# EXPERIMENTS OF LOSSLESS CROSSING - RESONANCE WITH TUNE MODULATION BY SYNCHROTRON OSCILLATIONS<sup>\*</sup>

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### Abstract

It had become a standard practice to constrain particle's tune footprint while designing the storage ring lattice so that the tunes fit between harmful resonances that limit ring dynamic aperture (DA). However, in recent ultrabright light source design, the nonlinearities of storage ring lattices are much enhanced as compared with the 3rd generation light source one. It is becoming more and more difficult to keep the off-momentum tune footprint confined and even more, the solution cannot be found to confine off-energy tune footprint in certain cases. The questions have been asked whether crossing of a resonance stopband from off-momentum particle will necessarily lead to particle loss. In NSLS-II, we modified the lattice working point to mimic machine tune footprint crossing half integer with beam synchrotron oscillation excitation and demonstrated that beam can cross a resonance without loss with control of stopband width and high order chromaticity.

### INTRODUCTION

It had become a standard practice to constrain the particle's tune footprint while designing storage ring lattice so that the particle tunes fit between harmful resonances that limit ring dynamic aperture (DA) [1]. This approach, known as "tune confinement", puts tight limits on the magnitude of the tune shifts with amplitude and with momentum. The latter requires labor-intensive optimization of the off-momentum DA and the corresponding tune footprint for the large momentum deviations to achieve reasonable lifetime.

As nonlinearities of the modern ring lattices are much enhanced as compared with the previous generation synchrotrons, it is becoming more and more difficult to keep the off-momentum tune footprint confined [2, 3]. In certain cases when the lattice solution with the confined offmomentum tune footprint cannot be found, one may ask whether the crossing of a resonance stopband from an offmomentum particle leads to a beam loss.

Recently modern synchrotrons advanced to Multi-Bend Achromat lattices featuring small dispersion and low beta functions, and, as a result, high nonlinearity of the particle motion. In certain cases, the tune spread for on-energy beam was successfully minimized, but the off-momentum tunes swing across the major resonances. Surprisingly, the tracking result does not show particle loss. Authors of [2] explained this phenomenon by rapid transition through the stopband together with substantial tune shifts

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with amplitude that help to drive particles off the resonance during the transition. Results of our studies presented in the paper indicate that this explanation is adequate.

Significance of the finding from the analysis presented in [2] advances the modern lattice design beyond the principles of the "tune confinement", while posing an important question: under which conditions does the chromatic tune footprint not need to be confined? In NSLS-II, we modified the lattice working point to mimic machine tune footprint crossing half integer with beam synchrotron oscillation excitation and demonstrated that beam can cross a resonance without loss with control of stopband width and high order chromaticity.

# DYNAMICS OF THE PARTICLE CROSS-ING A STATIC RESONANCE STOPBAND

We will consider a storage ring model with large chromatic tune shift with momentum deviation  $\delta = \Delta p/p$  up to the second order as:

$$\nu(\delta) = \nu_0 + \xi_1 \delta + \xi_2 \delta^2 + O(\delta^3), \tag{1}$$

where  $\xi_1$  and  $\xi_2$  are linear and 2<sup>nd</sup> order chromaticities. In the following we constrain our analysis to the 2dimensional case of y and  $\delta$ . For our experiments we maximized the 2<sup>nd</sup> order chromaticity of the lattice, so that  $\xi_{1y}$ =+1 and  $\xi_{2y}$ =+300 (similar to [2, 3]) by changing ring sextupoles while maintaining small tune shifts with amplitude.

Next we assume that the particle energy oscillates with the maximum deviation  $\delta_0$  and this synchrotron oscillation for simplicity is taken as  $\delta(t) = \delta_0 \sin(2\pi\nu_s n)$ , where  $v_s$  is the synchrotron tune and *n* is the number of turns around the ring. A cartoon illustrating the problem under consideration is shown in the plot of Fig. 1. As one can see, the betatron tune of a longitudinally oscillating particle crosses the resonance  $v = v_R$ , which is not infinitely thin in presence of quadrupole errors. The resonance is characterized by a stopband with the width  $\Delta v_R$ , which is heuristically defined as the boundary of the tune range where the peak beta-beat  $\Delta\beta/\beta = \frac{\beta-\beta_0}{\beta_0}$  reaches 100% [4]. Here  $\beta_0$  is the reference beta function calculated from the unperturbed lattice model, and  $\beta$  is the measured beta function obtained from coherent beam oscillations excited by a pulse kick and measured by beam position monitors distributed around the ring [5].

We define  $\delta_R = \delta(v_R)$  as the value of energy deviation where the particle's tune crosses the resonance  $v_R$ . The energy boundaries corresponding to the resonance stopband  $\Delta v_R$  are (neglecting the contribution from  $\xi_I$ ):

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$$\delta_R \pm \frac{\Delta \delta_R}{2} = \sqrt{\frac{(\nu_R - \nu_0) \pm \Delta \nu_R/2}{\xi_2}}$$

For calculating the number of turns the particle will take to cross the stopband we get:

$$\Delta n_R = \left( \operatorname{asin}\left( \left( \delta_R - \frac{\Delta \delta_R}{2} \right) \delta_0^{-1} \right) - \operatorname{asin}\left( \left( \delta_R + \frac{\Delta \delta_R}{2} \right) \delta_0^{-1} \right) \right) / 2\pi \nu_s, \text{ when } \delta_0 > \delta_R + \frac{\Delta \delta_R}{2}$$
$$\Delta n_R = \operatorname{acos}\left( \left( \delta_R - \frac{\Delta \delta_R}{2} \right) \delta_0^{-1} \right) / \pi \nu_s, \text{ when } \delta_0 \le \delta_R + \frac{\Delta \delta_R}{2}$$



Figure 1: Cartoon of particle executing synchrotron oscillations with amplitude  $\delta_0$  and crossing resonance at  $v_R$  with the stopband width of  $\Delta v_R$ . The lower plot refers to the energy deviation versus the RF phase  $\varphi(t)$  of the particle executing synchrotron oscillations.

## CONTROLLING THE RESONANCE STOPBAND WIDTH

Quadrupole imperfections of the linear lattice lead to a betatron tune shift together with forming a finite bandwidth of half-integer resonances on the tune diagram. The tune shift caused by the quadrupole errors and the corresponding stopband width are determined by the amplitudes of zeroth and second Fourier harmonics of the quadrupole perturbations around the machine:

$$\Delta \nu_t = \frac{1}{4\pi} \sum_q \beta_q (\Delta k_1 L)_q$$
$$\Delta \nu_R = \frac{1}{2\pi} |\sum_q \beta_q (\Delta k_1 L)_q e^{-2i\nu\phi_q}$$

where q runs over the lattice quadrupoles,  $\beta$  and  $\phi$  are betatron amplitude and phase and  $\Delta k_1 L = \Delta B' L/(B\rho)$  is the perturbed quadrupole focusing strength.

The way to control the resonance stopband width  $\Delta v_R$ is to act on the 2<sup>nd</sup> harmonic of  $(\Delta k_1 L)_q$  while maintaining the 0<sup>th</sup> harmonic caused by the same  $(\Delta k_1 L)_q$  equal to zero. Methods of minimizing the stopband width were presented in [6].

In our experiments we characterized the resonance stopband at NSLS-II using the following approach. We moved the vertical tune in the vicinity of resonance  $v_y=16.5$  and measured stopband width by measuring the beta-beat (Fig. 2). Then we changed ring quadrupoles to move the tune across  $\frac{1}{2}$  resonance while maintaining the stopband width constant and measured beta beat at every

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step. Next we repeated the scan across the resonance starting by changing beam energy by shifting the RF frequency, which changed the energy and, in turn, the vertical tune due to the high chromaticity. We characterized the resonance stopband by the two tune scans described above, resulting in the measured  $\Delta v_R$  of 0.016.



Figure 2: measurement of the  $\frac{1}{2}$  resonance stopband width  $\Delta v_R$  at NSLS-II, the scan of quadrupole gradients  $(\Delta k_1 L)_q$  (blue points) and the scan of RF frequency  $\Delta f_{RF}$  (red points) and their fits.

To increase the stopband width we selected  $N_q=17$  quadrupoles separated by  $n \cdot \pi + \pi/2$  in betatron phase advance and changed their strength by  $(\Delta k_1 L)_q$  yielding the increase in the stopband width of  $\frac{1}{2\pi} \sum_{q=1}^{N_q} \beta_q (\Delta k_1 L)_q$ .

### **CONCEPT OF THE EXPERIMENT**

We moved the tune  $v_y$  to a proximity of half integer resonance located at 16.5 and excited beam vertical betatron oscillations with transverse pulsed kicker (pinger). Turn-by-turn (TBT) beam transverse positions as well as beam relative intensity were measured with beam position monitors (BPMs). The vertical TBT data showed a modulated betatron oscillations, which provided a convenient tool for independent measurement of the detuning  $\delta v$  and  $\Delta v_R$ , together with well-controlled initial conditions. Rapidly changing the RF phase at some delay with respect to the pinger pulse induced beam energy oscillation large enough to cross the resonance stopband.

Beam TBT energy oscillation was retrieved from BPMs horizontal data located in dispersion region. We measured the first and second order dispersion and chromaticity by scanning RF frequency, which provided the necessary calibration of the energy and tune oscillations.

In our experiments we benefited from the small emittance and energy spread of the NSLS-II beam [7], which enabled us to study betatron motion of the whole beam as a single particle. Indeed the transverse and longitudinal pinger amplitudes ( $\Delta\alpha$ =200 µrad and  $\delta_0$ =1.5%) exceeded the natural beam divergence (1.5 µrad) and the energy spread (0.05%) by the ratio of about a hundred. The beam decoherence and subsequent filamentation, which masked the TBT BPM signals of the coherent particle motion, become significant a large fraction of the first synchrotron oscillation later. Specifically for this experiment, to modulate the offenergy tune, we developed a method of rapid excitation of coherent beam energy oscillations ("RF jump" or "RF pinger", [8]). The LLRF controller [9] was modified by adding an external timing trigger to control the phase transient.

The RF pinger timing was aligned with other timingdriven subsystems, such as transverse pinger and BPMs. The timing delay for the RF pinger trigger was synchronized with BPM sum signal so that RF pinger start is delayed by 100 turns relative to SR BPMs to monitor beam energy oscillations. Thus three planes pingers can be excited independently or simultaneously in any combination.

### **EXPERIMENTAL RESULTS**

In the experiment, we stored a few mA beam, switched to the highly chromatic lattice and then moved betatron tune to near half-integer ( $v_0 \sim 16.47$ ) resonance by controlling non-dispersive quadrupoles.

The beta-beat along the ring at different tunes was retrieved from BPM TBT data to measure the stopband width. The beta beat for the nominal lattice was corrected to  $\sim 3\%$  with stopband width at 0.016. We called these experimental conditions the "Small stopband" scenario.

With the same RF jump and transverse pinger settings we designed another experimental scenario in which the harmonic quadrupole strength was adjusted to expand the stopband width from 0.016 to 0.038, so that the beam tune stays within the stopband much longer during the RF jump. We called these experimental conditions the "Large stopband" scenario.

The measurement results are shown in Fig. 3. The energy oscillation amplitude is about  $\pm/-1.4\%$  (peak to peak). When the tune approaches the half integer resonance, *y* plane motion starts to show behavior that is typical for parametric resonance, i.e. modulation at the detuning frequency  $\delta v$ .



Figure 3: Turn by turn beam parameters (upper plot: calculated y-tunes with stopband in shadow area, lower plot: BPM measured turn-by-turn y-position in left axis and normalized beam intensity in right axis) during RF phase jump at several initial tunes. Left plot corresponds to the "Small stopband" and the right plot shows the "Large stopband" cases.

It is clear that when the beam is passing through the "Small stopband" we do not observe particle loss. We estimate that the beam takes a minimum of about 7 turns to cross the resonance for the stopband in this case. In the "Large stopband" scenario, the beam is moving through the resonance for about 40 turns resulting in the particles loss. The losses then repeat every following synchrotron oscillation. We estimate the maximum number of turns that the beam can spend within the "Small stopband" as 25 and that for the "Large stopband" as 49. The amplitude growth factor for few of our experiments and found an approximate agreement with the corresponding measurements: a factor of 1.5~2.5 for the "Small stopband"

#### **SUMMARY**

We carried out a study focused on a new non-standard approach in the lattice design for circular accelerators. We have shown that it may be unnecessary to confine the chromatic tune footprint between the resonance stopbands. The storage rings with a large chromatic tune swing can successfully reach large momentum aperture provided that two conditions are satisfied. The first condition states that the resonance stopbands need to be corrected well by manufacturing and aligning of magnets with small residual field errors and accurate cancellation of the harmful error harmonics that otherwise increase the stopband width. The other condition is in maintaining large magnitude of high order chromaticity. Combination of the two conditions above leads to a rapid crossing of the resonance stopband, which does not cause loss of the particles as demonstrated by our experiments.

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### REFERENCES

- E.D. Courant and H.S. Snyder, 1957, published in *Annals of Physics*, vol. 281, pp. 360-408, 2000.
- [2] APS-U Conceptual Design Report, http://www.aps.anl.gov/Upgrade/FlexPaper/cdr \_rev3/cdr.pdf
- [3] Y. Jiao and Z. Duan, https://arxiv.org/ftp/arxiv/papers/1606/1606 .07191.pdf
- [4] S.Y. Lee, Accelerator Physics, 3rd edition, Singapore: World Scientific, 2012.
- [5] V. Smaluk *et al.*, in *Proc. of IPAC 2016*, Busan, Korea, 2016, p. 3400.
- [6] J. Milutinovich, A.G. Ruggiero, S. Tepikian and W.T. Weng, RHIC Tech. Note No. 56, 1989.
- [7] G.M. Wang et al., Rev. Sci. Instrum., vol. 87, p. 033301, 2016.
- [8] G. M. Wang et al., in Proc. of IPAC 2016, Busan, Korea, 2016, p. 3161.
- [9] J. Rose et al., in Proc. of PAC 2011, New York, USA, 2011, p. 2583.

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