Linear Optics Characterization and Correction Method using Turn-by-Turn BPM Data based on Resonance Driving Terms with Simultaneous Calibration Capability

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DTBLOC (Driving-Terms-Based Linear Optics Characterization / Correction)

➢ A new fast linear lattice characterization / correction method based on turn-by-turn (TbT) beam position monitor (BPM) data in storage rings recently developed and demonstrated experimentally at NSLS-II.

- ✓ Input (Observables): 4 frequency components extracted from TbT data & dispersion functions
- ✓ Output (Fitting Parameters): normal & skew quadrupole errors, BPM errors (H/V gain, roll & deformation)
- ✓ Iterative least-square fitting via SVD w/ an analytical Jacobian matrix based on resonance driving terms (RDTs)
- ✓ Only ~5 min for data acq. & proc. and fitting (vs. ~1 hr to measure full ORM for LOCO) at NSLS-II
- ✓ Corrected to <1% beta-beating, dispersion errors of ~1 mm, emittance coupling ratio on the order of10^{-4}
- ✓ As a validation tool for estimated magnetic and BPM error values.

NSLS-II Bare Lattice

Energy	3 GeV
Circumference	791.96 m
# of DBA cells	30 (15×2)
RF frequency	499.68 MHz
Harmonic #	1320
Rev. period	2.64 μs
Ring tune: ν_x, ν_y	33.22, 16.26
Chromaticity: ξ_x, ξ_y	+2, +2
Mom. compaction α_c	3.6×10^{-4}
Damping time $\tau_{x,y}$	54 ms
Horiz. emittance ε_x	2.1 mm-rad

Experimental Lattice Correction

	Initial	Corr. #1	Corr. #2	Cycling
RMS $\Delta\beta_x/\beta_x$ [%]	4.3	0.8	0.4	0.5
RMS $\Delta\beta_y/\beta_y$ [%]	2.9	0.5	0.4	0.3
RMS $\Delta\eta_x$ [mm]	7.1	1.5	1.1	1.2
RMS $\Delta\eta_y$ [mm]	3.0	1.2	1.2	1.1
Lifetime [hr]	31.6	12.6	8.8	8.5
Avg. $\varepsilon_y/\varepsilon_x$ [%]	0.4	~0.04	N/A	N/A

➢ Lifetime reduction by 2.5 (31.6 to 12.6 hr) roughly agrees with expected Touschek lifetime reduction by ~3 (sqrt of coupling ratio reduction of 10 from 0.4% to ~0.04%)

Primary & Secondary Freq. Components as Func. of RDTs

Complex Courant-Snyder Variables $h_{x,-}$ and $h_{y,-}$

$$h_{x,-}(s,N)=\hat{x}-i\hat{p}_x=\sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{s,x,0})}-2i\sum_{jklm} jf^{(s)}_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}e^{i[(1-j+k)(2\pi\nu_x N+\psi_{s,x,0})+(m-l)(2\pi\nu_y N+\psi_{s,y,0})]}$$

$$h_{y,-}(s,N)=\hat{y}-i\hat{p}_y=\sqrt{2I_y}e^{i(2\pi\nu_y N+\psi_{s,y,0})}-2i\sum_{jklm} lf^{(s)}_{jklm}(2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m-1}{2}}e^{i[(k-j)(2\pi\nu_x N+\psi_{s,x,0})+(1-l+m)(2\pi\nu_y N+\psi_{s,y,0})]}$$

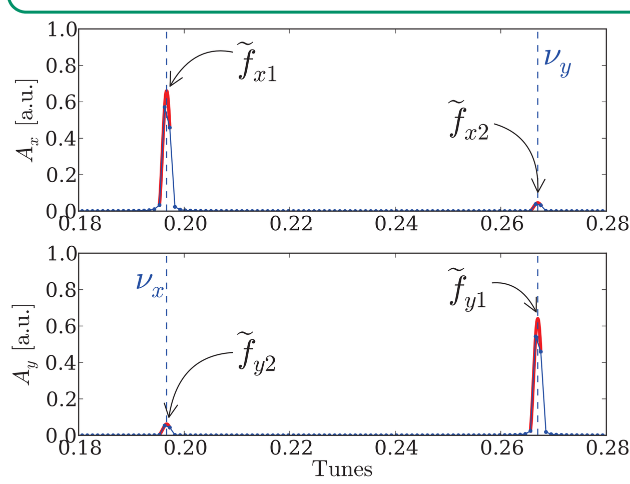
Linear RDTs

$$f_{2000}(s)=\frac{\sum_w(-\Delta b_2 L^w)\beta_x^w e^{i(2\Delta\phi_x^{w,s})}}{8(1-e^{2\pi i(2\nu_x)})}$$

$$f_{0020}(s)=\frac{\sum_w(+\Delta b_2 L^w)\beta_y^w e^{i(2\Delta\phi_y^{w,s})}}{8(1-e^{2\pi i(2\nu_y)})}$$

$$f_{\substack{1001\\1010}}(s)=\frac{\sum_w(\Delta a_2 L^w)\sqrt{\beta_x^w\beta_y^w}e^{i(\Delta\phi_x^{w,s}\mp\Delta\phi_y^{w,s})}}{4(1-e^{2\pi i(\nu_x\mp\nu_y)})}$$

➢ Starting from TbT complex CS variable expressions, add focusing errors only, followed by adding coupling errors.
➢ Obtain $(\hat{x},\hat{y})$ TbT expressions in terms of 2 sine-cosine pair terms with 2 diff. freq. as a func. of RDTs. Then convert them into (x,y) TbT to get $f_{x1,2}$ & $f_{y1,2}$



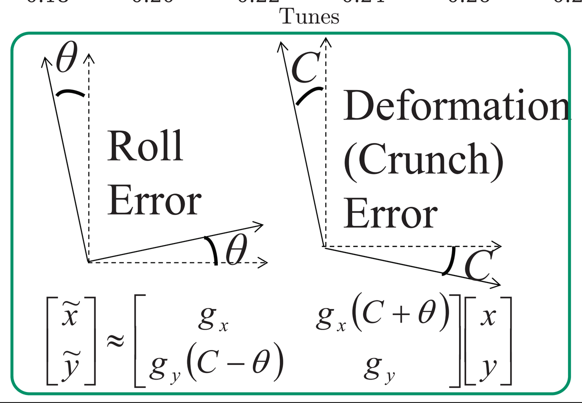
$$|f_{x1}|=\sqrt{\{C_x^{(x)}\}^2+\{S_x^{(x)}\}^2},\quad |f_{x2}|=\sqrt{\{C_y^{(x)}\}^2+\{S_y^{(x)}\}^2},$$
$$|f_{y1}|=\sqrt{\{C_y^{(y)}\}^2+\{S_y^{(y)}\}^2},\quad |f_{y2}|=\sqrt{\{C_x^{(y)}\}^2+\{S_x^{(y)}\}^2},$$

where

$$C_x^{(x)}=\sqrt{2I_x\beta_{x0}}(1+4\Im\{f_{2000}\}),\quad C_x^{(y)}\approx 2\sqrt{2I_x\beta_{y1}}(\Im\{f_{0110}\}+\Im\{f^V_{1010}\})$$
$$S_x^{(x)}=-\sqrt{2I_x\beta_{x0}}(4\Re\{f_{2000}\}),\quad S_x^{(y)}\approx 2\sqrt{2I_x\beta_{y1}}(\Re\{f_{0110}\}-\Re\{f^V_{1010}\})$$
$$C_y^{(x)}\approx 2\sqrt{2I_y\beta_{x1}}(\Im\{f_{1001}\}+\Im\{f^H_{1010}\}),\quad C_y^{(y)}=\sqrt{2I_y\beta_{y0}}(1+4\Im\{f_{0020}\})$$
$$S_x^{(x)}\approx 2\sqrt{2I_y\beta_{x1}}(\Re\{f_{1001}\}-\Re\{f^H_{1010}\}),\quad S_y^{(y)}=-\sqrt{2I_y\beta_{y0}}(4\Re\{f_{0020}\})$$
$$\beta_{x1}=\beta_{x0}\cdot(1+8\Im\{f_{2000}\})/(1+8\Im\{f_{0020}\})$$
$$\beta_{y1}=\beta_{y0}\cdot(1+8\Im\{f_{0020}\})/(1+8\Im\{f_{2000}\})$$

Adding BPM errors to $f_{x1,2}$ & $f_{y1,2}$ => Apparent freq. comp. $\tilde{f}_{x1,2}$ and $\tilde{f}_{y1,2}$ with $C_{x,y}$ and $S_{x,y}$ variables replaced by corresponding variables with ~:

$$\tilde{C}_x^{(x)}=g_x\{C_x^{(x)}+C_x^{(y)}(C+\theta)\},\quad \tilde{S}_x^{(x)}=g_x\{S_x^{(x)}+S_x^{(y)}(C+\theta)\},$$
$$\tilde{C}_y^{(x)}=g_x\{C_y^{(x)}+C_y^{(y)}(C+\theta)\},\quad \tilde{S}_y^{(x)}=g_x\{S_y^{(x)}+S_y^{(y)}(C+\theta)\},$$
$$\tilde{C}_x^{(y)}=g_y\{C_x^{(x)}(C-\theta)+C_x^{(y)}\},\quad \tilde{S}_x^{(y)}=g_y\{S_x^{(x)}(C-\theta)+S_x^{(y)}\},$$
$$\tilde{C}_y^{(y)}=g_y\{C_y^{(x)}(C-\theta)+C_y^{(y)}\},\quad \tilde{S}_y^{(y)}=g_y\{S_y^{(x)}(C-\theta)+S_y^{(y)}\}.$$



$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix}\approx\begin{bmatrix}g_x & g_x(C+\theta)\\ g_y(C-\theta) & g_y\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

H Primary

$$|\tilde{f}_{x1}|\cong g_x\sqrt{2J_x}\sqrt{\{\sqrt{\beta_{x0}}(1+4\Im\{f_{2000}\})+\sqrt{\beta_{y0}}(2\Im\{f_{0110}\}+2\Im\{f^V_{1010}\})(C+\theta)\}^2+\{\sqrt{\beta_{x0}}(-4\Re\{f_{2000}\})+\sqrt{\beta_{y0}}(2\Re\{f_{0110}\}-2\Re\{f^V_{1010}\})(C+\theta)\}^2}$$

H Secondary

$$|\tilde{f}_{x2}|\cong g_x\sqrt{2J_y}\sqrt{\{\sqrt{\beta_{x0}}\underbrace{(2\Im\{f_{1001}\}+2\Im\{f^H_{1010}\})}_{10^{-2}}+\sqrt{\beta_{y0}}\underbrace{(1+4\Im\{f_{0020}\})(C+\theta)}_{10^{-2}}\}^2+\{\sqrt{\beta_{x0}}\underbrace{(2\Re\{f_{1001}\}-2\Re\{f^H_{1010}\})}_{10^{-2}}+\sqrt{\beta_{y0}}\underbrace{(-4\Re\{f_{0020}\})(C+\theta)}_{10^{-4}}\}^2}$$

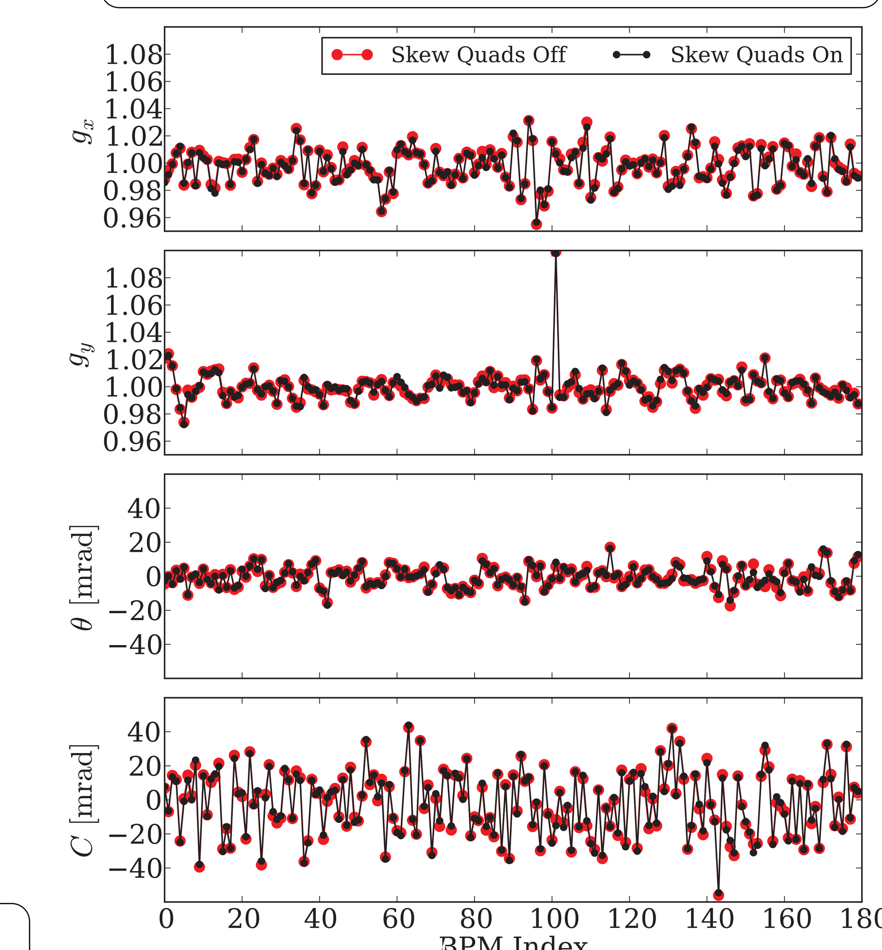
V Primary

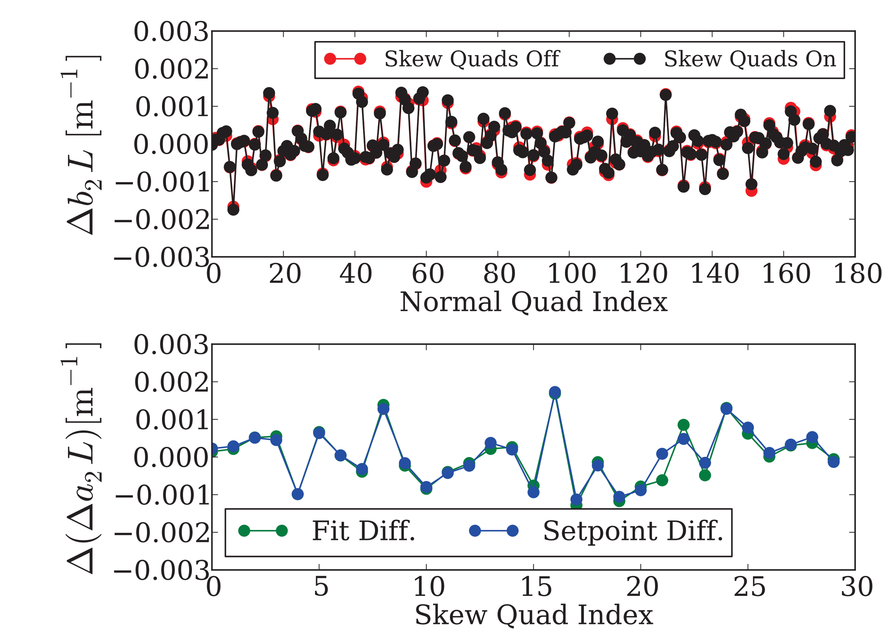
$$|\tilde{f}_{y1}|\cong g_y\sqrt{2J_y}\sqrt{\{\sqrt{\beta_{x0}}(2\Im\{f_{1001}\}+2\Im\{f^H_{1010}\})(C-\theta)+\sqrt{\beta_{y0}}(1+4\Im\{f_{0020}\})\}^2+\{\sqrt{\beta_{x0}}(2\Re\{f_{1001}\}-2\Re\{f^H_{1010}\})(C-\theta)+\sqrt{\beta_{y0}}(-4\Re\{f_{0020}\})\}^2}$$

V Secondary

$$|\tilde{f}_{y2}|\cong g_y\sqrt{2J_x}\sqrt{\{\sqrt{\beta_{x0}}\underbrace{(1+4\Im\{f_{2000}\})(C-\theta)}_{10^{-2}}+\sqrt{\beta_{y0}}\underbrace{(2\Im\{f_{0110}\}+2\Im\{f^V_{1010}\})}_{10^{-2}}\}^2+\{\sqrt{\beta_{x0}}\underbrace{(-4\Re\{f_{2000}\})(C-\theta)}_{10^{-4}}+\sqrt{\beta_{y0}}\underbrace{(2\Re\{f_{0110}\}-2\Re\{f^V_{1010}\})}_{10^{-2}}\}^2}$$

Exp. Lattice Characterization

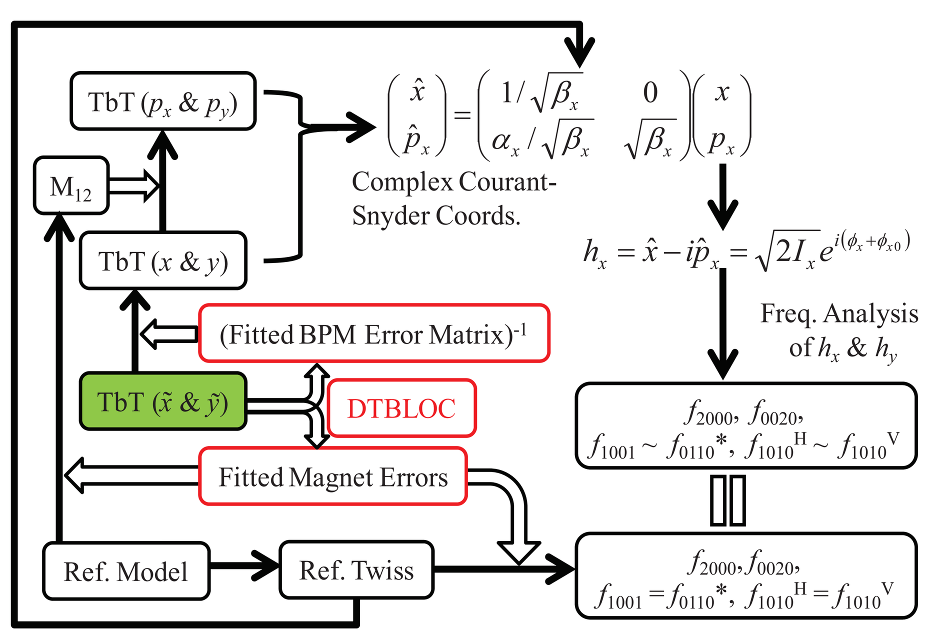




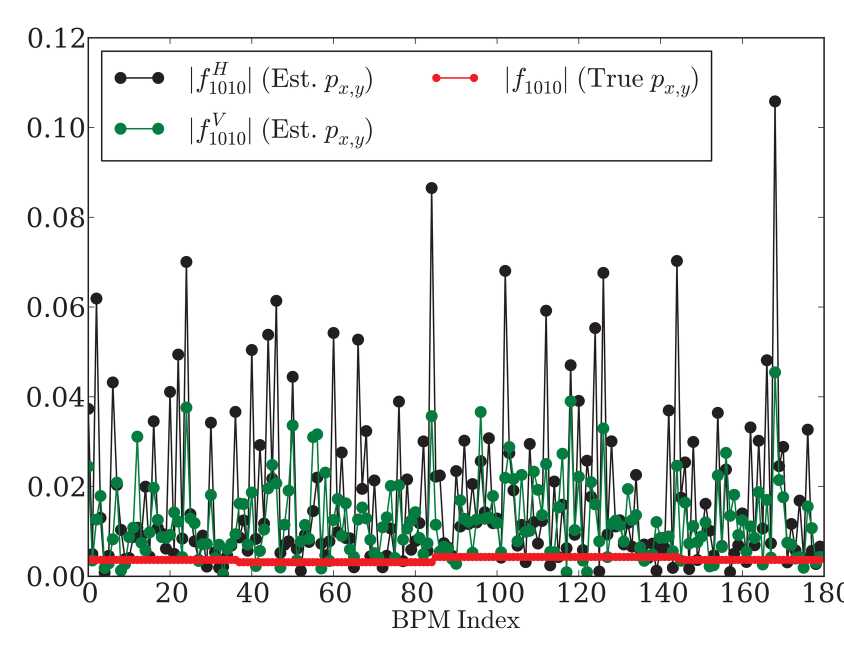
➢ Lattice comparison with skew quads On & Off. DTBLOC estimated correctly that

- ➢ BPM errors and normal quad errors to be almost the same between the 2 cases.
- ➢ skew quad diff. to be almost the same as the diff. expected from setpoint diff.

As Validation Tool of Fit Estimates

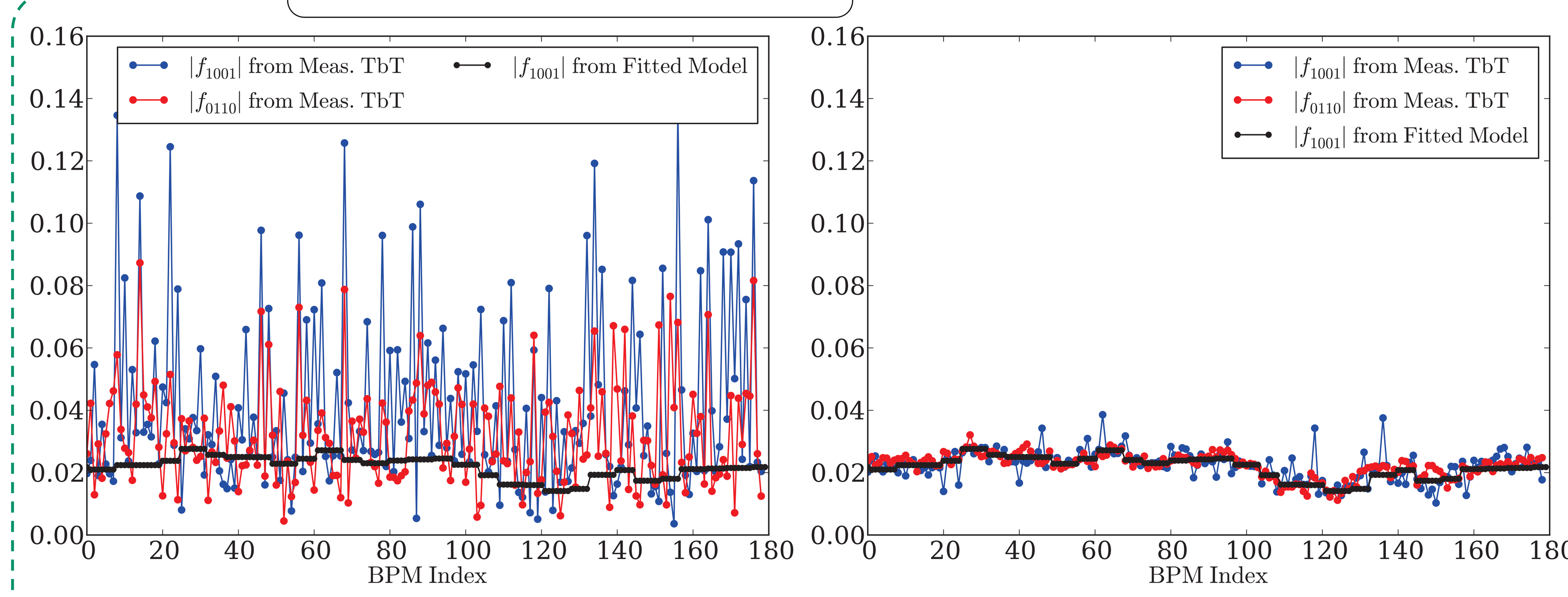


➢ 3 ways to compute coupling RDTs (2 from TbT & 1 from Twiss)
➢ If estimated errors are valid, all 3 estimates of coupling RDTs must agree.

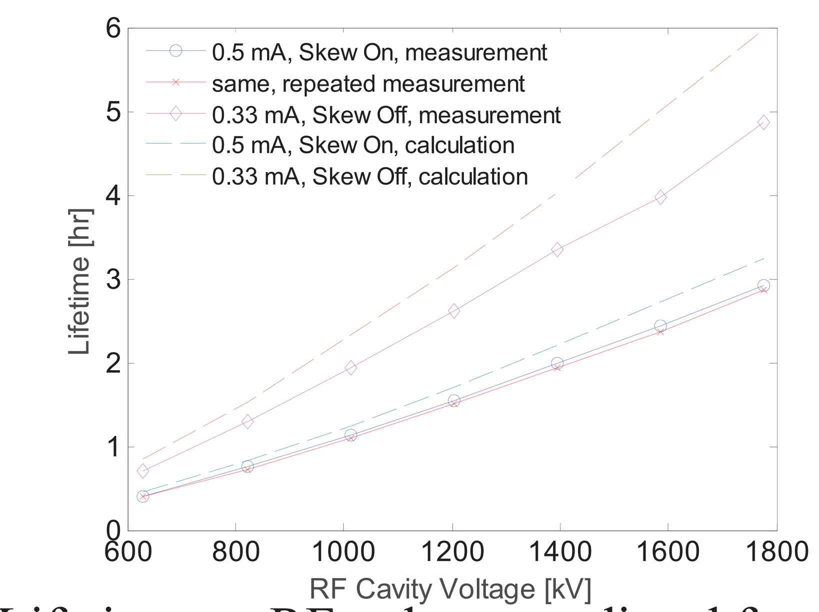


➢ Simulation: TbT-derived sum coupling RDTs f_{1010H} & f_{1010V} split by large factors with 10-mrad RMS BPM roll errors applied.

Experimental Validation



➢ Experiment: Without BPM error correction applied to TbT data, TbT-derived coupling RDTs split (left). But with BPM error correction applied, these TbT-derived coupling RDTs converge (right), indicating errors estimated by DTBLOC are physically consistent!



➢ Lifetime vs. RF voltage predicted from linear lattice models created from DTBLOC magnetic error estimates agreed well with the experimental curves.

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