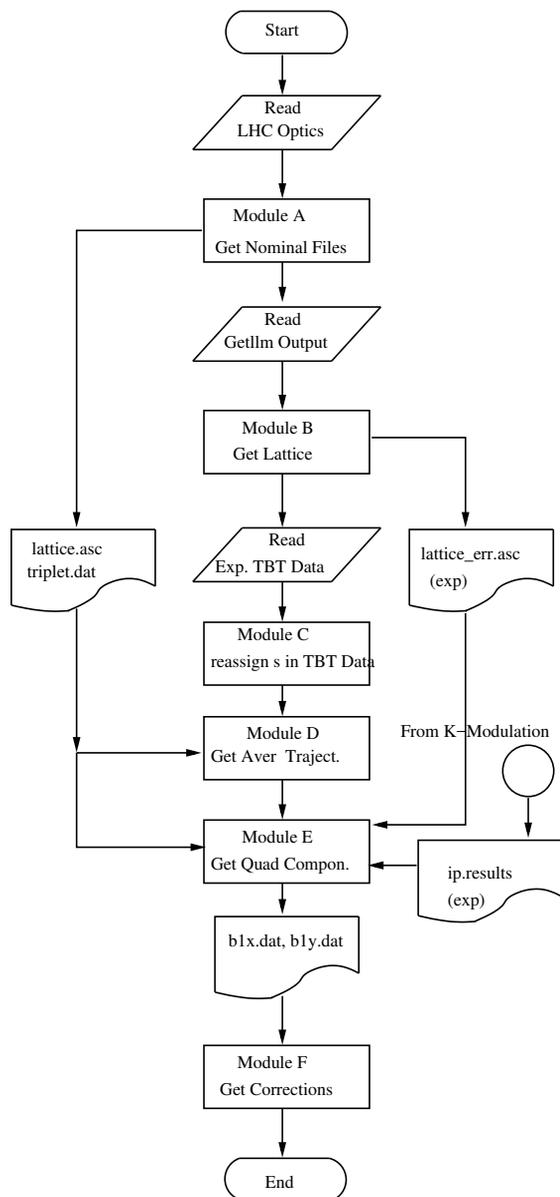


Figure 2: Flowchart of software modules to analyze an experimental TBT data set and estimate the corrections at a particular IR.

### Module G (Simulating Files with Errors)

Lattice functions are created in this module as in module A except that an arbitrary list of magnetic errors and corrector strengths can be added to the LHC magnet lattice resulting in a file called `lattice_err.asc`. Besides the lattice functions, simulations of the expected k-modulation variables are also generated (file `ip.results`). Simulated lattice functions with errors, k-modulation variables, and the TBT data generated through module H is used routinely to test the correct functioning of the complete software.

### Module H (Simulating TBT data)

Simulated TBT data with the same format of the TBT data coming out of module C is generated through this module. The simulated data is created with MADX through one of its modules called PTC track. The same magnet lattice used in module A is used here. In addition, an arbitrary list of magnetic errors and corrector strengths can be added.

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