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LONGITUDINAL BEAM DYNAMICS IN ARRAY OF EQUIDISTANT MULTICELL CAVITIES*

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Abstract

Linear accelerators containing the sequence of independently phase cavities with constant geometrical velocity along each cavity are widely used in practice. The chain of cavities with identical cell length is utilized within a certain beam velocity range, with subsequent transformation to the next chain with higher cavity velocity. Design and analysis of beam dynamics in this type of accelerator are usually performed using numerical simulations. In the present paper, we provide an analytical treatment of beam dynamics in such linacs. Expressions connecting beam energy gain and phase slippage along the cavity are implemented. The dynamics of the beam around the reference trajectory and matched beam conditions are discussed.

DYNAMICS IN ACCELERATING SECTION WITH EQUIDISTANT CELLS

Consider longitudinal beam dynamics in a structure with identical cells (see Figs. 1 and 2). Most of such structures in ion accelerators are π-structures with cell length $\beta_g\lambda/2$, where β_g is the geometrical velocity and $\lambda = 2\pi c/\omega$ is the RF wavelength. Acceleration of particles in such field can be considered as dynamics in an equivalent traveling wave propagating along with the structure with constant phase velocity β_g and with amplitude $E = E_o T(\beta)$, where E_o is the average field per accelerating gap, $T(\beta)$ is the transit time factor and φ is the phase of a particle in traveling wave [1]:

$$\varphi = \omega t - \int_o^z k_z\, dz\,, \qquad (1)$$

where $k_z = 2\pi/(\beta_g\lambda)$ is the wave number. The phase φ is also a phase of a particle in the standing wave at the moment of time when the particle crosses the center of the accelerating gap. Differentiation of Eq. (1) along the longitudinal coordinate z together with the equation for particle energy gain provides a set of equations for on-axis particle dynamics in traveling wave [2]:

$$\frac{d\varphi}{dz} = \frac{2\pi}{\lambda}\left(\frac{1}{\beta} - \frac{1}{\beta_g}\right), \qquad \frac{d\gamma}{dz} = \frac{qE}{mc^2}\cos\varphi\,, \qquad (2)$$

where m and q are mass and charge of particle, and $\gamma = (1-\beta^2)^{-1/2}$ is the normalized particle energy. Equations (2) can be derived from Hamiltonian

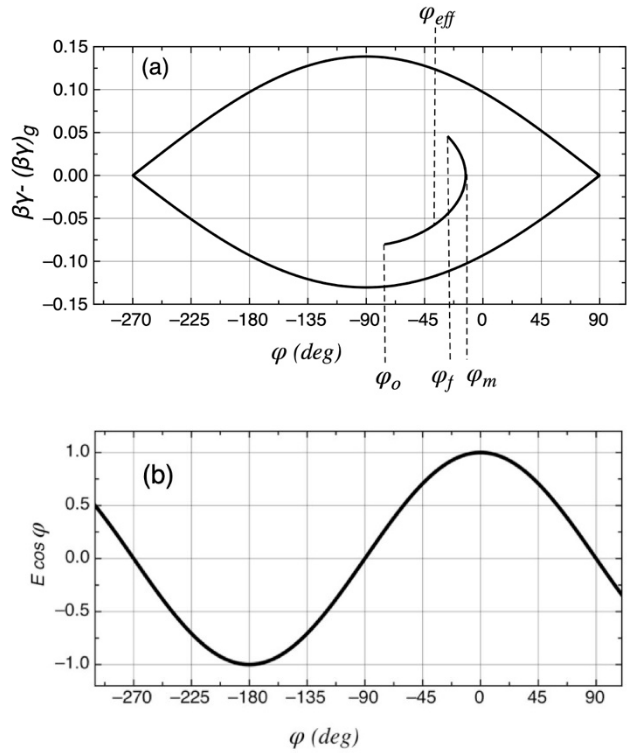

Figure 1: (a) Phase space trajectory of a particle in an RF structure with equidistant cells, (b) equivalent traveling wave with amplitude E.

$$H = \frac{2\pi}{\lambda}\left(\sqrt{\gamma^2 - 1} - \frac{\gamma}{\beta_g}\right) - \frac{qE}{mc^2}\sin\varphi\,, \qquad (3)$$

where Hamiltonian equations are $d\gamma/dz = -\partial H/\partial\varphi$, $d\varphi/dz = \partial H/\partial\gamma$. In the standing wave structure with identical cells, the average field per cell is constant, $E_o = const$ and variation of particle velocity along the cavity is typically small, $\Delta\beta/\beta << 1$, therefore, the amplitude of accelerating field can be approximated to be constant $E = E_o T(\beta) \approx const$. Because the geometrical velocity is also a constant, β_g = const, the Hamiltonian, Eq. (3), is a constant of motion. From Hamiltonian, Eq. (3), the integral of particle motion in such field, $C = H\lambda/(2\pi)$, is

$$\sqrt{\gamma^2 - 1} - \frac{\gamma}{\beta_g} - \frac{qE\lambda}{2\pi mc^2}\sin\varphi = C\,. \qquad (4)$$

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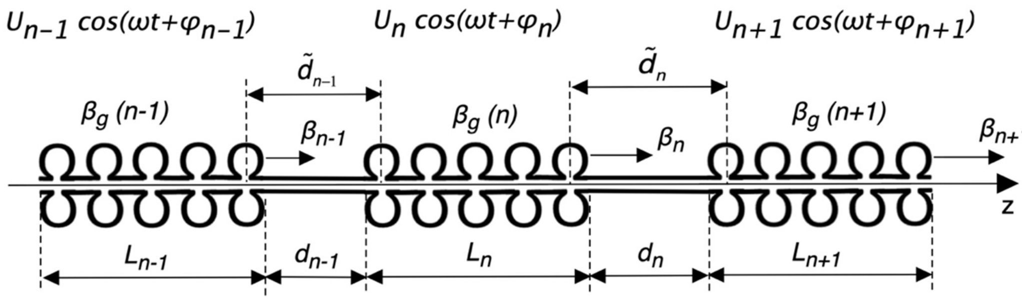

Figure 2: Accelerating structure of independently phased cavities.

In the accelerating section with $\beta_g < 1$, the synchronous phase in each individual accelerator structure is $\varphi_s = -90^o$, and acceleration is achieved as a rotation in phase space around the synchronous phase (see Fig. 1). To find the value of beam energy in closed form, let us express the constant C in Eq. (4) through the value of RF phase φ_m, at which the particle velocity is equal to geometrical velocity, $\beta = \beta_g$

$$C = \beta_g \gamma_g - \frac{\gamma_g}{\beta_g} - \frac{qE\lambda}{2\pi mc^2} \sin\varphi_m, \qquad (5)$$

where the energy corresponding to the geometrical velocity of the cavity is $\gamma_g = (1-\beta_g^2)^{-1/2}$. Using the expansion of particle momentum $\beta\gamma$ near $\beta_g \gamma_g$, Eq. (4) becomes

$$\frac{(\gamma - \gamma_g)^2}{(\beta_g \gamma_g)^3} = \frac{qE\lambda}{\pi mc^2} (\sin\varphi_m - \sin\varphi). \qquad (6)$$

Equation (6) explicitly connects particle energy along accelerating structure, γ, with the phase of a particle in RF field, φ. The value of φ_m is determined from Eq. (6) by the initial value of beam phase φ_o, and initial energy γ_o:

$$\sin\varphi_m = \sin\varphi_o + \frac{\pi}{(\beta_g \gamma_g)^3} \frac{mc^2}{qE\lambda} (\gamma_g - \gamma_o)^2. \qquad (7)$$

Equation (6) determines two values of particle energy for each phase, depending on the cavity length: larger, $\gamma_f \ge \gamma_g$, and smaller, $\gamma_f \le \gamma_g$, than the energy corresponding to the geometrical velocity of the cavity, γ_g. The values of the final energy, γ_f, corresponding to the final phase φ_f are:

$$\gamma_f = \gamma_g \pm \sqrt{\frac{qE\lambda(\beta_g \gamma_g)^3}{\pi mc^2}} \sqrt{\sin\varphi_m - \sin\varphi_f}, \qquad (8)$$

where the negative sign is taken when $\gamma_f < \gamma_g$, while the positive sign is taken when $\gamma_f > \gamma_g$. Energy gain in accelerator structure of length L_n can be expressed as $\Delta W = qE_o T(\beta) L_n \cos\varphi_{eff}$, where φ_{eff} is the effective phase of the particle in RF field of the cavity defined by:

$$\cos\varphi_{eff} = mc^2 (\gamma_f - \gamma_o) / (qE_o T(\beta) L_n). \qquad (9)$$

Let us determine the phase slippage of particles in cavity. From Eqs. (2), (6), the dimensionless time of particle acceleration in the structure, $\Delta(\omega t)$, is determined as [3]:

$$\Delta(\omega t) = \sqrt{\pi \beta_g \gamma_g^3 (\frac{mc^2}{qE\lambda})} \int_{\varphi_o}^{\varphi_f} \frac{d\varphi}{\sqrt{\sin\varphi_m - \sin\varphi}}. \qquad (10)$$

Expanding RF phase of particle φ around φ_m as $\sin\varphi \approx \sin\varphi_m + (\varphi - \varphi_m)\cos\varphi_m - 0.5(\varphi - \varphi_m)^2 \sin\varphi_m$, the integral, Eq. (10), can be approximated as

$$\Delta(\omega t) \approx \sqrt{\frac{2\pi \beta_g \gamma_g^3 mc^2}{qE\lambda |\sin\varphi_m|}} \{\arcsin[1 + (\varphi_m - \varphi_f)\tan\varphi_m] - \arcsin[1 + (\varphi_m - \varphi_o)\tan\varphi_m]\}. \qquad (11)$$

Equation (11) connects the dimensionless time of particle acceleration in the cavity, with the phase slippage in RF field from φ_o to φ_f. The right-hand side of Eq. (11) has a positive sign for $\varphi_f > \varphi_o$, and a negative sign for $\varphi_f < \varphi_o$. In case the particle trajectory in phase space passes the value of φ_m, like that illustrated in Fig. 1a, the time, $\Delta(\omega t)$, should be calculated as a sum of that required for phase variation from the initial value of φ_o to φ_m, and then from φ_m to final value φ_f:

$$\Delta(\omega t) = \Delta(\omega t)\Big|_{\varphi_o}^{\varphi_m} + \Delta(\omega t)\Big|_{\varphi_m}^{\varphi_f}. \qquad (12)$$

For accelerating structures working on π-mode, the number of accelerating cells is $N_{cell} \simeq \Delta(\omega t)/\pi$, and the length of the cavity is $L_n = N_{cell} \beta_g \lambda / 2$.

DYNAMICS IN AN ARRAY OF CAVITIES

The dynamics of the beam in an array of accelerating cavities can be described in classical terms of particle oscillations around the synchronous phase $\varphi_s(z)$ of reference (synchronous) particle, which velocity is equal to that

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of the effective traveling wave $\beta_s(z)$. The dynamics of the reference particle is determined by the geometry of the accelerating channel and shifts of RF phases between cavities. While the reference particle travels from the center of the last cell of the cavity (n) to the center of the first cell of the cavity ($n+1$) separated by the distance $\tilde{d}_n$ (see Fig. 2), the phase of RF field is changed in each cavity by the value $\phi = \omega t_d$, where $t_d = \tilde{d}_n / (\beta_s c)$. Consequently, the velocity of reference particle after cavity (n) is

$$\beta_n = 2\pi \tilde{d}_n / [\lambda(\varphi_n - \varphi_{n+1})], \quad (13)$$

where $\varphi_n - \varphi_{n+1} = 2\pi m - \Delta\varphi_n$, $m = 0, 1, 2,..$ is the difference in RF phases of cavities, which includes the integer number of RF periods and a fractional part $\Delta\varphi_n$.

The effective synchronous phase of the linac is determined by the rate of increase of velocity of the reference particle along with the machine. From Eq. (2), taking into account that $d\gamma = \beta\gamma^3 d\beta$, the expression for synchronous phase is

$$\cos\varphi_s(z) = \beta_s \gamma_s^3 (d\beta_s / dz) mc^2 / (q\bar{E}), \quad (14)$$

where $\bar{E}$ is the amplitude of equivalent traveling wave propagating along the linac. Within the cavity, the velocity of reference particles can be approximated as $\beta_{s\_n} = (\beta_{n-1} + \beta_n)/2$. The amplitude $\bar{E}$ is the ratio of the cavity voltage $U_n = E_{o\_n} L_n$ to the effective length occupied by the cavity, $L_n + 0.5(d_n + d_{n+1})$, which includes the cavity length, and the halves of drift spaces between cavities (see Fig. 2)

$$\bar{E} = E_{o\_n} T_n(\beta_{s\_n}) L_n / [L_n + 0.5(d_n + d_{n+1})], \quad (15)$$

where E_{o_n} and $T_n(\beta_s)$ are the average fields in RF gaps and the transit time factor in cavity (n), correspondingly. The velocity of the reference particle is changing within the cavity from β_{n-1} to β_n, therefore, the rate of increase of velocity of the reference particle in the cavity with the number (n) is $d\beta_s / dz \approx (\beta_n - \beta_{n-1}) / [L_n + 0.5(d_n + d_{n+1})]$. Therefore, the synchronous phase of the linac at the cavity (n) is determined as:

$$\cos\varphi_{s\_n} = \frac{mc^2}{qE_{o\_n} T_n L_n} \beta_{s\_n} \gamma_{s\_n}^3 (\beta_n - \beta_{n-1}). \quad (16)$$

The values of $\beta_s(z)$, $\varphi_s(z)$, $\bar{E}(z)$ define the dynamics of the reference particle in the equivalent traveling wave and are entirely determined by the accelerator channel. The beam velocity and effective phase φ_{eff}, Eq. (9), do not nesseseraly coincide with $\beta_s(z)$, $\varphi_s(z)$, creating a mismatch between the beam and accelerating wave.

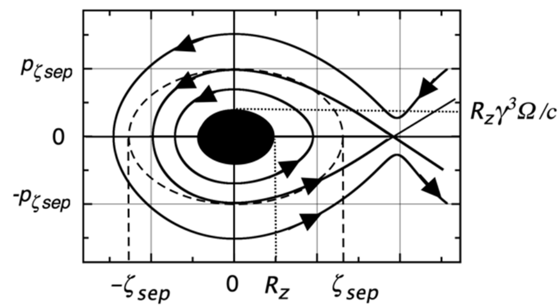

Figure 3: Longitudinal phase space trajectories: (dotted) approximation of separatrix by the ellipse, (bold) normalized longitudinal emittance of matched beam.

The performed analysis allows us to determine normalized acceptance of accelerator and matched conditions for the beam in linac. The linear particle oscillations in phase space of canonical-conjugate variables $p_\zeta = p_z - p_s$, $\zeta = z - z_s$, are determined by the Hamiltonian

$$H = \frac{p_\zeta^2}{2m\gamma^3} + m\gamma^3\Omega^2\frac{\zeta^2}{2}, \quad (17)$$

where Ω is the frequency of small-amplitude oscillations

$$\frac{\Omega}{\omega} = \sqrt{\frac{q\bar{E}\lambda}{mc^2}\frac{|\sin\varphi_s|}{2\pi\beta_s\gamma_s^3}}. \quad (18)$$

and p_ζ, ζ are deviation from momentum and position of synchronous particle, correspondingly. The separatrix can be approximated by an ellipse with half-width in momentum, $p_{\zeta sep}$, determined by actual separatrix, and the longitudinal half-size of seperatirx, ζ_{sep}, determined by Eq. (17), see Fig. 3:

$$\frac{p_{\zeta sep}}{mc} = 2\beta_s\gamma_s^3\frac{\Omega}{\omega}\sqrt{1 - \frac{\varphi_s}{tg\varphi_s}}, \quad \zeta_{sep} = 2\frac{\beta_s c}{\omega}\sqrt{1 - \frac{\varphi_s}{\tan\varphi_s}}. \quad (19)$$

The normalized longitudinal acceptance, $\varepsilon_{acc} = \zeta_{sep} p_{sep} / (mc)$, is specified as

$$\varepsilon_{acc} = \frac{2}{\pi}\lambda\beta^2\gamma^3(\frac{\Omega}{\omega})(1 - \frac{\varphi_s}{\tan\varphi_s}). \quad (20)$$

Equation (17) determines zero-intensity averaged matched beam with given longitudinal emittance ε_z, where longitudinal beam radius R_z, and beam half-momentum spread, p_ζ are (see Fig. 3):

$$R_z = \sqrt{\frac{\varepsilon_z\lambda}{2\pi\gamma^3}(\frac{\omega}{\Omega})}, \qquad \frac{p_\zeta}{mc} = \sqrt{2\pi\gamma^3\frac{\varepsilon_z}{\lambda}(\frac{\Omega}{\omega})}. \quad (21)$$

In presence of space charge forces, the matched conditions are modified (see Ref. [4]). More details on beam dynamics in independently phase cavities are presented in Ref. [5].

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