ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-THYBA5

STUDY OF FLUCTUATIONS IN UNDULATOR RADIATION IN THE IOTA RING AT FERMILAB*

I. Lobach†, University of Chicago, Chicago, Illinois, USA
K. J. Kim[1], ANL, Argonne, Illinois, USA
T. Shaftan, BNL, Upton, New York, USA
V. Lebedev, S. Nagaitsev[1], A. Romanov, G. Stancari, A. Valishev, Fermilab, Batavia, Illinois, USA
A. Murokh, RadiaBeam, Los Angeles, California, USA
A. Halavanau, Z. Huang, V. Yakimenko, SLAC, Menlo Park, California, USA
[1]also at University of Chicago

Abstract

We study turn-by-turn fluctuations in the number of emitted photons in an undulator, installed in the IOTA electron storage ring at Fermilab, with an InGaAs PIN photodiode and an integrating circuit. In this paper, we present a theoretical model for the experimental data from previous similar experiments and in our present experiment, we attempt to verify the model in an independent and a more systematic way. Moreover, in our experiment we consider the regime of very small fluctuation when the contribution from the photon shot noise is significant, whereas we believe it was negligible in the previous experiments. Accordingly, we present certain critical improvements in the experimental setup that let us measure such a small fluctuation.

INTRODUCTION

Reference [1] reports on the results of experimental studies of statistical properties of wiggler and bending-magnet radiation in an electron storage ring at BNL. A silicon PIN photodiode combined with an amplifier and an integrator were used to obtain a signal (the number of photoelectrons $\mathcal{N}$) representing the number of detected synchrotron radiation photons per turn. Then, the average amplitude of this signal $\langle\mathcal{N}\rangle$ was varied by a set of neutral density (ND) filters, and the dependence of $\mathrm{var}(\mathcal{N})$ on $\langle\mathcal{N}\rangle$ was studied. Experimental data from this experiment are plotted in Fig. 1. In this plot, the noise of the apparatus (shown by the red line) was subtracted. The authors concluded that for the bending-magnet radiation $\mathrm{var}(\mathcal{N}) \propto \langle\mathcal{N}\rangle$, and for the wiggler radiation $\mathrm{var}(\mathcal{N}) \propto \langle\mathcal{N}\rangle^2$. Qualitative explanation of the results was provided in [1]. Here, we present a theoretical model for the effect, which can predict the fluctuations very precisely, and then we repeat the BNL experiment in IOTA [2] with several major improvements in the setup.

THEORETICAL MODEL

It was shown in [3,4] that any classical current produces radiation with Poisson statistics. Since a bunch of electrons in a bending magnet, wiggler, or undulator constitute a classical current (negligible electron recoil), one may argue that the turn-by-turn statistics for photoelectron count for any of these kinds of radiation in a storage ring is Poisson, i.e., $\mathrm{var}(\mathcal{N}) = \langle\mathcal{N}\rangle$, where $\mathcal{N}$ is the number of detected photons (photoelectrons). However, it is not correct, because every turn relative positions of the electrons in the bunch change and hence, every turn, it is a new classical current. That is, the incoherent sum of electromagnetic fields produced by the electrons is slightly different from turn to turn, producing different amounts of emitted power. These effects result in the following equation for variance of the number of detected photons

$$\mathrm{var}(\mathcal{N}) = \langle\mathcal{N}\rangle + \frac{1}{M}\langle\mathcal{N}\rangle^2, \quad (1)$$

where M can be identified with the number of coherent modes [5], which depends on the kind of radiation (bending-magnet, wiggler, undulator, etc.), the bunch parameters, and the detection configuration. Although the Poisson contribution in Eq. (1) is related to the quantum nature of emitted light, the incoherence contribution (the second term) is purely classical [6]. The expression for $1/M$ takes the form

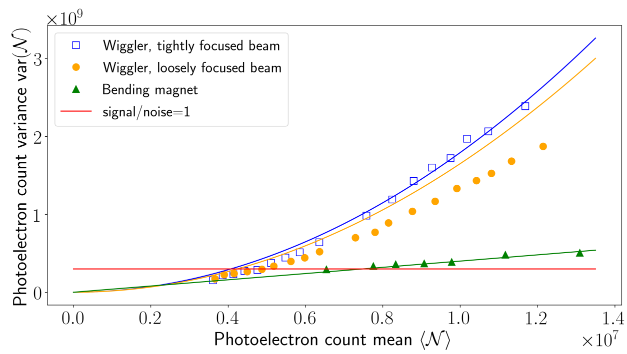

Figure 1: Experimental data from [1], predictions for wiggler radiation fluctuations made by our theoretical model, and a linear fit for the bending-magnet radiation.

---
* This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.
† ilobach@uchicago.edu

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-THYBA5

$1/M = \Delta/\langle\mathcal{N}\rangle^2$, with $\langle\mathcal{N}\rangle = N_e\mathcal{N}^{(1)}_{\text{q.c.}}$ and

$$\Delta = N_e\,(N_e-1)\,\frac{\sqrt{\pi}}{\sigma_z}\int k^4 dk\,d\Omega_1 d\Omega_2 \times I^{(1)}_{k\boldsymbol{n}_1} I^{(1)}_{k\boldsymbol{n}_2} e^{-k^2\sigma_x^2(\theta_{1x}-\theta_{2x})^2-k^2\sigma_y^2(\theta_{1y}-\theta_{2y})^2}, \quad (2)$$

where the direction unit vectors $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ are defined as $\boldsymbol{n}_1 \approx (\theta_{1x},\theta_{1y},1)$, $\boldsymbol{n}_2 \approx (\theta_{2x},\theta_{2y},1)$, N_e is the number of electrons in the bunch, k is the magnitude of the wave-vector, $d\Omega_1$ and $d\Omega_2$ are infinitesimal elements of solid angle, σ_x, σ_y, σ_z are transverse and longitudinal sizes of the bunch (Gaussian shapes are assumed), $I^{(1)}_{\boldsymbol{k}}$ is the quasi-classical spectral-angular distribution for the number of detected photons $\mathcal{N}^{(1)}_{\text{q.c.}}$ in the case when there is only one electron in the ring:

$$I^{(1)}_{\boldsymbol{k}} = \frac{d\mathcal{N}^{(1)}_{\text{q.c.}}}{d\boldsymbol{k}}, \qquad \mathcal{N}^{(1)}_{\text{q.c.}} = \int d\boldsymbol{k}\, I^{(1)}_{\boldsymbol{k}}, \quad (3)$$

with $d\boldsymbol{k} = dk_x dk_y dk_z = k^2 dk d\Omega$. In the above derivations, we considered only one polarization, the beam divergence was assumed negligible, and $k\sigma_z \gg 1$.

We are now in position to compare experimental data from [1] with our model's predictions, see Fig. 1. We will disregard the bending-magnet data (green triangle points with a linear fit in Fig. 1), since the authors of [1] suspect that the data actually represent the statistics of secondary photons produced in the Pyrex vacuum chamber window rather than the statistics of the original bending-magnet radiation. As to the wiggler radiation data, we used the values of parameters from [1] and performed the integration in Eq. (2) numerically with expressions for $I^{(1)}_{\boldsymbol{k}}$ for wiggler radiation from [7]. Our model's predictions are plotted in Fig. 1 along with the experimental data. It is noteworthy, that in this experiment the quantum Poisson contribution (the first term in Eq. (1)) was negligible. In general, the agreement is good. However, the details of the experimental conditions are not available anymore. In particular, it is difficult to analyze the reasons for the small systematic discrepancy for the "loosely focused beam" data. These considerations, combined with the fact that fluctuations of the same nature are present in SASE FELs [5, 6, 8–10], motivated us to carry out an independent experiment and to study the fluctuations in a more detailed and systematic way.

MEASUREMENTS

Our studies were performed in the IOTA ring and only concerned undulator radiation for now. Main parameters of the experiment are given in Table 1. We used an InGaAs PIN photodiode (Hamamatsu G11193-10R) connected to an op-amp based transimpedance amplifier with a regular RC low-pass filter ($R_\text{f} = 10\,\text{k}\Omega$, $C_\text{f} = 2\,\text{pF}$, $\tau = 20\,\text{ns}$) in the feedback. Thus, the number of photoelectrons generated in the photodiode $\mathcal{N}$ can be extracted from the signal amplitude A by $\mathcal{N} = C_\text{f}A/e$. To considerably improve signal-to-noise ratio, we used a comb filter with a delay equal to exactly one revolution in IOTA (133 ns). That is, we used a signal splitter to obtain two copies of the initial signal from the photodetector, then one of the copies was delayed by one IOTA revolution, and then the two copies went to a hybrid, which yielded their sum and difference, which, in their turn, went to two channels of a Rohde&Schwarz RTO 1044 scope @20 GSa/s. We will refer to them as Σ- and Δ-channels, respectively. One experimental dataset constituted a scope waveform of about 11000 IOTA revolutions.

Table 1: Experiment Parameters

IOTA circumference	40 m (133 ns)
Beam energy	100 MeV
Max average current	4.0 mA
σ_x, σ_y @1.3 mA	815 µm, 75 µm
σ_z @1.3 mA	38 cm
Undulator parameter K	1.0
Undulator period	55 mm
Number of periods	10
First harmonic wavelength	1077 nm
Photodiode diameter	1 mm
Quantum efficiency @1077 nm	80 %

We will denote the signal from the photodetector averaged over many revolutions by $S(t)$ (where t is within one turn). Consider two consecutive revolutions with relative amplitude deviations δ_1 and δ_2, so that the signals going to the comb filter are given by $S_1 = (1+\delta_1)S(t)$ and $S_2 = (1+\delta_2)S(t)$. Irrelevant pre-factors due to attenuation in the comb filter and the hybrid will be omitted hereinafter. Then the two outputs of the hybrid are $\Sigma = S_1 + S_2 \approx 2S(t)$ and $\Delta = S_1 - S_2 = (\delta_1 - \delta_2)S(t)$. Basically, our goal was to measure the turn-by-turn variance of the amplitude of the signal from the photodetector $A = (1+\delta)S(t_{\max})$, where δ is different for each revolution, and $t_{\max}$ corresponds to the maximum of $S(t)$. Since $\text{var}(a-b) = \text{var}(a) + \text{var}(b)$ for independent a and b, it follows that $\text{var}(\delta_1 - \delta_2) = 2\text{var}(\delta)$. Hence, $\text{var}(A)$ can be found as $\text{var}(A) = \text{var}(\Delta(t_{\max}))/2$; $\text{var}(\mathcal{N})$ can be determined by using $\mathcal{N} = C_\text{f}A/e$.

The above formula for var(A) can be used when noise is negligible. However, in the actual experiment with the undulator, in Δ-channel, signal/noise $\lesssim$1. Therefore, we had to develop a special algorithm to subtract noise. Its idea is that if we take the equation for Δ-channel at a fixed time t

$$\Delta(t) = (\delta_1 - \delta_2)S(t) + \text{noise}, \quad (4)$$

and compute variance over many IOTA turns, then we get

$$\text{var}(\Delta(t)) = \text{var}(\delta_1 - \delta_2)S^2(t) + \text{var}(\text{noise}), \quad (5)$$

where variance of noise is just a constant. On top of this constant level there is a peak $\propto S^2(t)$, and one can deduce var(A) from the height of this peak.

To test the proposed algorithm and find its error we took measurements for a test light pulse source (laser diode @1064 nm modulated by a pulse generator) with almost identical to IOTA's time structure, but with larger fluctuations,

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-THYBA5

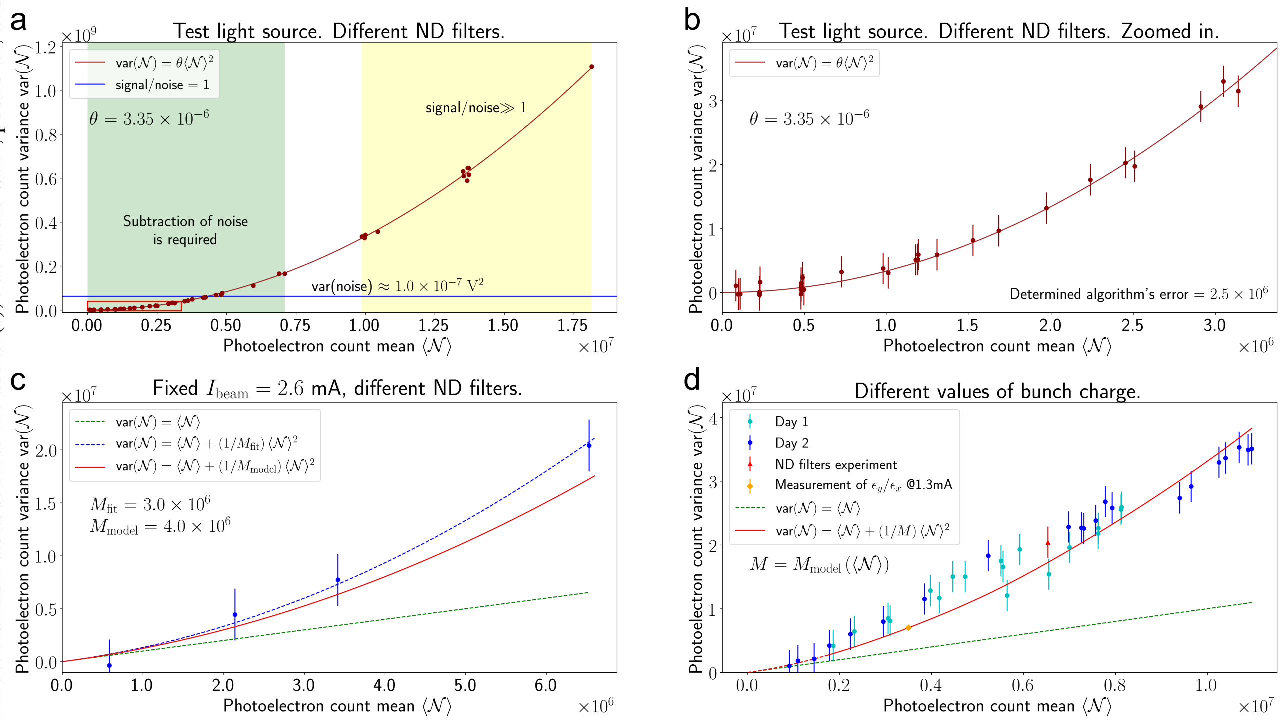

Figure 2: Results of measurements with the test light source (a,b) and with the undulator in IOTA (c,d).

determined by errors in the modulating pulse generator (the laser diode's own fluctuations were negligible, possibly even Sub-Poisson [11, 12]). The fluctuations were much larger than the noise in Δ-channel and could be reliably measured, they also remained constant with time. Then, we used a number of different ND filters to reduce $\langle\mathcal{N}\rangle$. In this case, rms fluctuations of $\mathcal{N}$ decreased in proportion with $\langle\mathcal{N}\rangle$, since ND filters did not change relative fluctuations. Thus, we had a very accurate measurement of relative fluctuations at large $\langle\mathcal{N}\rangle$, and a prediction for when we used ND filters to reduce $\langle\mathcal{N}\rangle$: $\mathrm{var}(\mathcal{N}) = \theta\langle\mathcal{N}\rangle^2$, where θ is the relative fluctuation measured at large amplitude. The idea of the test was that if fluctuations at small $\langle\mathcal{N}\rangle$ (when signal/noise $\lesssim 1$) measured by the algorithm with noise subtraction agree with the expected curve, then we can trust this algorithm. In Fig. 2a,b, one can see that experimental points indeed follow the expected curve very well. Figure 2b depicts the region of fluctuations that were measured in the actual experiment with the undulator in IOTA. Therefore, three standard deviations from the expected curve in Fig. 2b were used as an error bar for the plots for the undulator radiation in IOTA in Fig. 2c ($\langle N\rangle$ varied by ND filters) and in Fig. 2d ($\langle\mathcal{N}\rangle$ varied by changing bunch charge).

DISCUSSION

To make a theoretical prediction for M and, consequently, for $\mathrm{var}(\mathcal{N})$, we had to know the dimensions of the electron bunch in IOTA. It was determined experimentally with a wall-current monitor that the longitudinal bunch size was approximately constant for $I_{\mathrm{beam}} > 0.65$ mA, $\sigma_z = 38$ cm. We believe that vertical emittance was determined solely by multiple scattering in the background gas, and, hence, it was independent of the beam current as well. We were able to determine the ratio of vertical and horizontal emittances from pictures of bending-magnet radiation at $I_{\mathrm{beam}} = 1.3$ mA. In addition, we could compute the theoretical radiation damping rates for all three directions. Thus, the only unknown parameters at 1.3 mA were horizontal emittance ϵ_x and momentum spread σ_p. We found these unknowns by balancing the radiation damping rates for ϵ_x and σ_p with corresponding simulated [13] intrabeam scattering growth rates. Thus, at 1.3 mA we knew all parameters of the bunch, including ϵ_y. At other values of I_{beam}, we used the value of ϵ_y found at 1.3 mA, and computed values of ϵ_x and σ_p by equating the radiation damping and intrabeam scattering rates again. Hence, we fully characterized the bunch dimensions and predicted $\mathrm{var}(\mathcal{N})$ at all values of $I_{\mathrm{beam}} > 0.65$ mA, see the red curves in Fig. 2c,d. However, there is still some uncertainty in a number of parameters of the bunch in IOTA, and also in the detection configuration. Therefore, the red curves should be understood as our best guess, rather than a reliable prediction.

Nonetheless, the agreement between our best guess and the experimental data is fairly good. We collected data for different values of bunch charge (see Fig. 2d), which was not done in [1]. We worked with such bunch parameters, that the Poisson contribution (dashed green lines in Fig. 2c,d) was comparable with the incoherence contribution to the fluctuations. Whereas in the BNL experiment, for wiggler light, the Poisson contribution was negligible. This also implies that the fluctuations in our experiment were smaller than in [1] and we had to improve the precision of our measurements by using the comb filter with one turn delay and the noise subtraction algorithm. It is noteworthy, that the effect of

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-THYBA5

fluctuations in undulator light may find an application in beam diagnostics. Since these fluctuations depend on the bunch size (see Eq. (2)), they can assist one in determining dimensions of the electron bunch [14–17]. This technique can be especially useful for very small bunches, when other methods, e.g., camera images, cease to be reliable. However, it should be understood that for the proposed method to work, the electron bunch has to radiate incoherently.

REFERENCES

[1] M. C. Teich, T. Tanable, T. C. Marshall, and J. Galayda, "Statistical properties of wiggler and bending-magnet radiation from the Brookhaven vacuum-ultraviolet electron storage ring", *Phy. Rev. Lett.*, vol. 65, no. 27, p. 3393, Dec. 1990. doi:10.1103/PhysRevLett.65.3393

[2] S. Antipov *et al.*, "IOTA (Integrable Optics Test Accelerator): facility and experimental beam physics program", *J. Instrum.*, vol. 12, no. 3, p. T03002, Mar. 2017. doi:10.1088/1748-0221/12/03/T03002

[3] R. J. Glauber, "Some notes on multiple-boson processes", *Phy. Rev. Lett.*, vol. 84, no. 3, p. 395, Nov. 1951. doi:10.1103/PhysRev.84.395

[4] R. J. Glauber, "Coherent and Incoherent States of the Radiation Field", *Phys. Rev.*, vol. 131, no. 6, pp. 2766-2788, Sep. 1963. doi:10.1103/PhysRev.131.2766

[5] K. J. Kim, Z. Huang, and R. Lindberg, *Synchrotron Radiation and Free-Electron Lasers*, Cambridge, United Kingdom: Cambridge University Press, 2017.

[6] E. L. Saldin, E. V. Schneidmiller, and M. V. Yurkov, *The Physics of Free Electron Lasers*, Berlin, Germany: Springer-Verlag, 2000.

[7] J. A. Clarke, "Undulator angular flux distribution", in *The Science and Technology of Undulators and Wigglers*, New York, NY, USA: Oxford University Press Inc., 2004, pp. 58–73.

[8] C. Pellegrini, A. Marinelli, and S. Reiche, "The physics of x-ray free-electron lasers", *Rev. Mod. Phys.*, vol. 88, no. 1, p. 015006, Mar. 2016. doi:10.1103/RevModPhys.88.015006

[9] K.-J. Kim, "Start-up noise in 3-D self-amplified spontaneous emission", *Nucl. Instrum. Methods A*, vol. 393, no. 1-3, pp. 167-169, Jul. 1997.

[10] S. Benson and J. M.J. Madey, "Shot and quantum noise in free electron lasers", *Nucl. Instrum. Methods A*, vol. 237, no. 1-2, pp. 55-60, Jun. 1985.

[11] P. R. Tapster, J. G. Rarity, and J. S. Satchell, "Generation of Sub-Poissonian Light by High-Efficiency Light-Emitting Diodes", *Europhys. Lett.*, vol. 4, no. 3, pp. 293-299, Aug. 1987.

[12] S. Machida and Y. Yamamoto, "Observation of sub-poissonian photoelectron statistics in a negative feedback semiconductor laser", *Opt. Commun.*, vol. 57, no. 4, pp. 290-296, Mar. 1986. 10.1016/0030-4018(86)90100-8

[13] S. Nagaitsev, "Intrabeam scattering formulas for fast numerical evaluation", *Phys. Rev. ST Accel. Beams*, vol. 8, no. 6, p. 064403, Jun. 2005. doi:10.1103/PhysRevSTAB.8.064403

[14] F. Sannibale, G. Stupakov, M. Zolotorev, D. Filippetto, and L. Jägerhofer, "Absolute bunch length measurements by incoherent radiation fluctuation analysis", *Phys. Rev. ST Accel. Beams*, vol. 12, no. 3, p. 032801, Mar. 2009. doi:10.1103/PhysRevSTAB.12.032801

[15] P. Catavas *et al.*, "Measurement of Electron-Beam Bunch Length and Emittance Using Shot-Noise-Driven Fluctuations in Incoherent Radiation", *Phy. Rev. Lett.*, vol. 82, no. 26, p. 5261, Jun. 1999. doi:10.1103/PhysRevLett.82.5261

[16] V. Sajaev, "Measurement of bunch length using spectral analysis of incoherent radiation fluctuations", in *Proc. 11th Beam Instrumentation Workshop*, Knoxville, TN, May 2004 p. 73. doi:10.1063/1.1831134

[17] V. Sajaev, "Determination of Longitudinal Bunch Profile using Spectral Fluctuations of Incoherent Radiation", in *Proc. EPAC'00*, Vienna, Austria, Jun. 2000, paper WEP1B04, pp. 1806–1808.