MEDSI 2016
MECHANICAL ENGINEERING DESIGN OF SYNCHROTRON RADIATION EQUIPMENT AND INSTRUMENTATION
MAX IV
LUND UNIVERSITY

Dynamically Isotropic Hexapods for High-Performance 6-DOF Manipulation

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Abstract

Recent advances in synchrotron facilities have led to a growing need for 6-DOF precise manipulation. Hexapods are the most widely used parallel robots which provide 6 DOFs. To obtain high precision and high dynamic performance in hexapods, it is necessary to design them in such a way that low eigenfrequencies are avoided (while the eigenfrequencies are also functions of the complex 3D geometry of hexapods). Theoretically, maximizing the lowest eigenfrequency leads to a condition where multiple eigenfrequencies become equal, which is known as (complete or partial) dynamic isotropy. Thus, one may consider a dynamically isotropic hexapod as the optimal design solution, where precision and dynamic performance is a goal. In this work, we analytically address this problem and establish a practical guideline in order to design generalized hexapods with complete dynamic isotropy. The findings are based on the recently defended PhD dissertation by Behrouz Afzali-Far.

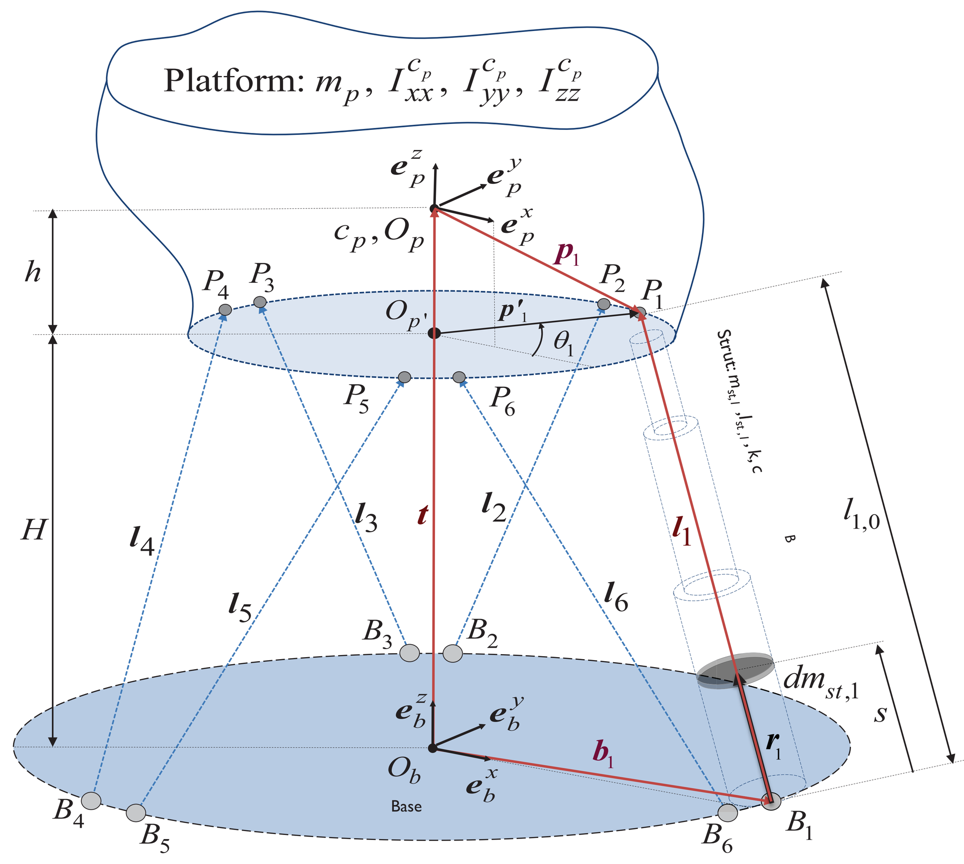
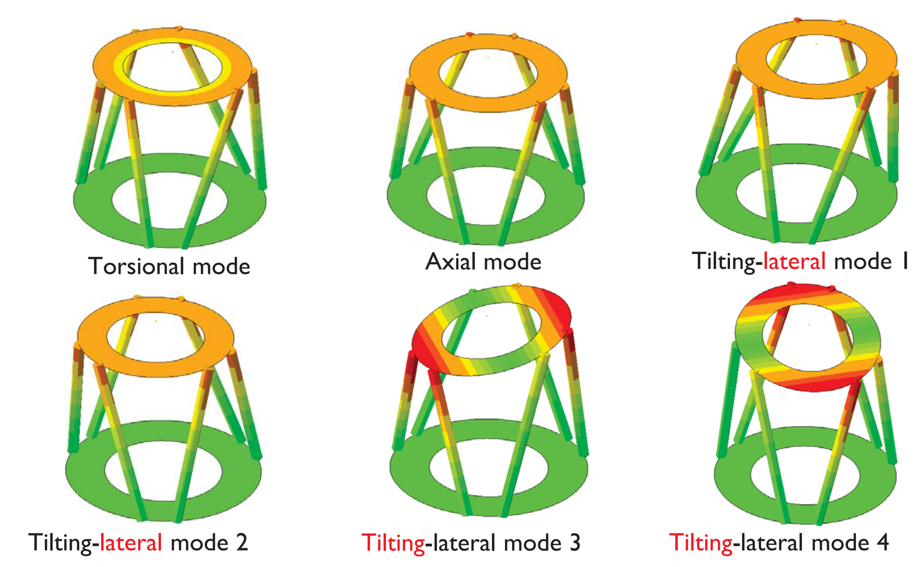


Fig. 1 Standard hexapod

Eigenfrequencies and eigenvectors (analytically formulated):

$$
\begin{cases}
\omega_\gamma = \sqrt{\dfrac{k_{\gamma\gamma}}{M_{\gamma\gamma}}}, \quad \omega_z = \sqrt{\dfrac{k_{zz}}{M}} \\
\omega_{x\beta,1} = \sqrt{\dfrac{Mk_{\beta\beta} + M_{\beta\beta}k_{xx} - 2M_{x\beta}k_{x\beta} - \Psi_{x\beta}}{2MM_{\beta\beta} - 2M_{x\beta}^2}} \\
\omega_{y\alpha,1} = \sqrt{\dfrac{Mk_{\alpha\alpha} + M_{\alpha\alpha}k_{yy} - 2M_{y\alpha}k_{y\alpha} - \Psi_{y\alpha}}{2MM_{\alpha\alpha} - 2M_{y\alpha}^2}} \\
\omega_{x\beta,2} = \sqrt{\dfrac{Mk_{\beta\beta} + M_{\beta\beta}k_{xx} - 2M_{x\beta}k_{x\beta} + \Psi_{x\beta}}{2MM_{\beta\beta} - 2M_{x\beta}^2}} \\
\omega_{y\alpha,2} = \sqrt{\dfrac{Mk_{\alpha\alpha} + M_{\alpha\alpha}k_{yy} - 2M_{y\alpha}k_{y\alpha} + \Psi_{y\alpha}}{2MM_{\alpha\alpha} - 2M_{y\alpha}^2}}
\end{cases}
$$



What is dynamic isotropy?

Dynamic isotropy is obtained when the following relation is satisfied

$$
M^{-1}K = \sigma I = \begin{pmatrix} \sigma & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma \end{pmatrix} \Leftrightarrow K = \sigma M
$$

from which it follows that all the eigenfrequencies are equal

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = \omega_6 = \ldots = \omega_{iso}$$

It is analytically proven that for a hexapod (with massless struts), the isotropic eigenfrequency is the absolute maximum of the minimum eigenfrequency of the system

$$\boxed{\omega_{iso} \geqslant \omega_{\min}}$$

However, it is impossible to obtain dynamic isotropy in hexapods

One of the isotropic conditions (known as the *classical isotropic constraint*): $\boxed{I_{zz}^{c_p} = 4I_{yy}^{c_p} = 4I_{xx}^{c_p}}$ Platform's moments of inertia

due to the fact that $I_{zz}^{c_p} \leq 2I_{yy}^{c_p}$ for a single (rigid) body.

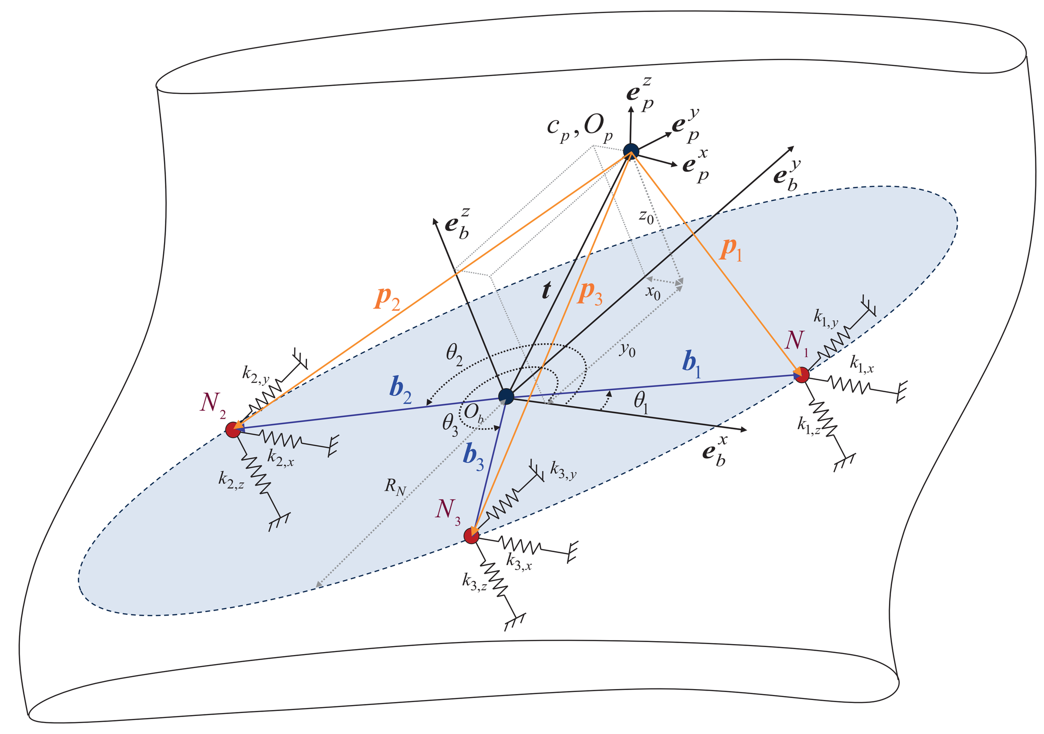


Fig. 2 A general 3D platform constrained at three arbitrary nodes

Generalized studies

To remove the classical isotropic constraint, we have generalized our studies, as shown in Fig.2. Our generalized studies formulate the conditions of dynamic isotropy in hexapods as well as the isotropic conditions for the two kinematic arrangements which are shown in Fig.3.

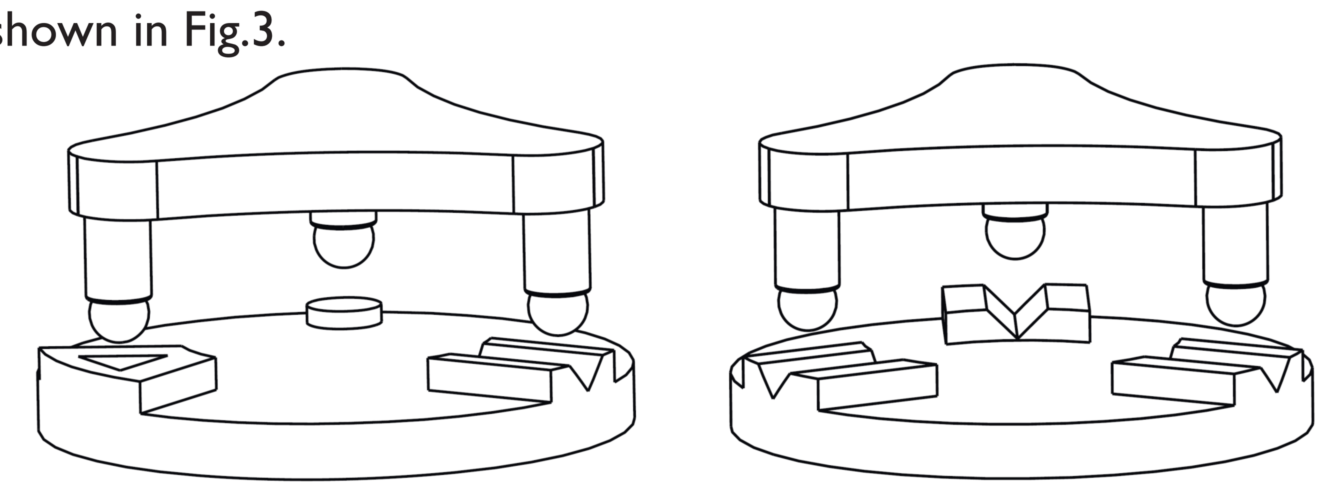


Fig. 3 Arrangements of kinematic couplings:
the 3-2-1 arrangement (left); the 2-2-2 arrangement (right).

Novel isotropic architecture of hexapods

Based on the generalized studies, a novel architecture of hexapods (GGSPs, see Fig. 4) is proposed, in which the classical isotropic constraint is removed

$$\cancel{I_{zz}^{c_p} = 4I_{yy}^{c_p} = 4I_{xx}^{c_p}}$$

and replaced by

$$\boxed{I_{zz}^{c_p} = \left[4\cos^2(\gamma_M)\right] I_{yy}^{c_p} = \left[4\cos^2(\gamma_M)\right] I_{xx}^{c_p}}$$

Main advantages:

- Dynamic isotropy for a wide range of inertia conditions
- Static and dynamic isotropy at the same time
- The struts (actuators) can still be identical, which is easy to design and fabricate

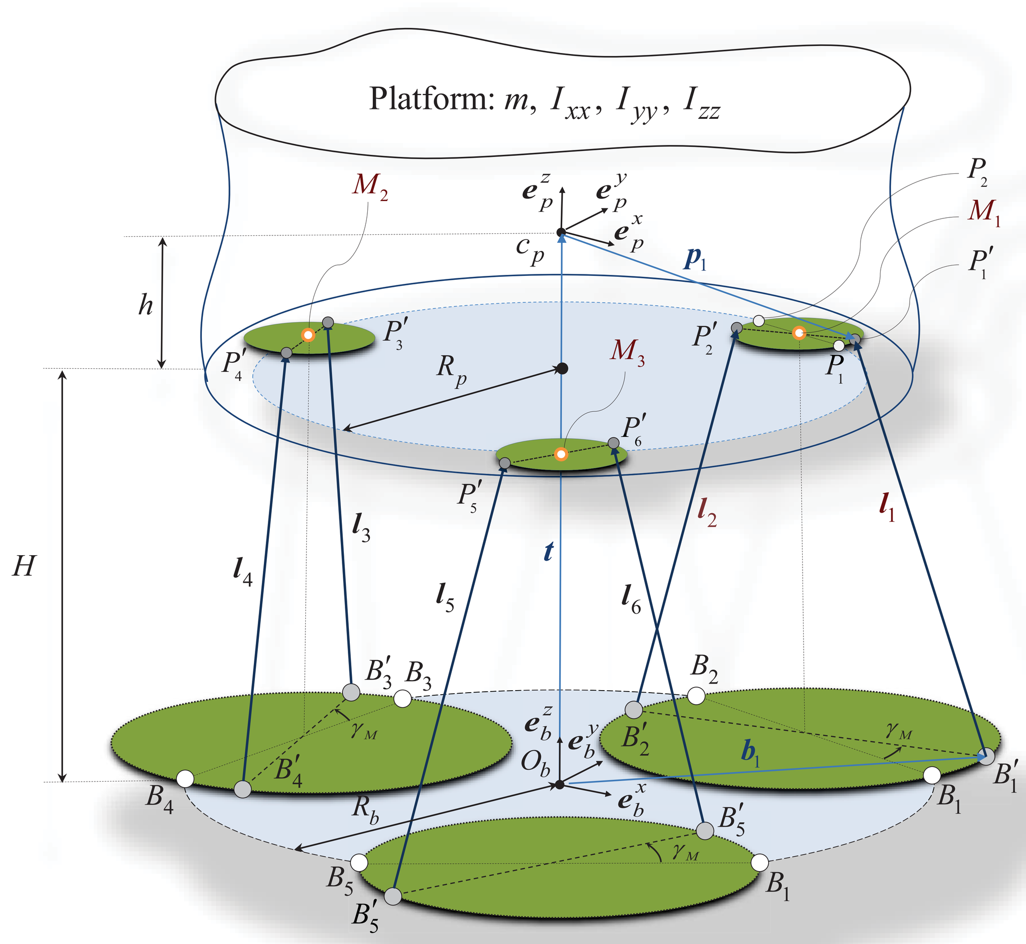


Fig. 4 The proposed generalized hexapod (GGSP)

Background of the project

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[6] B. Afzali-Far and P. Lidström, "A Joint-Space Parametric Formulation for the Vibrations of Symmetric Gough-Stewart Platforms," Advances in Intelligent Systems and Computing, vol. 1089, Springer International Publishing, Cham, 2015.