# EFFECT OF NUMBER OF MACRO PARTICLES ON TIME EVOLUTION OF PHASE SPACE DISTRIBUTION*

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*Abstract*

In particle tracking simulation with space charge effect, the macro-particle model, which has same mass-to-charge ratio, is widely used, since it does not require any symmetry of beam shape. However, selection of proper number of macro-particles is important, because the accuracy depends on it. Emittance, which is calculated by phase-space distribution, is especially affected by the number of macro-particles. In order to study the relation between the number of macro-particles and the resolution in the phase space, we defined a transformation, which describes reduction process of macro-particle number, and analyzed static phase space distribution. As a next step, we studied the effect of the macro-particle number on the dynamics of the phase space distribution for 1D charged particle distribution in the rest frame. The numerical result shows that the number of macro-particles affected the phase space distribution around the head and the tail of the bunch.

## INTRODUCTION

In an accelerator, the motion of a charged particle beam which consists of $N$ particles can be described by a trajectory on $6N$ dimension phase space. Since an actual beam contains enormous number of particles, for example $N \sim 4.8 \times 10^8$ electrons in an Energy Recovery Linac with 77 pC operation, degree of freedom in a simulation and a theoretical analysis is reduced using an approximation. In order to analyze the charged particle beam, a beam model is used to reduce number of degrees of freedom, e.g. charged disk model, charged cylinder model and macro-particle model [1]. The macro-particle model, which does not depend on the symmetry in the beam, is versatile method to describe it.

In numerical simulation, the macro-particle model, which has same mass-to-charge ratio, is widely used, since it does not require any symmetry of beam shape. However, the estimation of proper number of macro-particles is one of the important issues, since it affects the resolution of the phase space distribution of the beam. Then, we defined a transformation, particle pair transformation, to reduce macro-particles. Using the transformation, we studied the relation between the number of macro-particles and the resolution in the phase space for 1D and 2D charged particle distributions in the rest frame [2]. In the previous study, the static electric field was calculated from the transformed particle distributions with the reduced macro-particle number by the particle pair transformation. The result shows that the strength of the electric field at the edge of the distribution is affected by the number of macro-particles.

The next topics is to study the effect of the macro-particle number on the dynamics of the phase space distribution. In this paper, we report the time evolution of the 1D charged particle distribution in the rest frame with different the number of macro-particles.

## MACRO PARTICLE MODEL

The equation of motion of an electron in electro-magnetic field, $\boldsymbol{E}$ and $\boldsymbol{B}$, is

$$c\frac{m_e}{e}\frac{d(\gamma\boldsymbol{\beta})}{dt} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}, \quad (1)$$

where, $m_e$ and $e$ are the mass and the charge of an electron, $c$ is the speed of light, $\boldsymbol{v}$ is the speed of the electron, $\beta = v/c$ and $\gamma = 1/\sqrt{\beta^2 - 1}$. Here, the electron beam consists of $N$ electrons. In order to describe the electron beam by the macro-particle model with $M$ macro-particles, we have to preserve the mass-to-charge ratio. The macro-particle contains $a(= N/M)$ electrons, and the mass and charge are $m_m = am_e$, $q_m = ae$, respectively. The equation of motion of the single macro-particle is

$$c\frac{m_m}{q_m}\frac{d(\gamma\boldsymbol{\beta})}{dt} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}. \quad (2)$$

It is the same equation as the single electron, Eq. (1), because $m_e/e = m_m/q_m$. Therefore, the description of the beam motion by macro-particle model corresponds to an approximation by $M$ charged particles with the mass, $am_e$, and the charge, $ae$.

## PARTICLE PAIR TRANSFORMATION

In this section, we describe a particle pair transformation to describe the replacement of an particle distribution by macro-particles [2]. The original distribution consists of $n_0$ macro-particles with the mass, $m_{m0}$, and the charge, $q_{m0}$. The procedure of the transformation has the following five steps.

1. Calculate the center of the original particle distribution.
2. Choose the most distant particle from the center, and the nearest neighbor particle from it.
3. Calculate the average position about the above two particles.
4. Replace the two particles by a new macro-particle with $m_{m1} = 2m_{m0}$ and $q_{m1} = 2q_{m0}$ on the average position.
5. Repeat step. 2 to step. 4 until all the original particles are replaced.

Figure 1 shows the particle pair transformation for one dimensional particle distribution. After the transformation, the number of transformed particles is reduced to $n_1 = n_0/2$. After $i$-th transformation, the number of transformed particles is reduced to $n_i = n_0/i$.

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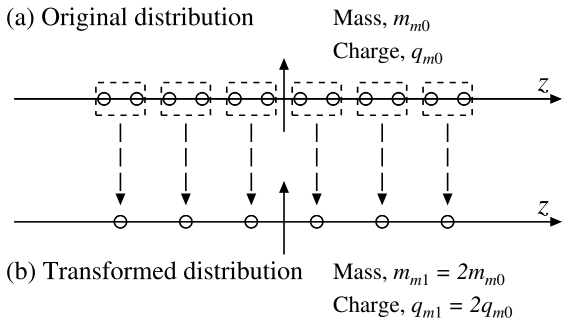


Figure 1: Particle pair transformation for one dimensional particle distribution.

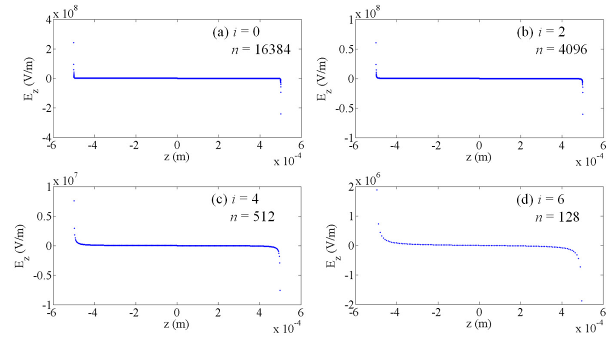


Figure 2: Static electric field of one dimensional particle distribution before and after particle pair transformations. The numbers of particles are 16384, 4096, 512, and 128.

## 1D CHARGED PARTICLE DISTRIBUTION

In the this section, we apply the transformation to 1D charged particle distributions in the rest frame, and calculate the static electric field caused by the transformed particle distribution. Based on the static electric field, we calculate the time evolution of the transformed phase space distribution.

### *Static Electric Field*

We think an one dimensional uniform particle distribution on $z$ axis. The initial distribution consists of $n_0$ macro-particles with $m_{m0} = a_0 m_e$ and $q_{m0} = a_0 e$, and the particle distance is tha same. The full width and the total charge of the distribution are 1.0 mm and $-100$ pC, respectively. The number of macro-particles is $n_0 = 16384$, and $a_0$ is 38333 electrons.

The static electric field of the one dimensional distribution is calculated by point-to-point model. The electric field at the $j$-th particle after the $i$-th transformation is

$$E_z(z_j) = \frac{q_{mi}}{4\pi\varepsilon_0} \sum_{k \neq j} \frac{z_j - z_k}{|z_j - z_k|^3}. \tag{3}$$

Figure 2 shows the static electric fields with the number of transformations, $i = 0$, 2, 4 and 6. As shown in Fig. 2, the strengths of the electric field at the head and the tail decrease, when the number of macro-particles is reduced. After the transformation, the particle distance is stretched, and it reduces the electric field at the head and the tail.

### *Time Evolution of Phase Space Distribution*

As a next step, we calculate the time evolution of the transformed phase space distribution based on the above static field. The time evolution of the macro-particle is described by

$$\frac{d(\gamma\beta_z)}{dt} = \frac{1}{c}\frac{e}{m_e} E_z(z), \tag{4}$$

$$\frac{dz}{dt} = \frac{c\gamma\beta_z}{\sqrt{(\gamma\beta_z)^2 + 1}}. \tag{5}$$

The equations of motion were integrated using 4th order Runge-Kutta method. The original distribution has 4096 macro-particles, and the reduced particle distributions, which has 2048, 1024, 512, 256 and 128 macro-particles, are generated by the particle pair transformation. Figure 3 shows the time evolution of phase space distribution with 4096 macro-particles. At $t = 0$ s, the all macro-particles have zero velocity, and the particles are moved by the static electric field. Figure 4 shows the time evolution of phase space distribution with 256 macro-particles. In order to compare the phase space distributions, we plot the phase space distributions with 4096, 1024, 256 macro-particles at 4 ps as shown in Fig. 5. Figure 5 shows that the distribution around the center does not depends on the number of particles, and both the edge positions, head and tail, approach the center for smaller macro-particle number.

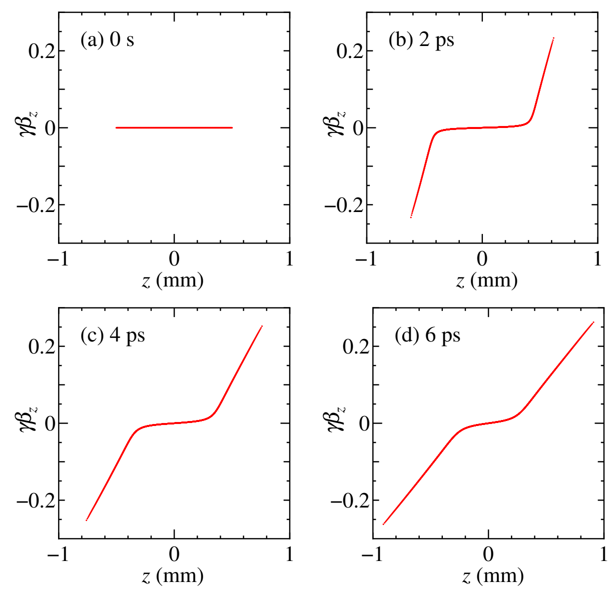


Figure 3: Time evolution of phase space distribution with 4096 macro-particles. The times are 0, 2, 4 and 6 ps.

In order to quantitatively analyze the effect of macro-particle number, we calculate rms bunch length and rms emittance,

$$\sigma_z = \sqrt{\langle z_c^2 \rangle}, \tag{6}$$

$$\varepsilon_z = \sqrt{\langle z_c^2 \rangle \langle (\gamma\beta_z)_c^2 \rangle - \langle z_c (\gamma\beta_z)_c \rangle^2}. \tag{7}$$

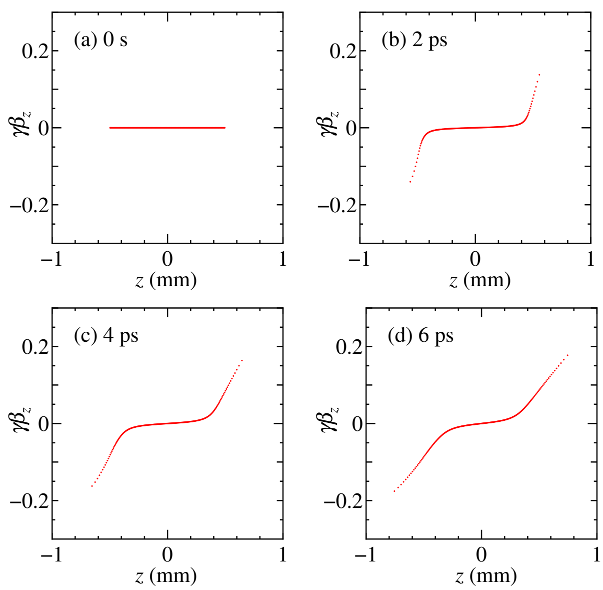


Figure 4: Time evolution of phase space distribution with 256 macro particles. The times are 0, 2, 4 and 6 ps.

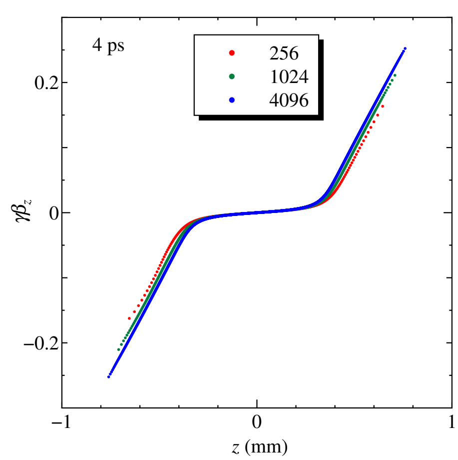


Figure 5: Phase space distributions at $t$ = 4 ps. The number of macro particles are 4096, 1024 and 256.

Here, $z_c = z - \langle z \rangle$, $(\gamma\beta_z)_c = (\gamma\beta_z) - \langle \gamma\beta_z \rangle$, and $\langle\rangle$ denotes distribution average. Figure 6 shows the time evolution of the rms bunch length. The rms bunch length decreases for smaller macro-particle number. Figure 7 shows the time evolution of the rms emittance. Note that the rms emittance is not the area on the phase space distribution. When the phase space distribution curves, the rms emittance increases. The emittance is maximum around $t$ = 3 - 4 ps as shown in Fig. 7.

## SUMMARY

In order to study the effect of the number of macro-particles on charged particle beam, we calculated the time evolution of the phase space distribution with different macro-particle number using the particle pair transformation. The numerical result for 1D charged particle distribution shows that the number of macro-particles affects the phase space distribution around the head and the tail of the bunch. As the next study, we plan to apply the particle pair transformation and the time evolution analysis for 2D and 3D distributions.

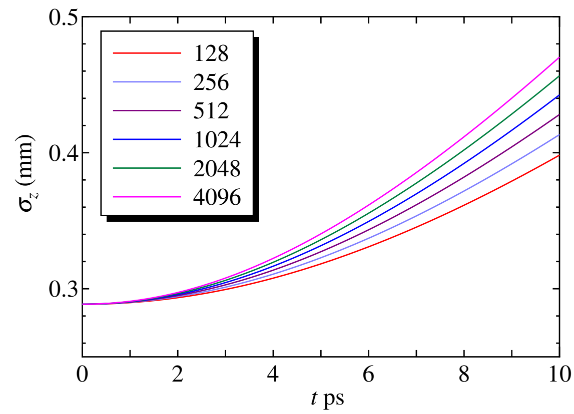


Figure 6: Time evolutions of bunch length with 4096, 2048, 1024, 512, 256 and 128 macro particles.

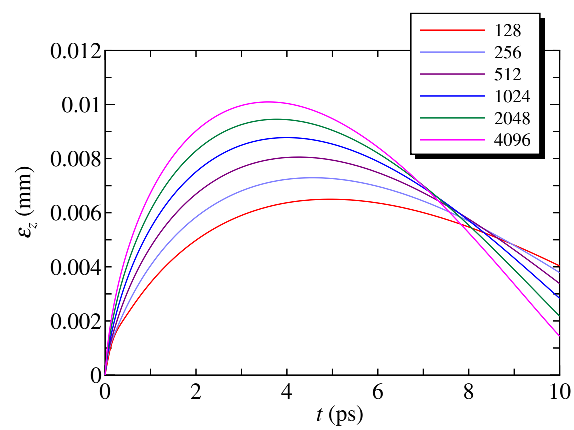


Figure 7: Time evolutions of rms emittance with 4096, 2048, 1024, 512, 256 and 128 macro particles.

## ACKNOWLEDGMENT

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