



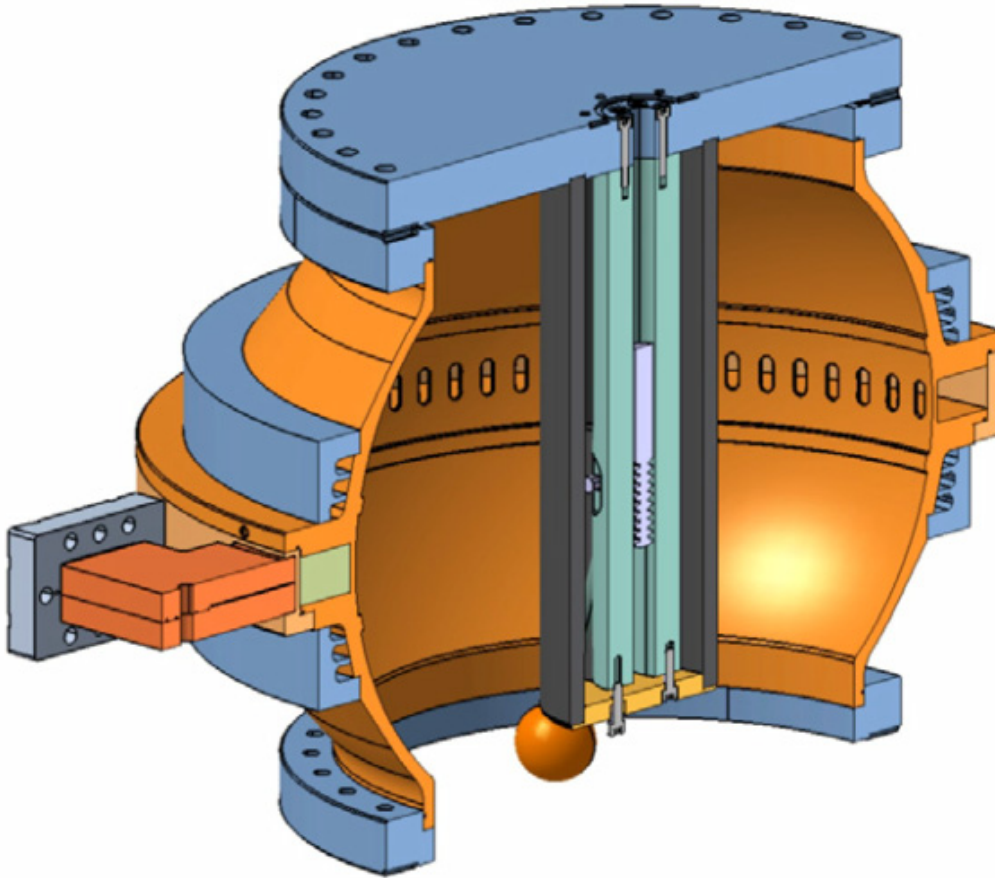
A New Spherical Pulse Compressor Working with Degenerated “Whispering Gallery” Mode

Zongbin Li (SINAP), Alexej Grudiev (CERN),
Wencheng Fang (SINAP), Qiang Gu (SINAP), Zhentang Zhao (SINAP)

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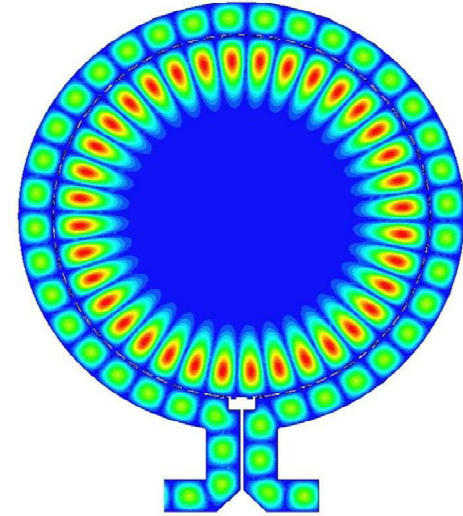
LINAC2018, Beijing, China



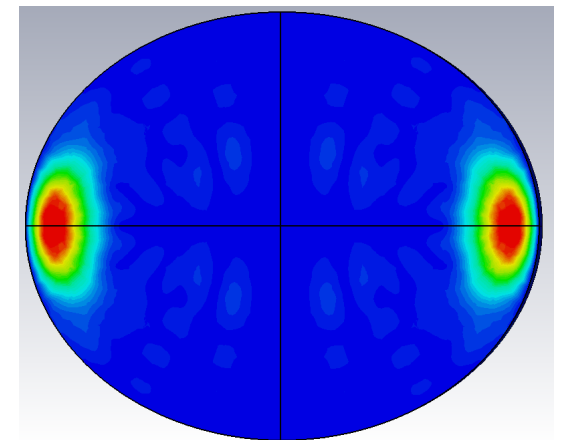


Barrel-shape Open Cavity

Working with “Whispering Gallery” mode.

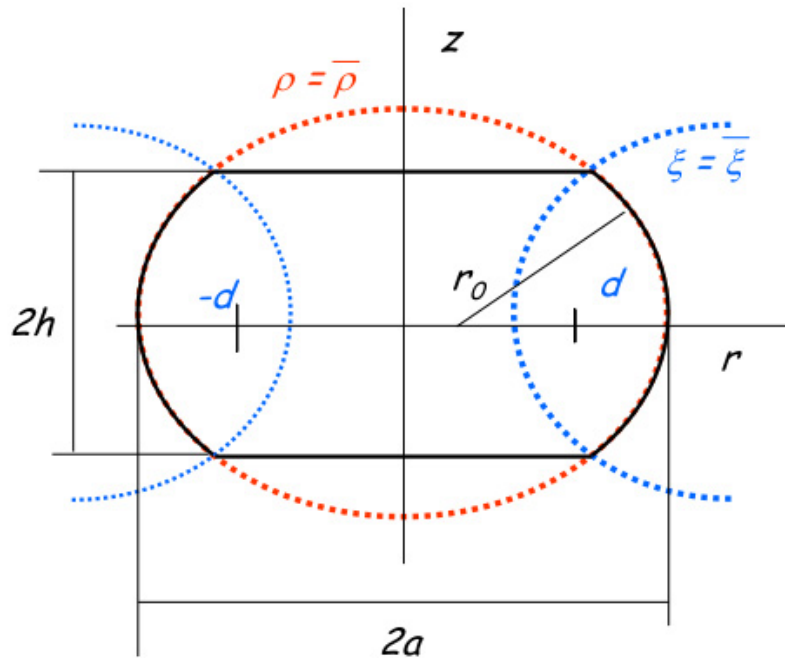


E-field: Top-view



E-field: Side-view

Barrel cavity short theory.



Cavity profile

$$z = \sqrt{ar_0 \left\{ 1 - \left(\frac{r}{a} \right)^2 \right\}}$$

Elliptical cavity

The eigen-frequency of the Barrel cavity with E_{mnq} oscillation is the solution of the next equation:

$$ka = v_{mn} + \frac{(q-1/2)\alpha}{\sin \theta}$$

v_{mn} is a root of the Bessel function that for the big m can be approximated as:

$$v_{mn}^o = m - \mu t_n^0 \quad (n = 1, 2, \dots),$$

$$-t_n^0 = [(n - 0.25)1.5\pi]^{2/3}, \quad \mu = \left(\frac{m}{2} \right)^{1/3}.$$

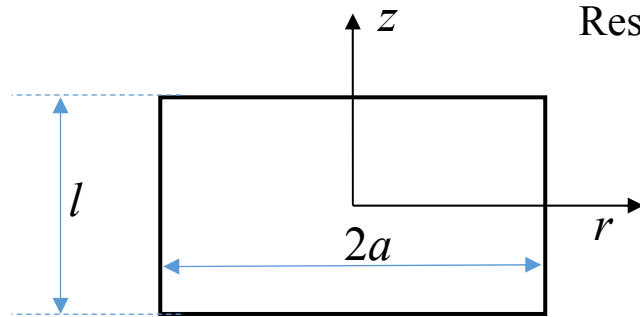
The optimal radius r_0 , when the external caustic has the smallest height comes from: $r_0 = 2a \sin^2 \theta$ where α and θ are derived from:

$$\sin \alpha = \sqrt{\frac{a}{r_0}} \sin \theta \quad \cos \theta = \frac{m}{v_{mn}}$$

Finally the height of the external caustic and Q-factor of the cavity are:

$$z_{q-1} = 2\sqrt{(q-1/2) \frac{a \sin \theta}{k \sin 2\alpha}}$$

$$Q_E = \frac{a}{\sigma_s}$$

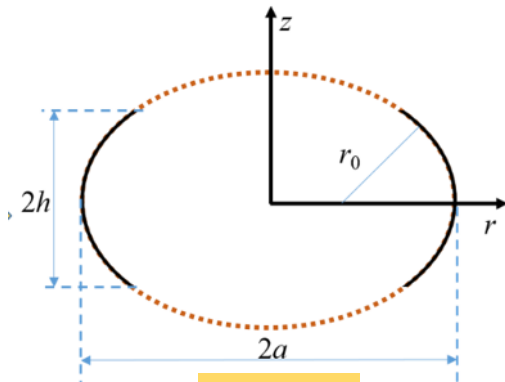
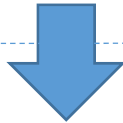


Cylindrical Cavity

Resonant frequency of TM_{mnp} :

$$k^2 a^2 = v_{mn}^2 + \left(\frac{p\pi a}{l}\right)^2$$

m : ϕ direction (T)
 n : r direction (T)
 p : z direction (L)

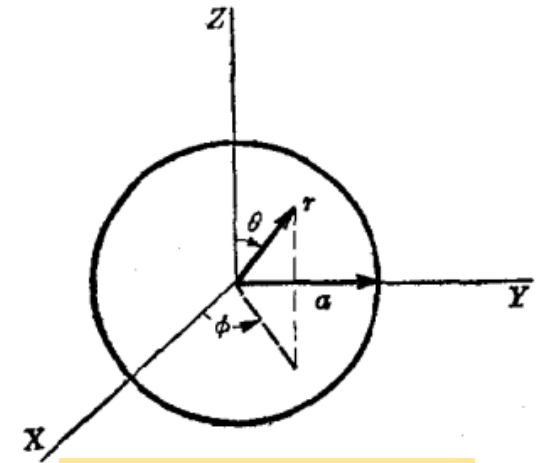


BOC

Resonant frequency of TM_{mnp} :

$$ka = v_{mn} + \frac{(p - 1/2)\alpha}{\sin \theta}$$

m : ϕ direction (T)
 n : r direction (T)
 p : z direction (L)



Spherical Cavity

Resonant frequency of TE_{mnp} :

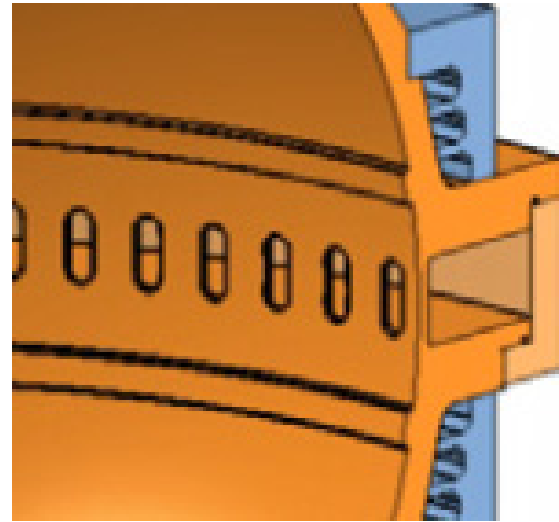
$$f = \frac{u_{np}}{2\pi a \sqrt{\epsilon\mu}}$$

m : ϕ direction (T)
 n : θ direction (T)
 p : r direction (L)

$$Q_0 = \frac{a}{\delta}$$

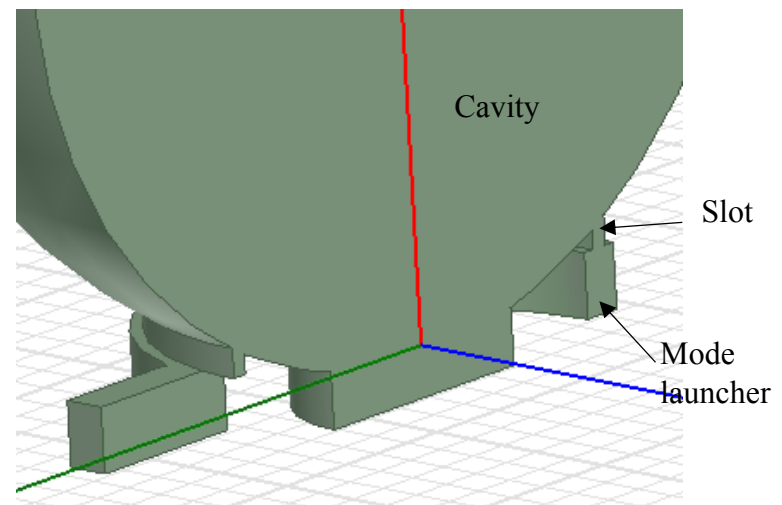
Traditional BOC:

- ❖ Cavity is surrounded by rectangular waveguide to couple power.
- ❖ Power is coupled through coupling apertures.



New design:

- ❖ We plan to use a coupling slot instead of the apertures, which can be machined more precisely.
- ❖ The cavity and the mode launcher can be designed separately, which can simplify the design.



For **Whispering Gallery** mode, both BOC and SC theory can be used:

Resonant frequency of TM_{mnq} for BOC:

$$ka = v_{mn} + \frac{(p - 1/2)\alpha}{\sin \theta}$$

When $m=24$,
 $n=1, q=1$

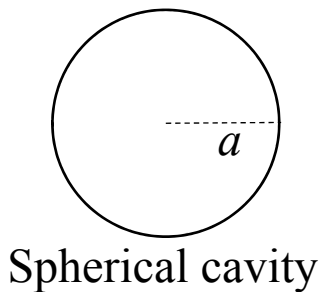
$$a = 120.3 \text{ mm}$$

$$Q_0 = 200,000$$

Resonant frequency of TE_{mnp} for spherical cavity:

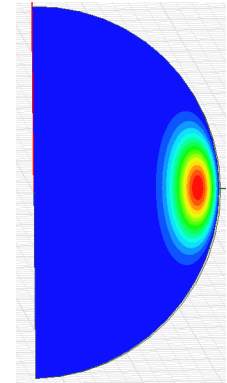
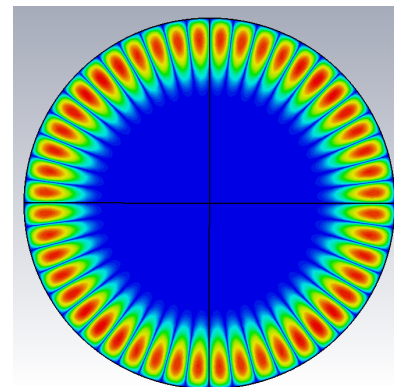
$$f = \frac{u_{np}}{2\pi a \sqrt{\epsilon\mu}}$$

When $n=24, p=1$



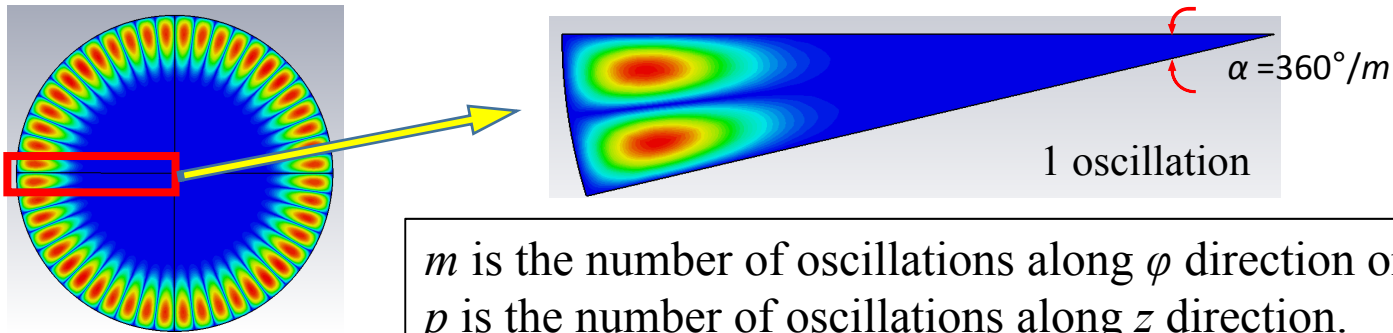
Simulation result:

$TM_{24,1,1}$	
Radius /mm	120.3
Frequency /MHz	11995.8
Q_0	199,374

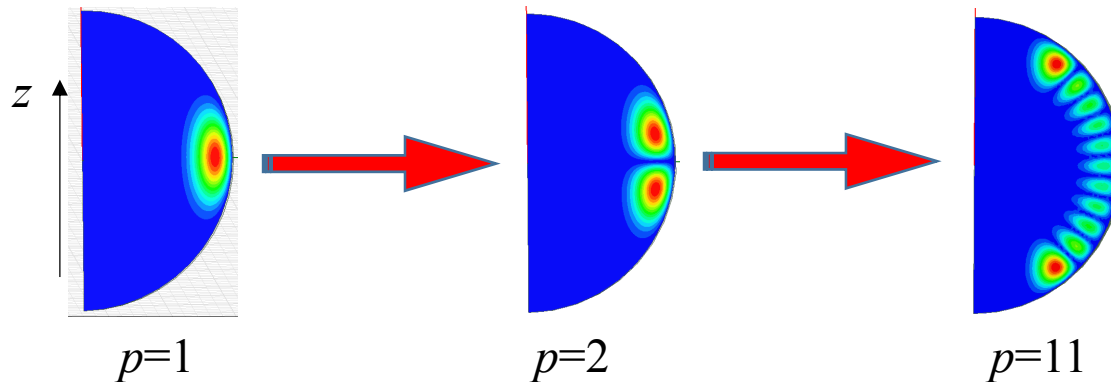


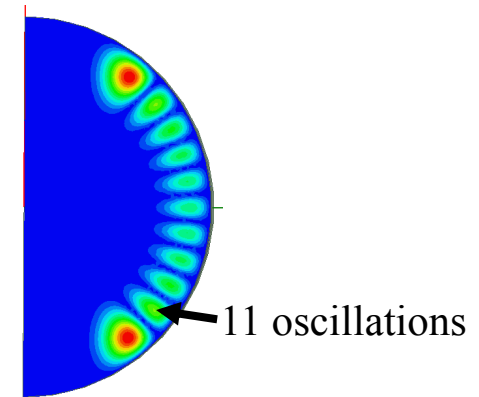
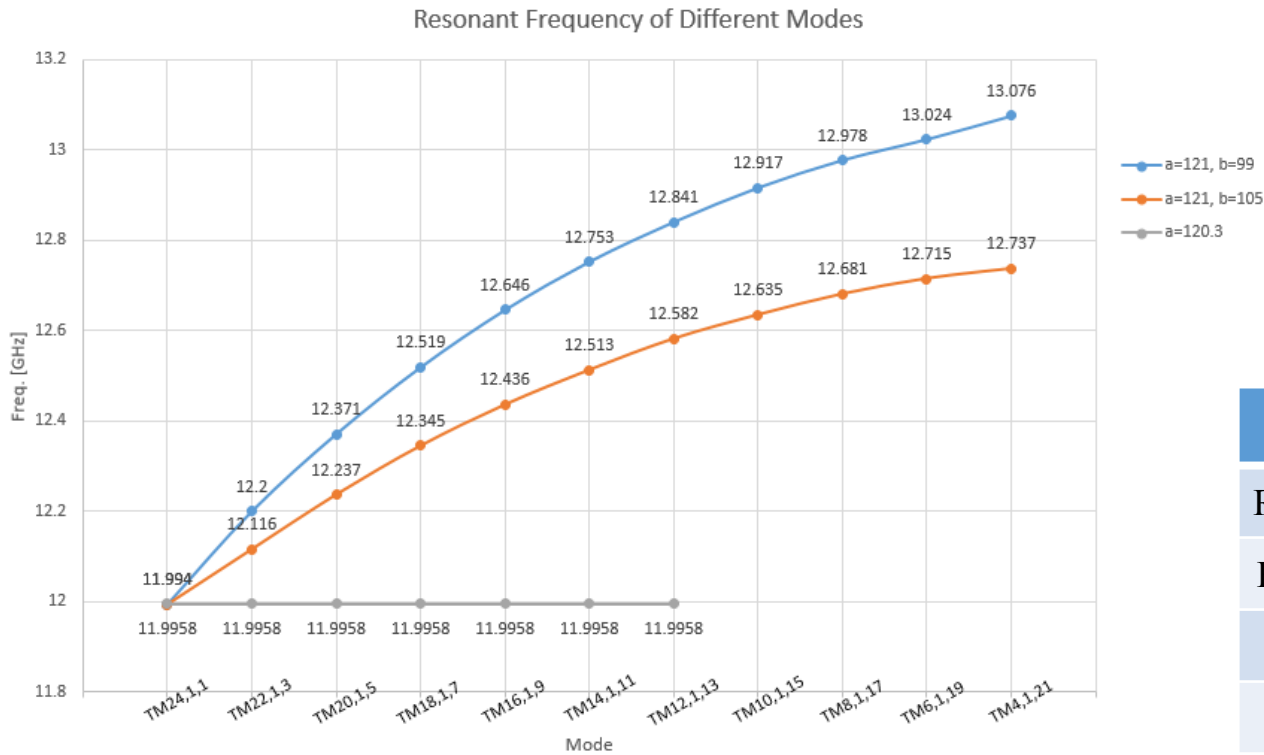
- ❖ We need to reduce the size of mode launcher, but there's a limitation because of the index m . For $m=24$, the minimum radius of the input port is **104.85 mm**, the cut-off radius of $TE_{24,1}$ mode.
- ❖ We have to keep the Q-factor high enough, so the cavity radius should not be reduced.

The solution is to **decrease the index m** , but **increase the index p** correspondingly.



m is the number of oscillations along φ direction on equator plane.
 p is the number of oscillations along z direction.





$TM_{14,1,11}$	
Radius [mm]	120.3
Freq. [MHz]	11995.8
Q_0	199376
α	25.714°

For different modes in BOC, as long as $n=1$, $m+p=25$:

- ❖ For elliptical BOC, their frequencies are different.
- ❖ The smaller the difference between a and b is, the smaller the frequency difference will be.
- ❖ If $a=b$, the frequency difference becomes 0. All these modes are degenerated.

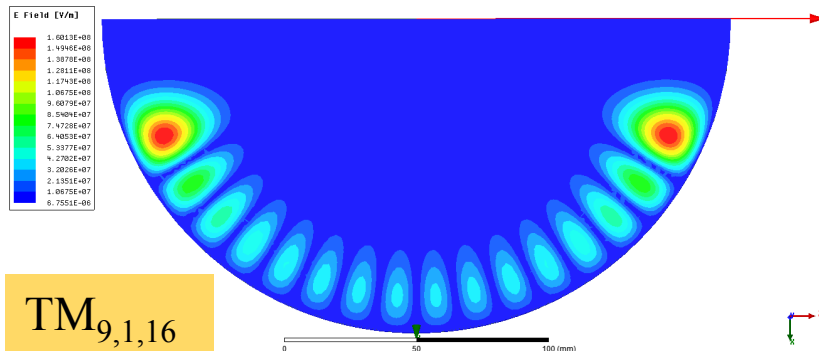
The mode launcher is still too big. So we continue to reduce index m .

But m cannot be too small, otherwise the surface field in mode launcher will be too high. Besides, the smaller the index m is, the more degenerated high-order modes there will be.

For example:

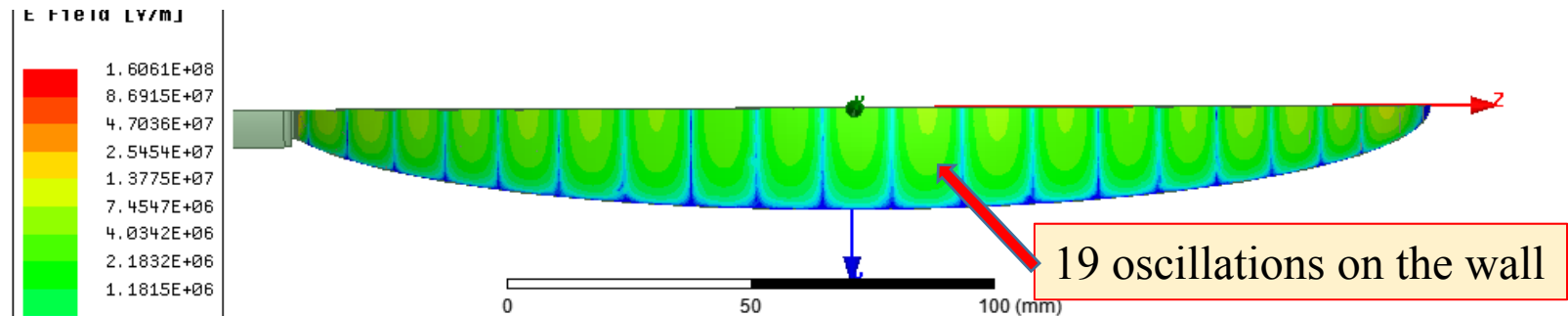
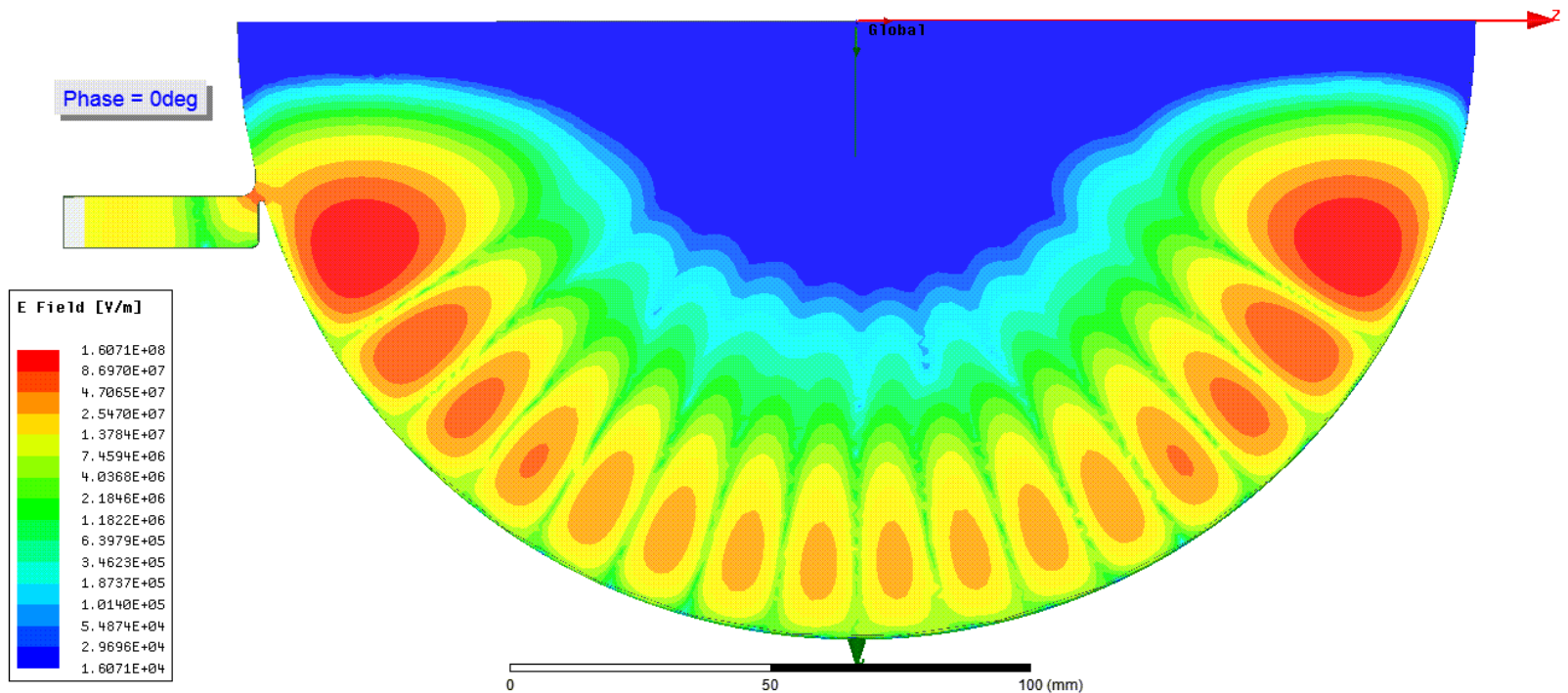
Working mode	High-order modes
$TM_{9,1,16}$	$TM_{18,1,7}$
$TM_{8,1,17}$	$TM_{16,1,9}, TM_{24,1,1}$
$TM_{7,1,18}$	$TM_{14,1,11}, TM_{21,1,4}$
$TM_{6,1,19}$	$TM_{12,1,13}, TM_{18,1,7}, TM_{24,1,1}$
$TM_{5,1,20}$	$TM_{10,1,15}, TM_{15,1,10}, TM_{20,1,5}$

For now, index m of 9 is chosen:

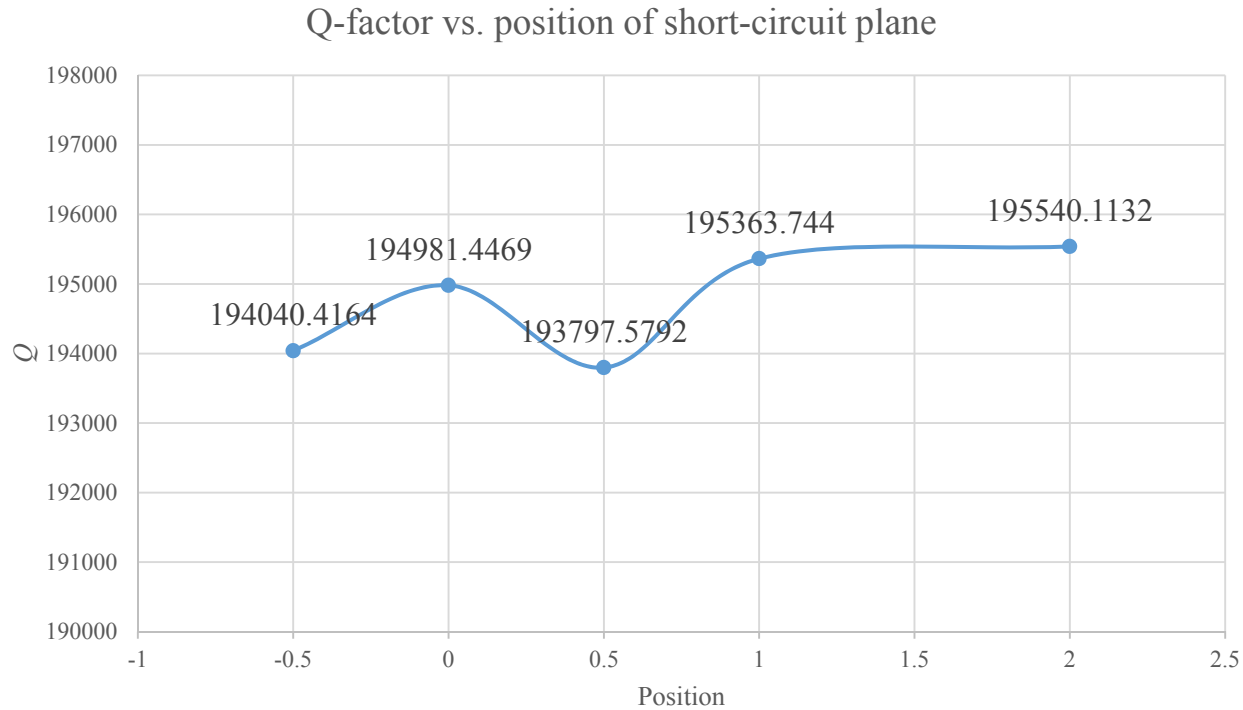


Neighboring modes.

	Eigenmode	Frequency (GHz)	Q
	Mode 1	11.9249 + j 0.000183232	32540.4
	Mode 2	11.9249 + j 0.000183227	32541.4
	Mode 3	11.9958 + j 3.00849e-05	99365.
	Mode 4	12.0073 + j 4.80147e-05	125038.
	Mode 5	12.0490 + j 3.62833e-05	166040.

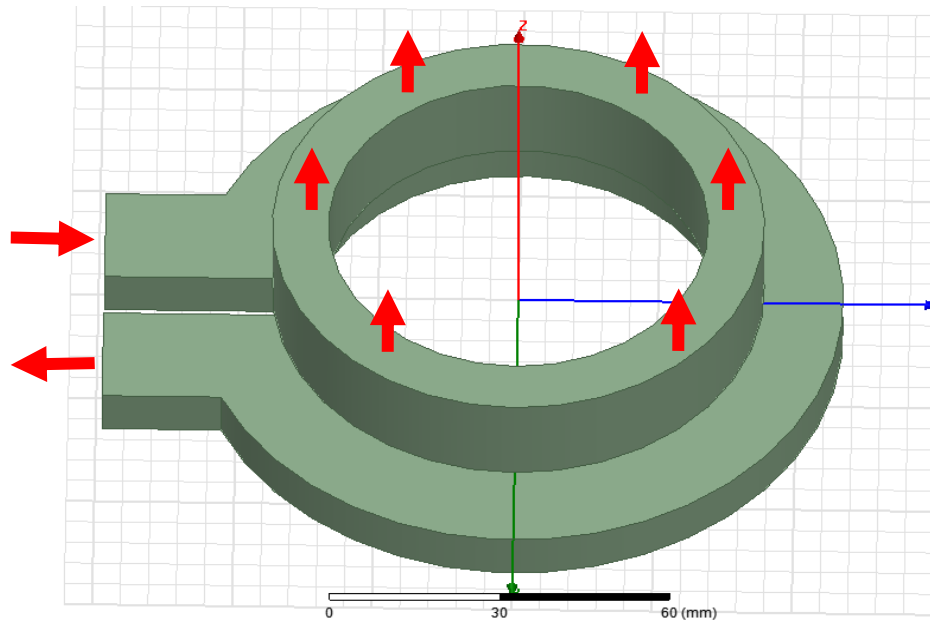


There's some mode mixture.

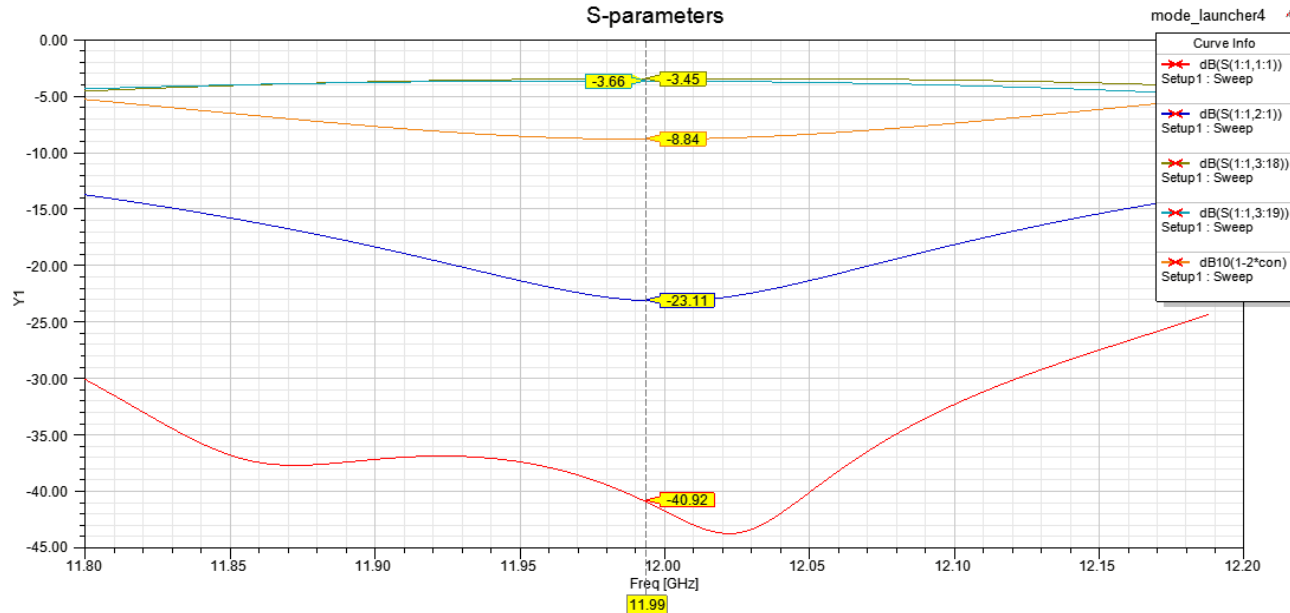


The Q-factor is around 194,000 (~2.5% reduced).

Model:

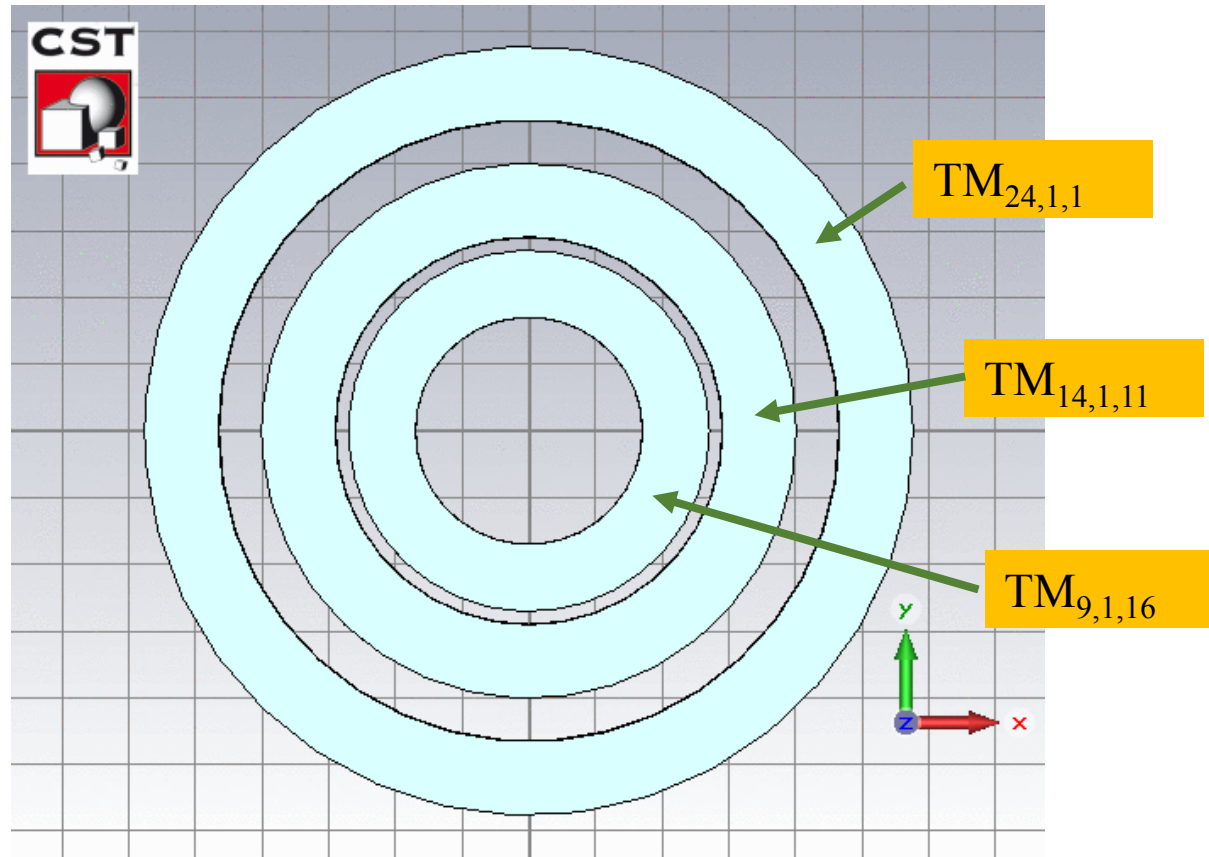


S-parameters

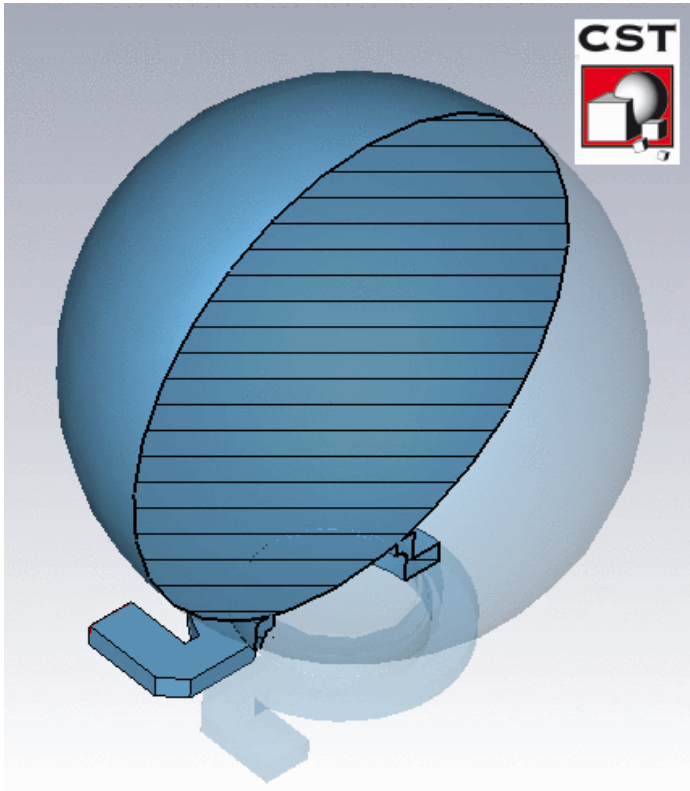


Best result for now:

Mode launcher circles of the three pulse compressor schemes:



The use of degenerated mode reduces the size of mode launcher significantly.



- ❖ This work is to design a spherical pulse compressor using degenerated “whispering gallery” mode.
- ❖ Some investigations on “whispering gallery” mode and its degenerated modes are presented.
- ❖ The structure is being designed. Some preliminary results are presented.

Thank you!

Meet me at THPO109.



Thank you!

