DETERMINATION OF THE FIELD DEPENDENCE OF THE SURFACE RESISTANCE OF SUPERCONDUCTORS FROM CAVITY TESTS*

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ABSTRACT

Cryogenic tests of superconducting cavities yield an average surface resistance as a function of the peak sur-face magnetic field. An analytical formalism has been developed to extract the actual field dependence of the surface resistance from cavity tests and is applied to coaxial cavities and cavities of more complex geometries..

INTRODUCTION

The well-known expressions for the surface resistance of superconductors in electromagnetic fields, and its dependence on frequency, temperature, and a few materials parameters, were obtained as a perturbation theory under the assumption that the magnitude of the electromagnetic field is much smaller than the critical field. This resulted in a surface resistance independent of the magnitude of the electromagnetic field. Experimentally, cryogenic tests of superconducting cavities developed for particle accelerators have shown that superconductors can display a strong dependence of their surface resistance on the rf field. Furthermore, the field dependence can vary greatly depending on the history of the cavities: chemical treatment, high temperature and low heat treatment, impurity, ambient magnetic field during transition, cooling rate during, and so on.

Developing a full understanding and theory of the rf field dependence of the surface resistance of superconductors will require accurate knowledge of that surface resistance as a function of the rf field, and its dependence on preparation and processing parameters.

In this paper we present a method and formulae that allow determination of the actual dependence of the surface resistance from experiments where an "average" surface resistance is derived from tests of superconducting resonators. An underlying assumption is still that, while the surface resistance has a magnetic field dependence, it does not have a dependence on location. This is not always true as it is known that superconducting cavities can have "hot spots" where the surface resistance is higher and with a stronger field dependence than in the rest of the cavity

ANALYTICAL METHOD

The geometrical factor G of a cavity is defined as $G=\omega\mu_0\frac{\int_V |\mathbf{H}|^2\,\mathrm{d}V}{\int_S |\mathbf{H}|^2\,\mathrm{d}S}.$ (1)

If the surface resistance is constant, then $G = QR_s$. If it is not, then $G/Q(H_p)=\bar{R}_s(H_p)$ where $\bar{R}_s(H_p)$ is an "average" surface resistance. It is related to the actual one by

$$\bar{R}_s(H_p)\int_S |\mathbf{H}(\vec r)|^2\,\mathrm{d}S=\int_S R_s\left(H(\vec r)\right)|\mathbf{H}(\vec r)|^2\,\mathrm{d}S. \qquad (2)$$

We define the function $a(h)$ as the fraction of the total cavity surface where $|H|\le hH_p$.
The function $a(h)$ is continuous, monotonically increasing with $a(0)=0$ and $a(1)=1$.
It could also be interpreted as the probability distribution for the surface magnetic field, and its derivative as the probability density.
We make a change of variable in Eq. (2) and integrate over the magnetic field instead of the area.

$$\bar{R}_s(H)=\int_0^1 (hH)^2\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h=\int_0^1 R_s(hH)\,(hH)^2\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h. \qquad (3)$$

We assume that the experimentally measured average surface resistance can be modelled by a sum of powers of the magnetic field:

$$\bar{R}_s\left(\frac{H}{H_0}\right)=\bar{R}_0\sum_{\alpha_i} r_{\alpha_i}\left(\frac{H}{H_0}\right)^{\alpha_i}. \qquad (4)$$

The sum can be of any size and the coefficients α_i are not restricted to integers, they can be any non-negative real numbers. $\bar{R}_0$ is the zero-field surface resistance and H_0 is arbitrary; it is introduced to make the coefficients r_{α_i} dimensionless. We assume the same power expansion for the real surface resistance but with modified coefficients:

$$R_s\left(\frac{H}{H_0}\right)=R_0\sum_{\alpha_i}\beta(\alpha_i)\,r_{\alpha_i}\left(\frac{H}{H_0}\right)^{\alpha_i}. \qquad (5)$$

Replacing $\bar{R}_s(H/H_0)$ and $R_s(H/H_0)$ by their power expansions in Eq. (3) and equating identical powers we obtain the coefficients $\beta(\alpha_i)$ relating the average and actual surface resistance.

$$\beta(\alpha)=\frac{\int_0^1 h^2\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h}{\int_0^1 h^{2+\alpha}\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h}. \qquad (6)$$

The function $\beta(\alpha)$ is continuous, monotonically increasing with $\beta(0)=1$.
Equation (6) is more appropriate when the function $a(h)$ can be obtained analytically.
For most cavities, it can only be obtained numerically; in that case it is more convenient to use an equivalent relationship derived from Eq.(6) by integration by parts.

$$\beta(\alpha)=\frac{2\int_0^1 h\left[1-a(h)\right]\mathrm{d}h}{(2+\alpha)\int_0^1 h^{1+\alpha}\left[1-a(h)\right]\mathrm{d}h}. \qquad (7)$$

In Eq. (7), $\left[1-a(h)\right]$ now represents the fraction of the cavity surface where $|H|>hH_p$.

APPLICATION TO COMPLEX GEOMETRIES

For more complex geometries where $a(h)$ and its derivative cannot be obtained analytically, $\beta(\alpha)$ can be calculated using Eq. (7).
For geometries with axial symmetry, $a(h)$ can be calculated via line integration along the profile.
For fully 3-D geometries $a(h)$ is obtained by sampling the magnetic field over the whole surface, and building the probability distribution.

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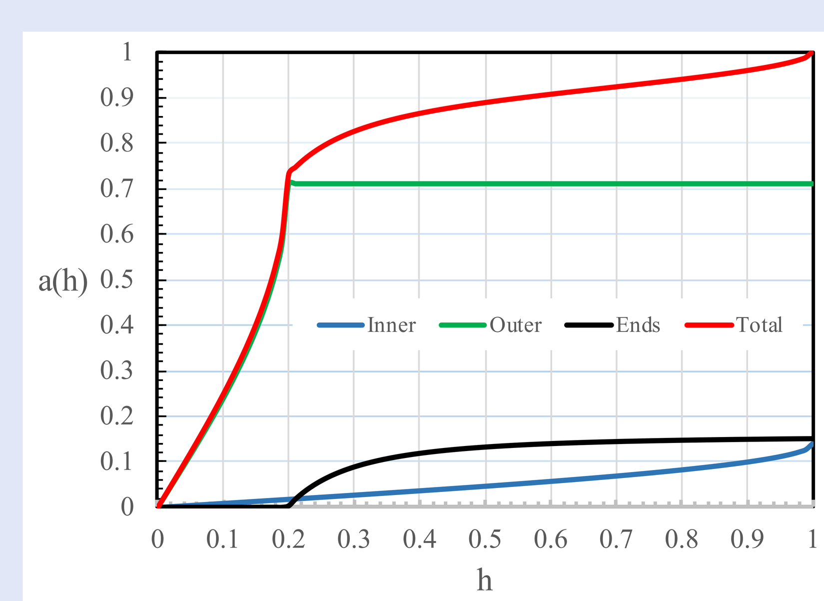
APPLICATION TO COAXIAL HALF-WAVE CAVITY

Coaxial half-wave cavity for the measurement of the temperature, frequency, and field dependence of the surface resistance of Nb [1,2]

Center conductor radius: a = 20 mm
Outer conductor radius: b = 101 mm
Length: L = 459 mm
Geometrical Factor: G = 59 Ω

For such a cavity the function $a(h)$ and its derivative can be obtained exactly for all the TEM modes, and the function $\beta(\alpha_i)$ can be calculated analytically from Eq. (6).

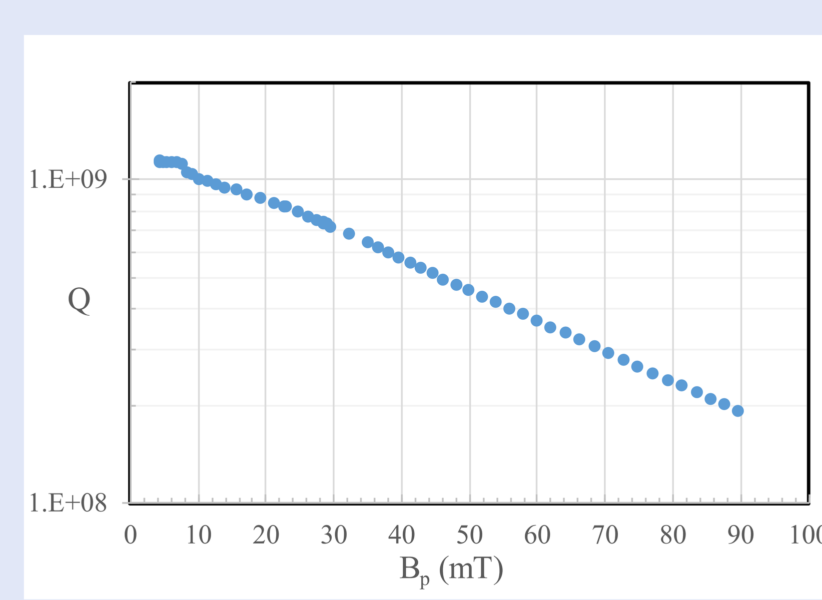


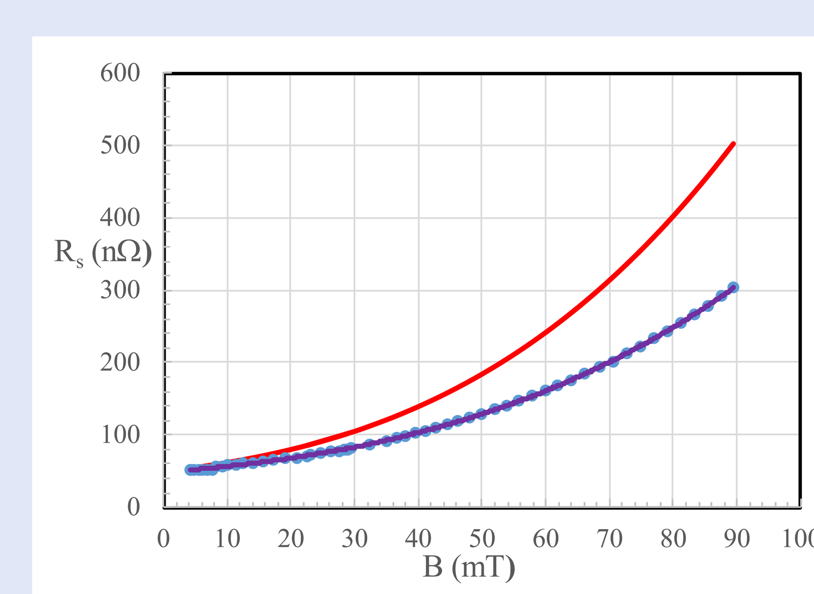
$$\beta(\alpha)=\frac{(1+\rho)/4+\rho\delta\ln(1/\rho)}{\frac{1+\rho^{1+\alpha}}{2\sqrt{\pi}}\frac{\Gamma(\alpha/2+3/2)}{\Gamma(\alpha/2+2)}+\frac{\rho\delta}{\alpha}(1-\rho^{\alpha})}, \qquad (8)$$

$$\rho=a/b,\quad \delta=b/L.$$

For the half-wave cavity shown above the correction coefficients calculated from Eq. (8) are:

$$\beta(0)=1,\quad \beta(1)=1.45,\quad \beta(2)=1.76,\quad \beta(3)=2.01 \qquad (9)$$





Q-curve for the 325 MHz mode at 4.35K

Surface Resistance

The average surface resistance obtained from the Q-curve is shown as the green dots and modeled by (purple curve, in nΩ with B_0=100 mT):

$$\bar{R}_s\left(B/B_0\right)=48.2\left[1+1.54\left(B/B_0\right)+2.03\left(B/B_0\right)^2+3.20\left(B/B_0\right)^3\right]$$

Using the correction coefficients of Eq. (9) the actual field-dependent surface resistance (red curve) is:

$$R_s\left(B/B_0\right)=48.2\left[1+2.24\left(B/B_0\right)+3.63\left(B/B_0\right)^2+6.52\left(B/B_0\right)^3\right]$$

REFERENCES

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