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HIGH-ACCURACY SMALL ROLL ANGLE MEASUREMENT METHOD BASED ON DUAL-GRATING DIFFRACTION HETERODYNE INTERFEROMETER*

S. Tang†, [1,2], M. Li†, [1,2], H. Liang[1], W. Sheng[1,2], J. Yang[2], Beijing Synchrotron Radiation Facility, Institute of High Energy Physics, Beijing 100049, P. R. China
[1]also at Laboratory of X-ray Optics and Technology, Institute of High Energy Physics, Beijing, P. R. China
[2]also at University of Chinese Academy of Sciences, School of Physics, Beijing, P. R. China

Abstract

Small roll angle (ROLL) is a crucial parameter for the motion performances of ultra-precision guide way often applied in fine mechanics and instruments of synchrotron radiation, such as long trace profiler (LTP). However, it is difficult to be measured by conventional methods including interferometer and autocollimator owing to their low sensitivities in axial direction. There is an orthogonal dilemma between measured direction and angular displacement plane for ROLL measurement. Therefore, a novel method based on dual-grating diffraction heterodyne interferometer is presented, which uses the combining scheme of diffraction grating and heterodyne interferometer to overcome the orthogonal problem. Moreover, the design of differential structure with dual-grating and grating interferometer instead of pure interferometer is adopted to improve the practicability against the environment, e. g. air fluctuation, inconstant rotation centre. It has inherited advantages of high-resolution up to 2nrad, high sampling rate up to 50kHz, and contactless by mathematical model and analysis. So, theoretical and experimental verifications are both implemented to its validation.

INTRODUCTION

Nano-radian accuracy small roll angle (ROLL) measurement method would be very urgent for the ultra-precision optical metrology and precision instrument [1-5], especially, long trace profiler (LTP). It will be benefit the development of two-dimensional LTP used for the test of X-ray mirrors. Besides, the ROLL is also important issue in the field of the industries, such as Numerical Control (NC) machine tools, Coordinate measuring machines (CMM), advanced manufacturing technology, and precision motion engineering. No matter which field, there is indeed a common technique which is application of precision linear motion guideway pair. But, it is very difficult to achieve high-performance measurement for the ROLL, especially field testing. Because the orthogonal problem of Roll displacement plane and linear motion direction of the guideway disables the typical methods of interferometer and autocollimator, compared with pitch or yaw measurement, as shown in Fig. 1.

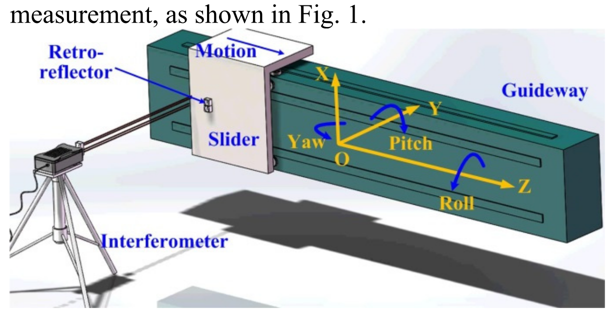

Figure 1: The diagram of angle deviations of the guideway.

So, there are exploration researches on it, such as:

1) Polarization detecting based on interferometry, including intensity [6, 7] and phase sensing [8-15].

2) Using special shape component based on interferometry, such as using wedge prism and its derivative [16, 17].

3) Geometrical transfer test using position sensitive detector (PSD) [18-20].

4) Synthetization of other optical methods [21-24], such as grating interferometer for ROLL measurement.

And there are maybe other instruments like level meter or inclinator. They are all proved the progress in laboratory. However, there is a little far way for practical application, especially, ROLL compensation of LTP and 3-axis NC machine tools or CMM. In this paper, a synthetization dual-grating heterodyne interferometric ROLL measurement method is presented, which uses grating for overcoming the orthogonal problem and heterodyne interferometer for high-accuracy, good stability, high-sampling rate.

CONFIGURATION AND MATHEMATICAL MODELING

As show in Fig. 2, the configuration of proposal method is composed of dual-frequency laser measuring head, polarizing beam splitter (PBS), right-angle prism (RAP), dual-grating, and dual-retroreflector (using RAPs), etc.

A beam emitted by the dual-frequency laser measuring head, containing two orthogonal linear polarized compounds with a stable frequency difference of ~3 MHz, which is split into two beams (called P-beam and

* Work supported by National Natural Science Foundation of China (NSFC) (No. 61505213).
† email address: Shanzhit@gmail.com and lim@ihep.ac.cn.

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S-beam) by PBS. The P-beam transmits the PBS and one of the dual-grating with diffractions. Its +1 order diffractive beam with a normal incidence is reflected by one of the dual-retroreflector, which transmits the grating and occur diffraction again. The +1 order beam of the second diffraction transmits the PBS, is lastly received by the laser head. This beam is considered as the measuring beam of heterodyne interferometer. The S-beam is reflected by the PBS and RAP, transmit the other of dual-grating with diffractions. Similarly, the +1 order diffractive beam reflected by the other of the dual-retroreflector, and return parallel to the incident beam, successively transmits this grating and rebounded by RAP and PBS to arrive at the laser head. The S-beam considered as the reference beam of heterodyne interferometer. Then, the two beams of measuring and reference have the interference which beat frequency regarded as measuring signal. It is used for getting difference by using reference beat signal offered by the laser head.

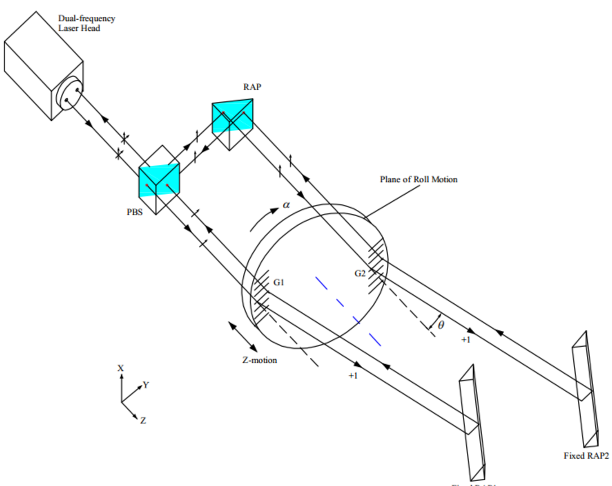

Figure 2: The principal diagram of the proposal method.

Thus, the mathematical model can be rigidly derived as follow.

For +1 order beam of the first diffraction of the measuring beam, and the grating equation is expressed as:

$$d \sin\theta = \lambda \tag{1}$$

Where, d means the grating constant or grating pitch; θ means the diffractive angle of the +1 order beam; λ means the wavelength of the laser.

Similarly, Eq. (1) is still applicable to the second diffraction of the measuring beam.

The slight arc displacement of the grating can be approximate to linear when the target has a motion with ROLL, which causes the Doppler frequency shift of the measuring beam. Due to the superimposed effect of the first and second the frequency shifts for two diffractions respectively. So, it can be expressed as:

$$\Delta f_M = \frac{2V_M}{\lambda}\cos\left(\frac{\pi}{2}-\theta\right) \tag{2}$$

Where, Δf_M means the total Doppler shift of the measuring beam; V_M means the velocity of the grating for the measuring beam.

From Eq. (1) and (2),

$$\Delta f_M = \frac{2V_M}{d} \tag{3}$$

For the reference beam, it can be similarly expressed as:

$$\Delta f_R = \frac{2V_R}{d} \tag{4}$$

Where, Δf_R means the total Doppler shift of the reference beam; V_R means the velocity of the grating for the reference beam.

From Eq. (3) and (4), the difference of the measuring and reference beams can be obtained as:

$$\Delta f = \Delta f_M - \Delta f_R = \frac{2}{d}(V_M - V_R) = \frac{2}{d}V \tag{5}$$

Where, Δf means the difference of the Doppler shifts of the two beams; V means the relative velocity of the two gratings each other.

Eq. (5) can be also written as:

$$V = \frac{d}{2}\Delta f \tag{6}$$

By time integral on the both sides of Eq. (6),

$$s = \int_0^T V \cdot dt = \frac{d}{2} \cdot \int_0^T \Delta f \cdot dt \tag{7}$$

Where, s means the relative displacement of the two gratings each other; *t and T* mean the time variable and particular time respectively.

Equation (7) reveals a grating interferometer for the measurement of the roll angular displacement with a natural resolution of $d/2$, which is similar to the pure optical displacement interferometer.

By the geometrical relationship, the ROLL is expressed as:

$$a = \frac{s}{L} = \frac{d}{2L} \cdot \int_0^T \Delta f \cdot dt \tag{8}$$

Where, a means the ROLL under test; L means the distance of the two gratings.

Therefore, the final measuring model is shown as Eq. (8). It inherits many advantages from the interferometer, such as high-accuracy, real-time, high sampling-rate, and so on.

THEORETICAL ANALYSIS

According to Eq. (8), there are two factors deciding the resolution of the ROLL measurement: one is the displacement measurement resolution based on the grating interferometer. The other is the distance of the two gratings, whose increase will enhance the final angle measurement resolution.

Apparently, the performance of the displacement grating interferometer is crucial for the ROLL test. We know, it is not enough for the high accuracy due to the nature resolution of $d/2$, which needs many subdivision techniques for higher resolution. Its resolution can be

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up to d/λ nano-meter or better if using by the same with electronic subdivision technique of the typical Michelson interferometer with 1nm resolution. So, the ROLL measurement resolution can be expressed as,

$$a_r = \frac{s_r}{L} = \frac{d}{\lambda L} \tag{9}$$

Where, a_r and s_r mean the resolutions of the ROLL with the unit of nano-radian and displacement measurements with the unit of nano-meter respectively. And L has the unit of meter.

From the geometrics as shown in Fig. 2,

$$\sin\theta = \frac{H}{D} \tag{10}$$

Where, H means the height of the RAP1 or RAP2; D means the maximum of linear displacement of the object under test.

From Eqs (1) and (10),

$$\sin\theta = \frac{H}{D} = \frac{\lambda}{d} \tag{11}$$

By the substitution of Eq.(11) into Eq.(9),

$$a_r = \frac{d}{\lambda L} = \frac{D}{HL} \tag{12}$$

Equation (12) implies that decrease of the grating constant d is an effective approach to enhance the test resolution when the wavelength λ is known (e.g. 633 nm), accordingly, the H should be expected to be increased unlimitedly for a better resolution. But the way will be restricted by the geometric dimension of the RAP (*H)* due to fabricating techniques. So, the resolution a_r should be optimized by feasible choice of H and D to match a better grating constant d.

For the conditions of $\lambda = 633nm$, *L=1000mm*, *H=500mm*, the theoretical resolution is up to 2nrad for the rangle of *D=1000mm* if the grating constant is 1.266 μm. Admittedly, the surface error will introduced a measurement uncertainty of several nrads if the height error is several nm because of beam moving on the optical surface of the RAP. But, it can be reduced or eliminated by the mapping the error.

EXPERIMENT

As shown in Fig. 3, a based-heterodyne interferometer test setup is built for the experimental verification, which consists of laser head of 5519A (Agilent, He-Ne laser with 633nm), PBS, Mirror, Gratings with 300 lines/mm (Thorlabs), and home-built RAPs with a length of 200mm. Additionally, a tilt stage with a scale is employed to implement ROLL motion on the guideway. The indicating values of the tilt stage are regarded as nominal ROLL though its resolution is not high. Correspondingly, the values of the proposal method are test values.

Before the test, the key systematic parameter, the distance of two gratings L, was accurately given by the angle calibration, which is equal to 96.20 mm. So, the resolution of the ROLL test system can be obtained as 55 nrad (0.01 arcsec) by the substitution of the parameters of λ, d, and L into Eq. (12).

Then, the ROLL tests are implemented for feasible validation in the common experimental room. By the measurement of many nominal ROLLs with bi-direction variable, the comparison results can be achieved as shown as in Fig. 4. Whether positive stroke or negative stroke of the tilt or ROLL motion, testing and nominal values coincide each other. The standard deviation of the two values between the proposal and tilt stage is 94.96 arcsec, but the average of relative errors of each point is 0.052%, and its standard deviation is also calculated, which is 0.069%.

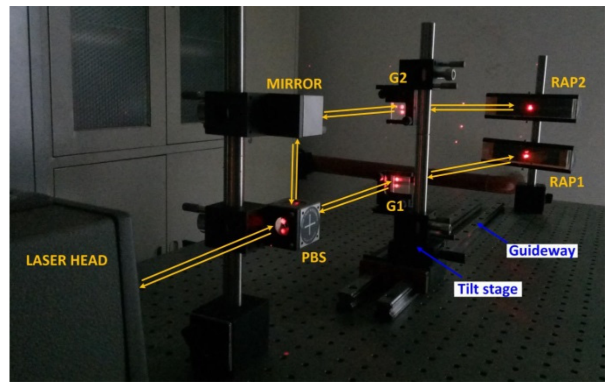

Figure 3: The prototype of the test setup based on the proposal method.

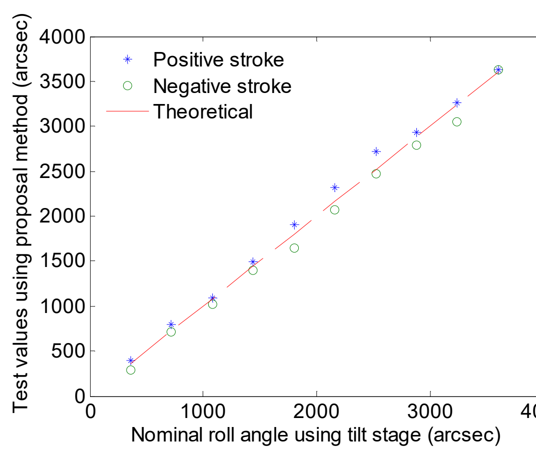

Figure 4: The results of the ROLL test.

Admittedly, the accuracy of the contrast could be much better if using a higher performance angle measurement instrument or comparator. However, it is indeed feasible for dual-grating heterodyne interferometric ROLL measurement method. Furthermore, it inherits lots of merits from pure Michelson displacement/angle interferometer, such as high-sampling rate. Figure 5 shows that the test curve of step-manual ROLLs using the proposal with simpling rate of 100Hz, which monitor the whole the motion process of the ROLL including the detailed step during the time of 500s. So, it features real-time and dynamical performance.

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Actually, it is simple, in which the dual-grating just moves with the motional target under test, and other components all keep static. The procedures and configurations look like the straightness test based on the commercial laser interferometer. Therefore, it is high-accuracy, very simple and stability.

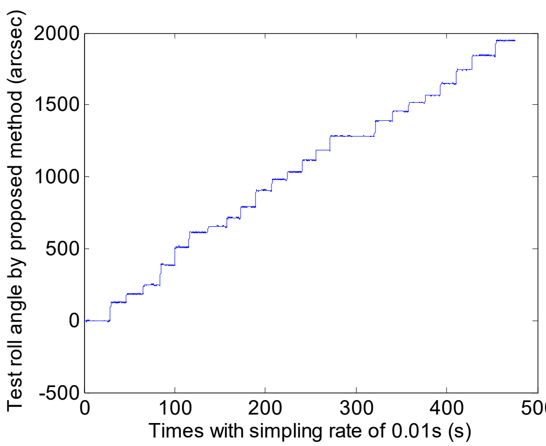

Figure 5: The test curve of step-manual ROLLs using the proposal with sampling rate of 100Hz.

CONCLUSION

A synthetization method for ROLL measurement based on dual-grating diffraction heterodyne interferometer has been presented and discussed in this paper. Both of theoretical model and experimental verification are implemented to validate the performance including high-resolution, high-sampling rate, and good operation of the proposal method. In a word, it offers a feasible and effective approach for ROLL monitoring and compensation for the R & D of advanced LTP, advanced manufacturing technology, and other relative fields.

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